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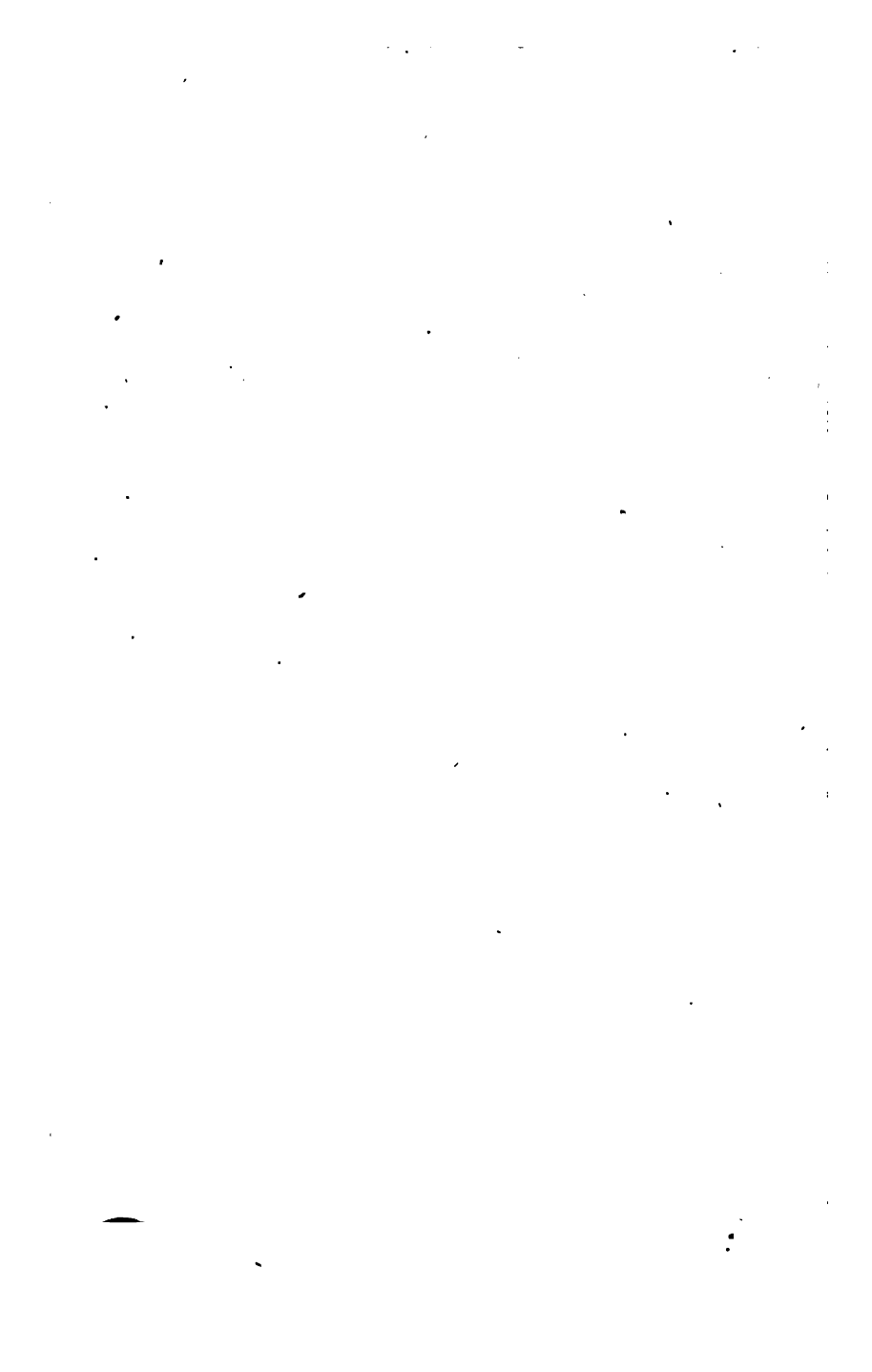
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## 71260 RECOMMENDATIONS.

*From Prof. E. A. Andrews.*

About two years ago, while on a visit to this place, I had the pleasure of meeting with Mr. Tracy, and of having some conversation with him on the subject of an Arithmetic, which he was preparing for schools. I was particularly pleased with that part of his system which related to *canceling*, and which appeared to me to possess great practical value. Within a few days, Mr. Tracy has put into my hands a part of his manuscript, that I might become more minutely acquainted with his system. My time has been so far occupied with my own business, that I have been able to examine a part only of the manuscript put into my hands; but with this part, I have been much gratified. It appears to me, that when carefully and thoroughly revised and perfected, as the author designs to do, it may become a *most valuable work*, inferior to none of the Arithmetics now used in our schools. Such is my confidence in the ability of the author, to complete and polish the work, that I look upon its success as quite certain.

*New Britain, Conn., Aug. 1st, 1839.*

*From E. H. Burritt, author of the Geography of the Heavens, &c.*

Through the politeness of Mr. Tracy, I have been favored with a perusal of an Arithmetic, in manuscript, which he is preparing for publication. The work is intended as a universal class book in elementary Arithmetic. It is the production of a gentleman of known abilities and experience in teaching, and he has, with great care, arranged its several parts, and given the rules, and selected the examples, step by step, in that natural order, and easy method, which his own judgment and experience approved. There are some excellencies in his Arithmetic—some facilities of dealing with figures, which, so far as I know, are entirely peculiar to this treatise, and which distinguish it from all others. On this ground especially, and that of its general merit, I think it a work which will commend itself to the attention of teachers.

*New Britain, August, 1836.*

I entirely coincide in the above opinion, having been particularly gratified with the ease and facility with which many difficult operations are performed by the new principle introduced by the author.

*J. P. BEACE, Principal of Hartford Female Seminary.*

*From Edward Strong, Principal of Bacon Academy, Colchester, Conn.*

I have had the pleasure of examining a system of Arithmetic, by Mr. Tracy, of Norwich Academy. Without speaking of its merits in other respects, which I am unable to do, from want of time to give a thorough perusal, I discover in it a distinctive feature—the method of *canceling*—which appears to me to be an important improvement on every other system with which I am acquainted. Should the other features of the work correspond with what may reasonably be expected from its author, I should regard it as a very important improvement upon our other systems of Arithmetic.

*Colchester, Oct. 8th, 1839.*

*From Rev. A. Bond, Pastor of the Second Congregational Church, in Norwich.*

Having examined the general plan of an Arithmetic, prepared by Mr. Tracy, Principal of Norwich Academy, I can cheerfully recommend it as a system possessing, in some important particulars, a superiority over any other system with which I am acquainted. The method of *canceling*, which is carried through the work, excepting the *Roots*, greatly facilitates the process of Arithmetical calculations, and will give it a decided advantage in the estimation of business men. The part on foreign exchanges will enhance its value with the commercial community. While its simplicity adapts it to the use of common schools, its comprehensiveness, and the ease and accuracy with which complicated problems may be solved, will be likely to secure for it a prominent place in the counting room.

*Norwich, August 7th, 1839.*

## RECOMMENDATIONS.

*From La Fayette S. Foster, Esq.*

Mr. Calvin Tracy, of this city, has submitted to my examination, in manuscript, an Arithmetic, prepared by himself, for publication. From the known ability of Mr. Tracy, as an instructor, I was prepared to entertain a high opinion of any treatise designed to facilitate the acquisition of knowledge, of which he might be the author, and from the attention which I have bestowed on his Arithmetic, I have no hesitation in bearing testimony to its high meritorious character. His plan appears to me to be highly judicious, and ably and skillfully executed. The work, in my opinion, will be a valuable addition to a very important branch of education.

Norwich, Dec. 24th, 1839.

*From Messrs. J. H. Gallup and G. Bushnel, Teachers of the Eclectic School, Norwich, Conn.*

Having examined Mr. Tracy's system of Arithmetic, we think it well calculated to answer the purposes for which it is intended. Mr. Tracy has illustrated, and happily combined with nearly all his operations, a method of abbreviating Arithmetical calculations, which, so far as we know, has never before been published in any common school Arithmetic. This we deem an essential improvement, and are of opinion, that the author has rendered, in this work, an essential service to the cause of education.

Norwich City, Oct. 22d, 1839.

*From Rev. L. N. Tracy, formerly Principal of New Britain Academy.*

I have spent considerable time in a careful examination of an Arithmetic prepared by Mr. C. Tracy, Principal of Norwich Academy. For my own benefit and pleasure, I have carefully examined every rule, and though I have daily used the best Arithmetics extant, while engaged for many years in teaching, I am led to believe that there is not a text book on Arithmetic in use which presents equal excellencies. Its grand feature—that which distinguishes it from every other Arithmetical treatise—is the principle of canceling, introduced and applied throughout the work. The extent and facility of its application to all operations in which Multiplication and Division are both concerned, are fully and clearly illustrated. It is safe to say that two thirds, and often four fifths of the labor and time usually required for arithmetical solutions, is saved. While it contains an amount of matter equal to any other Arithmetic in use, it is still a strictly elementary work.

Norwich, Oct. 12th, 1839.

*From E. C. Herrick, Esq.*

I have cursorily examined the manuscript of Mr. C. Tracy's treatise on Arithmetic. The most prominent feature of the work is the introduction of a peculiar mode of stating numerous classes of problems, which are then solved by an abridged process, called canceling. This appears to me an important improvement on the books in common use, and one which renders the publication of this treatise very desirable.

New Haven, Dec. 27th, 1839.

*From J. H. Rogers, Esq., Principal of Prospect Hill High School.*

Messrs. DURREN & PROX,

Gentlemen—From a hasty examination of Tracy's Arithmetic, I believe it worthy of being ranked among the best School Books. The method of canceling, very fully brought into practice in this work, greatly abridges many operations; and may be mentioned as one of its most valuable features.

Sincerely yours,

J. H. ROGERS.

East Haven, April 24th, 1840.

*At a meeting of School Visitors of the First School Society of New Haven, held May 2, 1840—*

The committee appointed at the previous meeting to examine and report on "A New System of Arithmetic," by Mr. C. Tracy,—reported, that in their opinion the work contains important improvements on the arithmetical treatises in common use, and recommend that it be introduced into the schools of this Society:—whereupon, it was

*Voted,* That Tracy's New System of Arithmetic be adopted for use in the schools of the First School Society of New Haven.

R. S. HINMAN, Chairman.

E. C. HERRICK, Clerk pro tempore.

NEW SYSTEM  
OF  
ARITHMETIC,

IN WHICH IS EXPLAINED AND APPLIED TO PRACTICAL PURPOSES, IN ADDITION  
TO THE ORDINARY RULES OF OPERATION,

THE PRINCIPLE OF CANCELING;

BEING AN

ABBREVIATED MODE

OF

ARITHMETICAL SOLUTION.

DESIGNED FOR SCHOOLS AND ACADEMIES.

BY <sup>*C. Calver*</sup>  
C. TRACY, A. M.

PRINCIPAL OF NORWICH ACADEMY.

NEW HAVEN:

PUBLISHED BY DURRIE & PECK.

NEW YORK: COLLINS, KEESE, & CO.

BOSTON: CROCKER & BREWSTER.

1840.

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## INTRODUCTION.

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It will readily be conceded, that all efforts in behalf of the general diffusion of useful knowledge, are in themselves commendable. There is, however, and probably ever will be, a difference of opinion relative to the extent to which books of any particular description, and treating upon the same general topic, may be multiplied, and the interests of education uniformly advanced thereby.

This difference of opinion exists especially in relation to books designed for the use of common schools and academies, and which treat upon the more common subjects of study. The multiplication of books of this description, to the extent realized at the present day, is regarded by many as injurious to the general good. That its tendency is to increase, in some small degree, the expense of education, at least in some parts of the country, will not be denied. But before sentence of final condemnation is pronounced, it always becomes those, who sit as umpires, to take as extended views of the subject before them, as the nature of the case will admit.

The question now presented is, *how far the general good is advanced by the multiplication of school books.*

To answer this, let it be supposed that only a single work in each department of science studied in common schools, had ever been presented to the public, and that each work were such as it should be. Books of this description would obviously find ample circulation, sufficient, perhaps, to satisfy both authors and publishers, without embracing one half of the ground to be occupied. The consequence would be, that the more recently settled parts of our country would be but poorly supplied with the means of education, for at least some considerable period of time. But as it now is, with such a multiplicity of school books constantly emanating from the press, a spirit of rivalry is created, a desire is excited on the part of both authors and publishers to give to their several works a more extended circulation than can be obtained without exploring the whole ground. As a natural consequence, the inhabitant of the less favored portion of our land, is scarcely settled in his log cabin, before books of every description necessary for the education of his sons and daughters, are presented him, as it were, at his own door. His attention is thus directed to a subject second in importance to none of a temporal nature; and one which, when duly presented, will be likely to be regarded, and to receive a consideration, which otherwise might be long neglected.

The truth of our supposition, that any one set of school books is such in all respects as is required, may, however, very reasonably be doubted. Many of them are unquestionably of a very high order, and very probably owe some degree of their merit to the fact, that other minds have been, are, and yet will be, traversing the same ground which their authors trod, and are preparing other works, to supersede them, if possible, in the estimation of the public.

The effect of the multiplication of school books is, therefore, *to render the means of education as perfect as the nature of their subjects will allow, and to convey these means, thus perfected, to every part of our entire country.*

From the preceding considerations, the author is inclined to regard the multiplication of school books as favorable to the cause of general education. It therefore only remains to point out some of the more important features of the following work, before introducing it to the ordeal of public opinion. That it is worthy of public attention and patronage, belongs not to him to decide. It certainly will be found to possess some *peculiarities*, which are of course regarded by him as *improvements*. Whether they are indeed such, remains for others to determine.

A peculiar feature of the following pages, and one which distinguishes this work from every other on the same subject, is the "System of Canceling," which, in connection with the ordinary mode of solution, is introduced throughout, and applied to such arithmetical problems as embrace in their operation both multiplication and division. This is regarded by the author, and by many others acquainted with his system, as a *decided improvement upon all Arithmetics heretofore presented to the public.*

The following are some of the advantages of the new system :

1st. The statement required, or, rather, recommended for canceling, *is itself a complete analysis of the sum proposed.* Suppose, for illustration, that 12 yards of cloth cost \$48, and that it is required to find the value of 15 yards of the same.

We analyze the preceding sum, either by first finding the value of one yard of the cloth, viz.  $\$48 \div 12 \text{ yd.} = \$4$ , and then multiplying that price by the number of yards, as,  $\$4 \times 15 \text{ yd.} = \$60$ , Ans.; or by finding the ratio of the number of yards of which the price is given, and of those of which the price is required, and then multiplying that ratio by the given cost. This ratio is  $\frac{15}{12} = \frac{5}{4}$ ; and  $\frac{5}{4} \times 48 = 60$ , the number of dollars required, and the same as above.

The same stated for canceling :

$$\begin{array}{r} 48. 15 \\ \hline 12 \end{array}$$

(See rule for canceling, in Single Proportion.) By the above statement, \$48 is to be increased by the ratio, 12 : 15; or  $\frac{15}{12} = \frac{5}{4}$ . It is, however, obvious that  $\frac{1}{4}$  of 48 multiplied by 15, is the same as  $\frac{1}{12}$  of 15 multiplied by 48. The above sum is therefore canceled and solved thus :

$$\begin{array}{r} 4 \\ 48. 15 \\ \hline 12 \end{array}, \text{ and } 15 \times 4 = 60, \text{ the dollars required.}$$



The application of the canceling principle is, however, more completely illustrated by the solution of sums in which the ratio of one of the given quantities to a required quantity of the same kind, is traced through several simple ratios. The following sum may serve as an illustration:

If 3 men, in 16 days, of 9 hours each, build a wall 20 feet long, 6 feet high, and 4 feet thick, in how many days, of 8 hours each, will 12 men build a wall 200 feet long, 8 feet high, and 6 feet thick?

It is obvious the ratio of the given days to the required number of days, is compounded of the ratios, 8 hours : 9 hours; 12 men : 3 men; 20 feet in length : 200 feet in length; 6 feet in height : 8 feet in height; and 4 feet in thickness : 6 feet in thickness. Or, these ratios may be fractionally expressed, thus:  $\frac{8}{9}$ ,  $\frac{12}{3}$ ,  $\frac{200}{20}$ ,  $\frac{6}{8}$ , and  $\frac{4}{6}$ .

Now the given days of 9 hours each are changed to days of 8 hours each, by the following statement:

$$\frac{16. 9}{8} = 18 \text{ days.}$$

The time, in days of 8 hours each, required for the 12 men to complete the work of 3 men, is obtained by uniting the second of the preceding ratios to the above statement; thus:

$$\frac{16. 9. 3}{8. 12} = 4\frac{1}{2} \text{ days.}$$

By introducing into the same statement, the third of the preceding ratios, we obtain the requisite time for completing the 200 feet of wall, allowing the height and thickness of each wall to be the same; thus:

$$\frac{16. 9. 3. 200}{8. 12. 20} = 45 \text{ days.}$$

If the fourth ratio be united, we obtain the time required, allowing each wall to be of the same thickness; thus:

$$\frac{16. 9. 3. 200. 8}{8. 12. 20. 6} = 60 \text{ days.}$$

Lastly, if the fifth and last ratio be introduced, the number of days required by all the conditions of the question, is obtained; viz.

$$\frac{16. 9. 3. 200. 8. 6}{8. 12. 20. 6. 4} = \text{the required days.}$$

The last statement canceled:

$$\frac{16. 9. 3. 200. 8. 6}{8. 12. 20. 6. 4} = 10 \times 9 = 90 \text{ days, answer.}$$

2dly. A second advantage to be derived from the canceling system, is the facility afforded by it for reducing several operations to a single statement. The following example will afford an illustration:

Bought 742 lb. of wool, a deduction of 5 per cent. from the gross weight being made, for dust, &c. For the net weight, I paid 9 s. New York currency, per lb., and for ready money, was allowed a deduction

of 6 per cent. Now, allowing that I sold the same so as to realize a gain of 20 per cent., how much money did I receive?

The ordinary mode of solving this sum requires the four following operations; viz.  $105:100::742$ : the net weight of the wool, which is 706½ lb. Again,  $706\frac{1}{2}\times 3+8=795$ , the number of dollars which the wool would have cost, if no deduction had been made for ready money. But a deduction of 6 per cent. was actually made; therefore,  $106:100::795$ : the money paid, viz. \$750. Now, on this last sum, I realized a gain of 20 per cent.; hence,  $100:120::750$ : the money received, viz. \$900, Ans. By canceling, these four statements are reduced to one, thus:

$$\frac{742. 100. 9. 100. 120}{105. 8. 106. 100};$$

that is, the 742 lb. is to be multiplied by the four succeeding ratios, and the number obtained will equal the number of dollars required.

The same canceled:

$$\begin{array}{r} 7 \quad 20 \quad 3 \quad 15 \\ 742. 100. 9. 100. 120 \\ \hline 105. 8. 106. 100 \end{array}, \text{ and } 20 \times 3 \times 15 = \$900, \text{ Ans.}$$

15  
8

3dly. A great advantage of the canceling system over all others, arises from the *expedition* it affords in arithmetical solutions. Instead of multiplying and dividing by all the numbers which the nature of the sum proposed would naturally require, the multipliers and divisors are made to cancel each other; that is, equal factors are rejected from both. Hence, they are all made to exert their appropriate influence in procuring the answer, while the labor of multiplying and dividing is avoided. The statement of each sum for canceling is a fractional answer of the same, and it is obvious, that the value of fractions is not affected by rejecting equal factors from their numerators and denominators.

The processes of reduction which occur very frequently in common Arithmetics, are mostly avoided by this system. Suppose it be required to find how many pounds sterling 5 hogsheads of wine would cost at 10 d. per pint. By the canceling system, it is necessary only to write down the numbers required to effect the reduction, and the question is then solved by canceling those numbers as far as practicable. Thus:

$$\frac{5. 63. 4. 2. 10}{12. 20}$$

The numbers above the line are obviously those, which, when multiplied together, will give the answer in pence. The numbers below the line, are those required to reduce pence to pounds sterling. The above sum canceled:

$$\begin{array}{r} 21 \\ 5. 63. 4. 2. 10 \\ \hline 12. 20 \end{array}; \text{ then, } 21 \times 5 = 105 \text{ £. Ans.}$$

8

That the above method of solving arithmetical problems is easily

comprehended and applied by the scholar, has been fully tested by the author. The experience of nine or ten years entirely devoted to the business of instruction, leaves him no room to doubt on this point. Being, however, fully aware that his Arithmetic might fall into the hands of some, who would not at once comprehend and apply the principle of canceling, he has introduced the ordinary rules of solution, in connection with those of canceling, and has endeavored to render both modes plain and familiar, by frequent and clear illustrations.

The constant aim of the teacher should be to prepare his pupils for the *active duties* of life; and, in the department of Arithmetic, this is accomplished only when the scholar has acquired correctness and *expedition* in effecting his solutions.

To make good arithmeticians, it is first necessary to acquire a correct and extensive comprehension of the simple or fundamental rules of Arithmetic. When this is done, their application will be obvious. The danger, therefore, is not, that the scholar will spend too much time on what is usually regarded as the more simple part of Arithmetic, but that he will leave it too soon.

In the use of this treatise, the author would recommend, that, when the pupil shall have passed the simple rules, and commenced those operations to which canceling may be applied, he be required to solve each problem both by the ordinary rule, and by the rule for canceling. More practice will thus be secured, and, consequently, greater expedition acquired.

In the illustrations, which are given in connection with the different rules, it has been the design of the author fully to acquaint the scholar with the nature of the subject presented, without carrying his explanations so far as to take the work which properly belongs to the scholar, out of his hands. No important acquisition can be made, without corresponding effort. This fact seems to have been overlooked, in the preparation of some Arithmetics now in use, and special effort made to render every thing as *easy as possible* for the scholar; that is, to enable him to effect the solutions with very little mental labor. The intellectual powers are, however, developed and strengthened only by being brought into *vigorous exercise*. "In Arithmetic, the young beginner should find just enough assistance to encourage and stimulate him to effort. That is not the best system, which enables the learner to advance from rule to rule with the least amount of study; but that, which, while it helps him over some difficulties, leaves him examples enough to task his powers to the utmost." (Dr. Humphrey's Thoughts on Education.) With these introductory remarks, the following work is commended to the candor of an enlightened public.

THE AUTHOR.

Norwich Academy, May, 1840.

## ERRATA.

Notwithstanding care has been constantly exercised to avoid errors, the following will be found to have escaped detection :

Page 42, line 25, read 6, instead of 5 hundred.

" 65, in the 4th sum, and in the first statement of that sum, read 21, instead of 4.

" 119, line 34, read  $\frac{9}{7}$ , instead of  $\frac{3}{7}$ .

" 136, line 14, read *integers*, for *integris*.

" 171, line 7, read \$420, instead of \$4.20.

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# ARITHMETIC.

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ARITHMETIC explains the properties and relation of numbers, and makes known their practical application.

There are six fundamental operations, with which the scholar must become perfectly familiar before he can advance successfully; viz. Notation, Numeration, Addition, Subtraction, Multiplication, and Division. These operations are called *fundamental*, because all others are founded upon, or are wrought by the application of one or more of them. They therefore require to be first clearly understood.

## NOTATION.

Notation is the art of expressing numbers by numerical characters. The characters employed to express numbers are, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, and are called figures. Each of these figures has its own *specific*, and also its *local* value, as will be learned from Numeration. Besides these characters, there are others used to express operations.

1st. The sign of addition, viz. +, (or plus, more;) requiring the numbers between which it is placed to be added:  $3 + 2$  are 5; that is, 3 added to 2 are 5; usually read, 3 plus 2 are 5.

2d. The sign of subtraction, viz. —, (or minus;) showing that the number following is to be taken from that which precedes it; thus,  $4 - 2$  is 2; that is, 2 taken from 4, 2 remains.

3d. The sign of multiplication, viz.  $\times$ ; requiring the number placed before it to be multiplied by that which follows; thus,  $3 \times 4$  is 12; that is, 3 multiplied by 4 is 12.

4th. The sign of division, viz.  $\div$ ; requiring the number preceding it to be divided by that which follows; thus,  $8 \div 2$  is 4; that is, 8 divided by 2 is 4.

In the use of each of the preceding signs, the figure preceding the sign is to be operated upon by that which follows it.

5th. The signs of proportion, viz.  $::$ ; showing that the numbers including and between these dots, are proportionals; thus,  $2 : 4 :: 6 : 12$ ; that is, 2 bears the same relation to 4 as 6 to 12. The numbers are thus read, 2 is to 4 as 6 is to 12.

6th. The sign of equality, viz.  $=$ ; expressing the equality of the numbers between which it is placed; or that the numbers on the right equal those on the left; thus,  $9 + 7 = 20 - 4$ .

7th. The characters  $\sqrt{\phantom{x}}$ ,  $\sqrt[3]{\phantom{x}}$ ,  $\sqrt[4]{\phantom{x}}$ ,  $\sqrt[5]{\phantom{x}}$ , &c. require some root of the number before which they stand to be extracted. The figure placed over the sign always shows what root is required. When the character is used without any figure, it then indicates the square root.

By the use of these characters, any arithmetical operation may be indicated. If it be required to add 9 to 16, from the amount to subtract 5, to divide the remainder by 4, and to multiply the quotient by 6, the operation would be thus expressed:  $9 + 16 - 5 \div 4 \times 6 = 30$ .

QUESTIONS.—What does Arithmetic explain? What application does it make of numbers? How many are the fundamental operations of Arithmetic? What are they? Why are these called fundamental operations? What is *Notation*? What are the characters used to express numbers called? What two-fold value has each figure? For what purposes are other characters used? What is the sign of addition, and for what is it used? The sign of subtraction? What does it require? The sign of multiplication? What does it require? The sign of division? What does it require? The signs of proportion? What do they show? The sign of equality? What does it show? What is the character used to express the extraction of roots called? Ans. The radical sign. What does the figure placed over the radical sign show?

## NUMERATION.

The scholar has seen, under Notation, the characters used to express the first nine numbers, viz. ; that to express one whole object or thing, 1 is used; to express two whole things, 2 is employed; and for three whole things, 3 is taken, &c., so that each character has its own specific value; and this it always expresses when it stands alone. But each figure has also a local value, that is, a value depending on the place it occupies;



thus, the value of 3 differs in each of the following numbers, viz. 003, 030, and 300. In the first number its value is three units, or ones; in the second number it is three tens, or thirty; and in the last, it is three hundreds. It will therefore be readily perceived, that the position of a figure materially affects its value. Numeration teaches how to determine what this value is; and thus it also enables us to determine the total value of any number of figures. It will be found that any figure is increased in a *ten-fold* ratio by having a single figure placed on the right of it; thus, 6 alone, is 6 units; but if another figure be placed on the right of this, its value is ten times as great as before; thus, in 63, the 6 is six tens, equal to 60, and the 3 is three units. This value is increased a *hundred-fold* by having two figures placed on the right; thus, 600; and a *thousand-fold* by having three figures on the right of it; thus, 6000. In the first of the last two examples, the value of 6 is six hundred, and in the second it is six thousand. Hence, the scholar will see the necessity of terms by which to designate this local value of figures, and will also readily see the appropriateness of those used, viz. Units, Tens, Hundreds, Thousands, Tens of Thousands, Hundreds of Thousands, Millions, Tens of Millions, Hundreds of Millions, &c. These nine terms are sufficient to express any number in common practice. The higher denominations are Billions, Tens of Billions, Hundreds of Billions; Trillions, Tens of Trillions, Hundreds of Trillions; Quadrillions, Tens of Quadrillions, Hundreds of Quadrillions; Quintillions, Tens of—Hundreds of—; Sextillions, Tens of—Hundreds of—; Septillions, Tens of—Hundreds of—; Octillions, Tens of—Hundreds of—; Nonillions, Tens of—Hundreds of—, &c.

It will be observed that as the first three figures, reckoning from the right, are units, tens, and hundreds, so every succeeding three are appropriated to the units, tens, and hundreds of the succeeding higher denominations. The following table will serve as an illustration:

| Hundreds<br>tens<br>units<br>of<br>Octillions. | Hundreds<br>tens<br>units<br>of<br>Septillions. | Hundreds<br>tens<br>units<br>of<br>Sextillions. | Hundreds<br>tens<br>units<br>of<br>Quintillions. | Hundreds<br>tens<br>units<br>of<br>Quadrillions. | Hundreds<br>tens<br>units<br>of<br>Trillions. | Hundreds<br>tens<br>units<br>of<br>Billions. | Hundreds<br>tens<br>units<br>of<br>Millions. | Hundreds<br>tens<br>units<br>of<br>Thousands. | Hundreds<br>tens<br>units<br>of<br>Units. |
|------------------------------------------------|-------------------------------------------------|-------------------------------------------------|--------------------------------------------------|--------------------------------------------------|-----------------------------------------------|----------------------------------------------|----------------------------------------------|-----------------------------------------------|-------------------------------------------|
| 369,                                           | 342,                                            | 900,                                            | 976,                                             | 368,                                             | 265,                                          | 371,                                         | 502,                                         | 634,                                          | 436.                                      |

This table will enable the scholar to see at a glance, that the names and value of figures are entirely dependent on their location. If they be counted from the right hand towards the left, the first figure in any line of figures is units; the second is tens; the third, hundreds; the fourth, thousands; the fifth, tens of thousands, &c.; and whatever station or place any figure may occupy, its value becomes ten times as great by being moved one degree farther to the left.

QUESTIONS.—What are the characters used to express the first nine numbers? Give an example. When does each figure express its own specific value? What other value has each figure? On what does the local value of a figure depend? Give an illustration. What, therefore, does Numeration teach us to do? What does it enable us to do? In what ratio is the value of any figure increased by having a single figure placed on the right? In what ratio is it increased by having two figures on the right of it? In what by having three placed on its right? In what ratio do numbers continue to increase from the right to the left? Ans. In a ten-fold ratio. What are the terms by which the local value of figures are expressed? To what are each three successive figures in any number appropriated? Ans. To the units, tens, and hundreds of each denomination. In tracing the figures from the right to the left, what is the first figure called? The second? The third? The fourth? &c. What effect is produced on the value of any figure by moving it one place to the left?

Enumerate the following numbers, viz. 6; 27; 467; 568; 4269; 13786; 27599; 367595; 1729567; 67596422; 586; 379872689; 278063; 596402606; 295; 336003; 300300303; 505050505; 467327986427; 585950876; 688001; 100000000; 9999999999; 6398742913; 4678.

The scholar should be taught to read these figures accurately; for example, suppose he be required to enumerate the last number, viz. 4678; let him commence and repeat thus: eight units, seven tens, six hundreds, and four thousands; and then unite them, thus: four thousand six hundred and seventy-eight. Let him also be required to give the value of any figure as it may vary by being written at different points under any line of figures.

After the scholar has become familiar with the preceding exercise, he may write the following numbers on his slate in figures, taking care to express each number accurately:—  
1. Thirty-five. 2. Three hundred and seventy-five. 3. Three hundred and five. 4. Seven thousand six hundred and thirty-five. 5. Seven thousand and thirty-five. 6. Seventy-five thousand four hundred and sixteen. 7. Seventy-five thousand and sixteen. 8. Seventy-five thousand and six. 9. Seventy-

five thousand. -10. Three hundred and thirty-three thousand three hundred and thirty-three. 11. Three hundred thousand and three. 12. Three hundred thousand three hundred and three. 13. Five millions and five. Six millions and seventy-five. One hundred and sixty millions. Forty-seven millions, one hundred and five thousand and sixty. 14. One hundred millions, one hundred and one. 15. One hundred and seven millions, one hundred and seven thousand, one hundred and seven. 16. Two billions, three hundred and three millions, five hundred and five thousand and six. 17. Seven hundred and seven trillions, six hundred and seventy-two billions, nine millions, three hundred and five thousand, six hundred and nine.

There is yet another method of expressing numbers; viz. the Roman method; in which the letters of the alphabet are used, as may be seen from the following table.

ROMAN TABLE.

|                |         |                 |                |
|----------------|---------|-----------------|----------------|
| I . . . . .    | One.    | LX . . . . .    | Sixty.         |
| II . . . . .   | Two.    | LXX . . . . .   | Seventy.       |
| III . . . . .  | Three.  | LXXX . . . . .  | Eighty.        |
| IV . . . . .   | Four.   | XC . . . . .    | Ninety.        |
| V . . . . .    | Five.   | C . . . . .     | One hundred.   |
| VI . . . . .   | Six.    | CC . . . . .    | Two hundred.   |
| VII . . . . .  | Seven.  | CCC . . . . .   | Three hundred. |
| VIII . . . . . | Eight.  | CCCC . . . . .  | Four hundred.  |
| IX . . . . .   | Nine.   | D . . . . .     | Five hundred.  |
| X . . . . .    | Ten.    | DC . . . . .    | Six hundred.   |
| XX . . . . .   | Twenty. | DCC . . . . .   | Seven hundred. |
| XXX . . . . .  | Thirty. | DCCC . . . . .  | Eight hundred. |
| XL . . . . .   | Forty.  | DCCCC . . . . . | Nine hundred.  |
| L . . . . .    | Fifty.  | M . . . . .     | One thousand.  |

ADDITION OF SIMPLE NUMBERS.

It is highly important that the scholar obtain clear and distinct views of the nature of what he has to perform. He is therefore recommended to make himself familiar with the fol-

following table in the first place; then to take the mental exercises that follow, and study them faithfully before commencing with a slate. Indeed, the author would advise, that the tables, together with the mental exercises, which stand at the beginning of each of the four simple rules, be first studied; and that the beginner then turn back and review the whole.

### ADDITION TABLE.

The signs plus and minus, &c. are introduced into the following tables, that the scholar may early learn the use of them. He will therefore turn back to Notation, if he does not recollect their use.

|                   |             |             |             |
|-------------------|-------------|-------------|-------------|
| 2 plus 1 equals 3 | 3 + 1 = 4   | 4 + 1 = 5   | 5 + 1 = 6   |
| 2 + 2 = 4         | 3 + 2 = 5   | 4 + 2 = 6   | 5 + 2 = 7   |
| 2 + 3 = 5         | 3 + 3 = 6   | 4 + 3 = 7   | 5 + 3 = 8   |
| 2 + 4 = 6         | 3 + 4 = 7   | 4 + 4 = 8   | 5 + 4 = 9   |
| 2 + 5 = 7         | 3 + 5 = 8   | 4 + 5 = 9   | 5 + 5 = 10  |
| 2 + 6 = 8         | 3 + 6 = 9   | 4 + 6 = 10  | 5 + 6 = 11  |
| 2 + 7 = 9         | 3 + 7 = 10  | 4 + 7 = 11  | 5 + 7 = 12  |
| 2 + 8 = 10        | 3 + 8 = 11  | 4 + 8 = 12  | 5 + 8 = 13  |
| 2 + 9 = 11        | 3 + 9 = 12  | 4 + 9 = 13  | 5 + 9 = 14  |
| 2 + 10 = 12       | 3 + 10 = 13 | 4 + 10 = 14 | 5 + 10 = 15 |
| 2 + 11 = 13       | 3 + 11 = 14 | 4 + 11 = 15 | 5 + 11 = 16 |
| 2 + 12 = 14       | 3 + 12 = 15 | 4 + 12 = 16 | 5 + 12 = 17 |
| 6 + 1 = 7         | 7 + 1 = 8   | 8 + 1 = 9   | 9 + 1 = 10  |
| 6 + 2 = 8         | 7 + 2 = 9   | 8 + 2 = 10  | 9 + 2 = 11  |
| 6 + 3 = 9         | 7 + 3 = 10  | 8 + 3 = 11  | 9 + 3 = 12  |
| 6 + 4 = 10        | 7 + 4 = 11  | 8 + 4 = 12  | 9 + 4 = 13  |
| 6 + 5 = 11        | 7 + 5 = 12  | 8 + 5 = 13  | 9 + 5 = 14  |
| 6 + 6 = 12        | 7 + 6 = 13  | 8 + 6 = 14  | 9 + 6 = 15  |
| 6 + 7 = 13        | 7 + 7 = 14  | 8 + 7 = 15  | 9 + 7 = 16  |
| 6 + 8 = 14        | 7 + 8 = 15  | 8 + 8 = 16  | 9 + 8 = 17  |
| 6 + 9 = 15        | 7 + 9 = 16  | 8 + 9 = 17  | 9 + 9 = 18  |
| 6 + 10 = 16       | 7 + 10 = 17 | 8 + 10 = 18 | 9 + 10 = 19 |
| 6 + 11 = 17       | 7 + 11 = 18 | 8 + 11 = 19 | 9 + 11 = 20 |
| 6 + 12 = 18       | 7 + 12 = 19 | 8 + 12 = 20 | 9 + 12 = 21 |

|                |                |                |
|----------------|----------------|----------------|
| $10 + 1 = 11$  | $11 + 1 = 12$  | $12 + 1 = 13$  |
| $10 + 2 = 12$  | $11 + 2 = 13$  | $12 + 2 = 14$  |
| $10 + 3 = 13$  | $11 + 3 = 14$  | $12 + 3 = 15$  |
| $10 + 4 = 14$  | $11 + 4 = 15$  | $12 + 4 = 16$  |
| $10 + 5 = 15$  | $11 + 5 = 16$  | $12 + 5 = 17$  |
| $10 + 6 = 16$  | $11 + 6 = 17$  | $12 + 6 = 18$  |
| $10 + 7 = 17$  | $11 + 7 = 18$  | $12 + 7 = 19$  |
| $10 + 8 = 18$  | $11 + 8 = 19$  | $12 + 8 = 20$  |
| $10 + 9 = 19$  | $11 + 9 = 20$  | $12 + 9 = 21$  |
| $10 + 10 = 20$ | $11 + 10 = 21$ | $12 + 10 = 22$ |
| $10 + 11 = 21$ | $11 + 11 = 22$ | $12 + 11 = 23$ |
| $10 + 12 = 22$ | $11 + 12 = 23$ | $12 + 12 = 24$ |

In reciting this and the following tables, the teacher should be careful that the scholar, as he repeats his answers, perform the necessary mental operation; he must teach his pupil to *think*.

#### MENTAL EXERCISES IN ADDITION.

If you borrow 6 dollars at one time and 3 at another, how many will you have? 6 and 3 are how many? A pays you 3 dollars, B 4, and C 5; how many do they all pay you?  $3 + 4 + 5 =$  how many? If you pay away 6 cents at one time, 4 at another, and 3 at another, how many do you pay away?  $6 + 4 + 3 =$  how many? If you buy twelve apples for 9 cents, and a pint of chesnuts for 4, how many cents do the apples and chesnuts cost?  $9 + 4 =$  how many? John gave Henry 8 apples at one time, 6 at another, and 4 at another; how many did he give him?  $8 + 6 + 4 =$  how many? Lent my brother 10 dollars at one time, 6 at another, and 4 at another; how many dollars did I lend him?  $10 + 6 + 4 =$  how many? Alfred has four brothers; to one, he gave 11 pears, to another, 9, to another, 6, and to another, 3; how many pears did he give away?  $11 + 9 + 6 + 3 =$  how many? He also gave his two sisters 5 each; how many did he give to both brothers and sisters?  $11 + 9 + 6 + 3 + 5 + 5 =$  how many? A man bought four bushels of corn for 3 dollars, six bushels of oats for 2 dollars, and 9 bushels of wheat for 12; how many bushels of grain did he buy, and how many dollars did the whole cost him?  $4 + 6 + 9 =$  how many?  $3 + 2 + 12 =$  how many? John has 9 dollars; James, 12; and Joseph, 7; how many have they all?  $9 + 12 + 7 =$  how many? Peter

gave a poor man 11 cents; Charles gave him 12 cents; Henry, 10; and George, 8; how many did they all give him;  $11 + 12 + 10 + 8 =$  how many? A man was three days performing a journey; the first day he traveled 13 miles; the second day, 14; and the third, 15; how far did he travel?  $13 + 14 + 15 =$  how many? How many sums will you have done, if you do 12 to-day, 15 to-morrow, and 18 the next day?  $12 + 15 + 18 =$  how many?

How many are  $3 + 4$ ?  $13 + 4$ ?  $23 + 4$ ?  $33 + 4$ ?  $43 + 4$ ?  $53 + 4$ ?  $63 + 4$ ?  $73 + 4$ ?  $83 + 4$ ?  $93 + 4$ ? How many are  $4 + 4$ ?  $14 + 4$ ?  $24 + 4$ ?  $34 + 4$ ? &c. How many are  $5 + 4$ ?  $15 + 4$ ?  $25 + 4$ ?  $35 + 4$ ? &c. How many are  $6 + 4$ ?  $16 + 4$ ?  $26 + 4$ ?  $36 + 4$ ? &c. How many are  $7 + 4$ ?  $17 + 4$ ?  $27 + 4$ ?  $37 + 4$ ? &c. How many are  $4 + 4$ ?  $4 + 14$ ?  $4 + 24$ ? &c.  $4 + 5$ ?  $4 + 15$ ?  $4 + 25$ ? &c.  $4 + 6$ ?  $4 + 16$ ?  $4 + 26$ ? &c.  $4 + 7$ ?  $4 + 17$ ?  $4 + 27$ ? &c. How many are  $5 + 5$ ?  $5 + 15$ ?  $5 + 25$ ? &c.  $5 + 6$ ?  $5 + 16$ ?  $5 + 26$ ? &c.  $6 + 6$ ?  $6 + 16$ ?  $6 + 26$ ? &c. How many are  $7 + 7$ ?  $7 + 17$ ?  $7 + 27$ ? &c.  $8 + 8$ ?  $8 + 18$ ?  $8 + 28$ ? &c.  $9 + 9$ ?  $9 + 19$ ?  $9 + 29$ ? &c. How many are  $10 + 10$ ?  $10 + 20$ ? &c.  $11 + 5$ ?  $11 + 15$ ?  $12 + 4$ ?  $12 + 14$ ? &c.

Questions of this character may be proposed to any extent, and should in no instance be omitted until the scholar can add without hesitation. Others of a more promiscuous character should likewise be proposed, such as the following:

$7 + 4 + 6 =$  how many?  $2 + 1 + 9 + 6 + 7 + 6 =$  how many?  $5 + 6 + 9 + 3 =$  how many?  $6 + 4 + 2 + 8 + 1 + 3 + 5 + 7 + 9 =$  how many?  $11 + 9 + 6 + 12 =$  how many?  $7 + 6 + 9 + 8 =$  how many?  $8 + 8 + 6 + 6 + 4 + 4 =$  how many?  $7 + 7 + 9 + 9 =$  how many?  $2 + 4 + 6 + 8 + 10 + 12 =$  how many?

The scholar may now commence the use of the slate and pencil, or the practice of written arithmetic.

The first consideration to which his attention should here be directed, is, that like things only can be added to or subtracted from each other. It would be absurd to attempt to add together books and chairs, to see how many books, or how many chairs the whole would make; the number of each would evidently remain unaffected. If, however, we add books to books, we obtain a number greater than either of the original numbers; that is, just equal to them both. Neither can we add units to tens; for the amount would be neither units nor tens; but units must be added to units, tens to tens, and hundreds to hundreds, and so on. But ten units make one ten, ten

tens make one hundred, and ten hundreds make one thousand, &c.; that is, simple numbers increase and decrease in a ten-fold ratio: Hence 10 in the column of units is equal to 1 in the column of tens; and 10 in the column of tens, is equal to 1 in the column of hundreds. If, then, in adding up the column of units the whole should amount to, just 10, it is obvious that nothing is lost, if these 10 units are converted into 1 ten, and the 1 ten added to the column of tens; for 10 units = 1 ten. The same is true of other denominations. Therefore the following General Rule will be found applicable to Simple Addition:

1st. Write down the numbers, placing units under units, tens under tens, &c.

2d. Draw a line underneath, and commence at the right hand and add together all the figures in the first column.

3d. If the sum be less than 10, set it down at the foot of that column; if it be 10, or more than 10, it will consist of two figures at least; set down the right hand one as before, and add the left hand one to the next column, it being in all cases so many tens, when compared with the figures added.

4th. Continue to perform the same operation with the remaining columns, observing only to write down the whole amount of the left hand column.

5th. To detect any error that may have been committed, commence again and add each column downwards; if the same numbers are obtained by each operation, the work is probably right.

Now to apply this rule, let us add together the four following numbers, viz. 1234; 2345; 6420; and 5796. The rule says, write these numbers with units under units, tens under tens, &c. thus:

|                                                      |   |   |   |   |
|------------------------------------------------------|---|---|---|---|
|                                                      | 1 | 2 | 3 | 4 |
| Now to add these numbers, I commence at the          | 2 | 3 | 4 | 5 |
| right hand, and first add together the unit figures; | 6 | 4 | 2 | 0 |
| viz. 6, 0, 5, 4; the sum I find to be 15 units,      | 5 | 7 | 9 | 6 |

equal to 1 ten and 5 units; I write down the 5 units, but add the 1 ten to the next column; thus, 1 added to 9 is 10, and 2 are 12 and 4 are

16 and 3 are 19; as before, I write down the 9, and add the 1 to the next column; this being added gives the amount 17; the 7 is written down and the 1 again carried to the next and last column; and here the amount is 15, which being the last column, the whole number is written down. This operation

gives the amount of the four numbers, 15795. The scholar will readily comprehend the nature of Simple Addition; viz. that it consists in uniting two or more numbers of the same denomination, so as to find their amount.

The scholar will carefully examine the following sums, which are added, to see if he obtains the same result.

2.

$$\begin{array}{r}
 847396 \\
 364869 \\
 482436 \\
 622439 \\
 \hline
 2317140
 \end{array}$$

In this second example, in the unit column, there are 3 to carry, because there are 3 tens or 30. For the same reason there are 2 to carry in all the remaining columns but one.

3.

$$\begin{array}{r}
 456789460 \\
 680246802 \\
 135791357 \\
 423650825 \\
 371574628 \\
 856456333 \\
 \hline
 2924509405
 \end{array}$$

4.

$$\begin{array}{r}
 123456789 \\
 987654321 \\
 123456789 \\
 987654321 \\
 123456789 \\
 987654321 \\
 \hline
 3333333330
 \end{array}$$

5.

$$\begin{array}{r}
 2468024 \\
 1357912 \\
 4208642 \\
 2135791 \\
 3355779 \\
 8866448 \\
 \hline
 22392596
 \end{array}$$

6.

$$\begin{array}{r}
 9796365 \\
 5636979 \\
 8246024 \\
 5963816 \\
 4674364 \\
 5796378 \\
 \hline
 40113926
 \end{array}$$

7.

$$\begin{array}{r}
 1798670 \\
 6124356 \\
 7212345 \\
 8463738 \\
 5739168 \\
 9156423 \\
 \hline
 \end{array}$$

8.

$$\begin{array}{r}
 9176435 \\
 5683214 \\
 2345678 \\
 0123456 \\
 \hline
 \end{array}$$

9.

$$\begin{array}{r}
 43457096 \\
 61728349 \\
 28394563 \\
 56831234 \\
 \hline
 \end{array}$$

10.

$$\begin{array}{r}
 70819634 \\
 64753278 \\
 86542365 \\
 23456789 \\
 \hline
 \end{array}$$



| 11.       | 12.       | 13.          |
|-----------|-----------|--------------|
| 81828384  | 39724638  | 45768903     |
| 18283848  | 97236452  | 78924683     |
| 99999999  | 15707934  | 13579246     |
| 77777777  | 65972109  | 97576887     |
| 14.       | 15.       | 16.          |
| 123456789 | 63840596  | 567347205379 |
| 987654321 | 27690903  | 275684467903 |
| 546372819 | 46379108  | 123456789012 |
| 951486804 | 55681396  | 378965842687 |
| 468046872 | 63705418  | 593748265379 |
| 608642109 | 36485074  | 916547681234 |
| 17.       | 18.       | 19.          |
| 795735198 | 369248754 | 1357924680   |
| 49157038  | 62958651  | 261595827    |
| 8930576   | 2581731   | 4230917546   |
| 697005    | 407347    | 6083419      |
| 29510     | 80478     | 682          |
| 4812      | 6184      | 8472166264   |
| 863       | 236       | 4462352      |
| 27        | 46        | 44605        |
| 4         | 8         | 210          |

20. A farmer sold his wheat for 320 dollars; his corn for 275 dollars; his oats for 78 dollars; his barley for 162 dollars; and one horse for 132 dollars. What was the amount of his sales? *Ans.* \$967.

21. A merchant owned four vessels, which were worth, the first, \$4800; the second, \$5200; the third, \$6000; and the fourth, \$6800; he had also goods on board one of these vessels, worth \$2700; besides \$3500 deposited in the bank. What is the amount of his property? *Ans.* \$29000.

22. Three farmers have each 562 acres of land; how many have they all? *Ans.* 1686 acres.

23. A farmer fattened and killed an ox for market; the hind quarters weighed, the one 182 pounds, and the other 177; each of the fore quarters weighed 163 pounds; the hide, 116 pounds; and the tallow, 120 pounds. What was the weight of the ox? *Ans.* 931 pounds.

24. In a certain town there are five schools, containing the following numbers of scholars, viz. 72, 28, 65, 84, and 91; how many children are attending these five schools? *Ans.* 340.

25. A steamboat performed in one week four trips from Hartford to New York; on the first trip, she took for passengers, \$378, for freight, \$175; on the second trip, for passengers, \$402, for freight, \$278; on the third, for passengers, \$263, for freight, \$147; on her fourth trip, for passengers and freight, \$500. What did her bills amount to for the week? *Ans.* \$2143.

26. A carpenter contracted for the building of five dwellings in one spring; for the first he was to receive \$1800; for the second, \$2100; for the third, \$2221; for the fourth, \$2850; and for the fifth, \$3172. To what did his contracts amount? *Ans.* \$12143.

27. In 1834, A traveled 1320 miles; in 1835, he traveled 1162 miles; in 1836, 2100 miles; in 1837, 1400 miles; in 1838, 1992 miles. How many miles did he travel from 1834 to 1838 inclusive? *Ans.* 7974.

28. Bought of my neighbor four loads of hay; the first weighed 1600 lbs.; the second, 2100 lbs.; the third, 1999 lbs.; and the fourth, 1709 lbs. What was the whole weight? *Ans.* 7408 pounds.

29. A wholesale dealer in grain has in one bin 242 bushels of wheat; in another he has 2356 bushels of rye; in a third, 1556 bushels of oats; in a fourth, 876 bushels of barley. How many bushels of grain has he of all kinds? *Ans.* 5030.

30. Suppose I am indebted to A, \$2560; to B, \$27; to C, \$169; to D, \$3470; and to E, \$17. How much do I owe in all? *Ans.* \$6243.

31. A man having three sons and two daughters, gave to each of his sons, \$879; and to each of his daughters, \$199. How much money did he give them all? *Ans.* \$1535.

32. A gentleman being asked how old he was, said, he was married when he was 29 years of age; that he lived with his wife 8 years before the birth of their son, who was now 27 years of age. What was the father's age? *Ans.* 64 years.

33. A man bought a horse for \$75; a chaise for \$150; and a harness for \$45; he then sold his horse for \$150; his chaise for \$125; and his harness for \$30. What did he pay for the whole, and what did he receive for the whole. *Ans.* Paid \$270; received \$305.

34. Add together three hundred and seventy-five thousand

and sixty-five; nine hundred thousand and three; one million six hundred thousand seven hundred and ninety-nine. *Ans.* 2875867.

35. Add also ninety-nine millions; seven hundred and fifty-five millions six hundred and thirty-three. *Ans.* 854000633.

36. There are 5784 apples in one pile; 588 in another; 84 in a third; and seven hundred and seventy-nine in a fourth. How many are there in all? *Ans.* 7235.

**QUESTIONS.**—What things only can be added to or subtracted from each other? In what does Simple Addition consist? Can we add units to tens? To what must units, &c. be added? For what number do we carry in Simple Addition? Why for 10? *Ans.* Because numbers increase in a ten-fold ratio. 10 units equal how many 10's? What is the Rule for Addition? How do you write down the numbers? Where do you commence to add? If the sum of the figures added be less than 10, what is to be done? What if it be 10, or more than 10? What is observed respecting the last or left hand column? How may errors in Addition be detected?

## SIMPLE SUBTRACTION.

This rule is directly the reverse of the preceding; while we are there taught to unite several numbers into one, we are here taught the operation by which one number is taken from another. A familiar acquaintance with the following table should be the first object of the scholar:

SUBTRACTION TABLE.

|         |         |         |         |
|---------|---------|---------|---------|
| 1—1=0   | 2—2=0   | 3—3=0   | 4—4=0   |
| 2—1=1   | 3—2=1   | 4—3=1   | 5—4=1   |
| 3—1=2   | 4—2=2   | 5—3=2   | 6—4=2   |
| 4—1=3   | 5—2=3   | 6—3=3   | 7—4=3   |
| 5—1=4   | 6—2=4   | 7—3=4   | 8—4=4   |
| 6—1=5   | 7—2=5   | 8—3=5   | 9—4=5   |
| 7—1=6   | 8—2=6   | 9—3=6   | 10—4=6  |
| 8—1=7   | 9—2=7   | 10—3=7  | 11—4=7  |
| 9—1=8   | 10—2=8  | 11—3=8  | 12—4=8  |
| 10—1=9  | 11—2=9  | 12—3=9  | 13—4=9  |
| 11—1=10 | 12—2=10 | 13—3=10 | 14—4=10 |
| 12—1=11 | 13—2=11 | 14—3=11 | 15—4=11 |
| 13—1=12 | 14—2=12 | 15—3=12 | 16—4=12 |

|         |         |         |         |
|---------|---------|---------|---------|
| 5—5=0   | 6—6=0   | 7—7=0   | 8—8=0   |
| 6—5=1   | 7—6=1   | 8—7=1   | 9—8=1   |
| 7—5=2   | 8—6=2   | 9—7=2   | 10—8=2  |
| 8—5=3   | 9—6=3   | 10—7=3  | 11—8=3  |
| 9—5=4   | 10—6=4  | 11—7=4  | 12—8=4  |
| 10—5=5  | 11—6=5  | 12—7=5  | 13—8=5  |
| 11—5=6  | 12—6=6  | 13—7=6  | 14—8=6  |
| 12—5=7  | 13—6=7  | 14—7=7  | 15—8=7  |
| 13—5=8  | 14—6=8  | 15—7=8  | 16—8=8  |
| 14—5=9  | 15—6=9  | 16—7=9  | 17—8=9  |
| 15—5=10 | 16—6=10 | 17—7=10 | 18—8=10 |
| 16—5=11 | 17—6=11 | 18—7=11 | 19—8=11 |
| 17—5=12 | 18—6=12 | 19—7=12 | 20—8=12 |

|         |          |          |          |
|---------|----------|----------|----------|
| 9—9=0   | 10—10=0  | 11—11=0  | 12—12=0  |
| 10—9=1  | 11—10=1  | 12—11=1  | 13—12=1  |
| 11—9=2  | 12—10=2  | 13—11=2  | 14—12=2  |
| 12—9=3  | 13—10=3  | 14—11=3  | 15—12=3  |
| 13—9=4  | 14—10=4  | 15—11=4  | 16—12=4  |
| 14—9=5  | 15—10=5  | 16—11=5  | 17—12=5  |
| 15—9=6  | 16—10=6  | 17—11=6  | 18—12=6  |
| 16—9=7  | 17—10=7  | 18—11=7  | 19—12=7  |
| 17—9=8  | 18—10=8  | 19—11=8  | 20—12=8  |
| 18—9=9  | 19—10=9  | 20—11=9  | 21—12=9  |
| 19—9=10 | 20—10=10 | 21—11=10 | 22—12=10 |
| 20—9=11 | 21—10=11 | 22—11=11 | 23—12=11 |
| 21—9=12 | 22—10=12 | 23—11=12 | 24—12=12 |

## MENTAL EXERCISES IN SUBTRACTION.

Charles had 10 marbles, and gave 5 of them to his brother; how many had he left? He then gave 3 to his sister; how many did there then remain? A boy had 15 cents, and lost 5 of them; how many had he left? John has 13 apples; how many will he have, if he gives away 5? Peter had 19 marbles, and gave away 9 of them; how many had he left? A boy purchased two toys, for the one he gave 20 cts. and for the other 12 cts; what was the difference in the cost of them? Out of 23 plumbs a boy selected 12; how many remained? A boy threw 15 pennies into the air, and caught 6 of them as they fell; how many reached the ground? Sold one cow for 19 dollars and another for 12; how much more did I receive for one than for the other? Charles has 19 marbles, and James has 11; how many has Charles more than James? Sold one

knife for 21 cents and another for 12 cents; what was the difference in price? 11 taken from 19, how many will remain? 8 from 19? 10 from 19? 7 from 19? 11 from 19? 6 from 19? 12 from 19? 5 from 19? 13 from 19? 4 from 19? 14 from 19? 3 from 19? If 9 be taken from 18, how many will remain? 8 from 18? 10 from 18? 7 from 18? 11 from 18? 6 from 18? 12 from 18? 5 from 18? 13 from 18? 4 from 18? 14 from 18? 3 from 18? 15 from 18? 2 from 18? 16 from 18? 1 from 18? 17 from 18? If 10 be taken from 20, how many will remain? 9 from 20? 11 from 20? 8 from 20? 12 from 20? 7 from 20? 13 from 20? 6 from 20? 14 from 20? 5 from 20? 15 from 20? 4 from 20? 16 from 20? 3 from 20? 17 from 20? 2 from 20? 18 from 20? 19 from 20? 11 from 21? 10 from 21? 12 from 21? 9 from 21? 13 from 21? 8 from 21? 14 from 21? 7 from 21? 15 from 21? 6 from 21? 16 from 21? 5 from 21? 17 from 21? 4 from 21? 18 from 21? 3 from 21? 19 from 21? 2 from 21? 11 from 22? 10 from 22? 12 from 22? 9 from 22? 13 from 22? 8 from 22? 14 from 22? 7 from 22? 15 from 22? 6 from 22? 16 from 22? 5 from 22? 17 from 22? 4 from 22? 18 from 22? 3 from 22? 19 from 22? 2 from 22? 12 from 23? 11 from 23? 13 from 23? 10 from 23? 14 from 23? 9 from 23? 15 from 23? 8 from 23? 16 from 23? 7 from 23? 17 from 23? 6 from 23? 18 from 23? 5 from 23? 19 from 23? 4 from 23? 20 from 23? 3 from 23? 21 from 23? 2 from 23? 22 from 23? 1 from 23? 12 from 24? 11 from 24? 13 from 24? 10 from 24? 14 from 24? 9 from 24? 15 from 24? 8 from 24? 16 from 24? 7 from 24? 17 from 24? 6 from 24? 18 from 24? 5 from 24? 19 from 24? 4 from 24? 20 from 24? 3 from 24? 21 from 24? 2 from 24? 22 from 24? 1 from 24? 23 from 24?

16 boys went on a sailing excursion, only 7 of them returned; the others were drowned; how many were lost? If from a pile of 20 apples, I take away 13, how many will be left? A man started on a journey of 23 miles; after he had traveled 16 miles, he stopped to feed his horse; how far had he then to travel? A man having 22 chickens, killed 13 of them; how many were left? From a stick of timber 19 feet long, 7 feet were cut off; what was the length of the remainder?  $19 - 7 =$  how many?  $21 - 9 =$  how many?  $17 - 9 =$  how many?  $23 - 5 =$  how many?  $15 - 7 =$  how many?  $24 - 9 =$  how many?  $11 - 7 =$  how many?  $22 - 16 =$  how many?  $12 - 7 =$  how many?  $13 - 5 =$  how many?  $14 -$

7 = how many?  $15 - 9 =$  how many?  $16 - 9 =$  how many?  
 $17 - 8 =$  how many?  $18 - 7 =$  how many?  $19 - 11 =$  how  
 many?  $20 - 9 =$  how many?  $21 - 7 =$  how many?  $22 - 9$   
 $=$  how many?  $23 - 13 =$  how many?

From the preceding table and examples, the scholar will comprehend the nature of Simple Subtraction; his next step will be to practice with his slate and pencil. It will already have been observed, that only two numbers are employed in a single operation of subtraction. The larger of these two numbers is called the minuend; and the smaller, the subtrahend. The object of the rule is to find the difference between the two; that is, to find how much will remain of the larger after the smaller is taken from it. The number obtained by the operation, is called the remainder.

The scholar may be guided by the following rule :

**RULE.**—1st. *Write the less of the two numbers under the greater, with units under units and tens under tens, &c. and draw a line beneath them.*

2d. *Commence with the right hand figure of the lower line or subtrahend, and take it from the figure which stands directly above it, if practicable. Do the same with the remaining figures in the subtrahend, if practicable, and the operation will be completed.*

3d. *But whenever this cannot be done, that is, when the lower figure is the larger, 10 should be added to the upper figure and the lower one taken from the sum.*

4th. *Whenever 10 is added to an upper figure, 1 must be carried or added to the next lower figure; that is, 1 is to be carried whenever 10 is borrowed.*

5th. *To prove the work, add the remainder to the subtrahend, and if the work be right, the amount will correspond with the minuend.*

The scholar will easily comprehend the nature of this rule, unless he should find difficulty in understanding why, when we borrow 10, we are required to carry only 1. He must however remember, that by the addition of this 10 to the upper number, he has increased the value of that number 10 units, 10 tens, 10 hundreds, or 10 thousands, according to the place the figure occupies. If he add it to the units, the value of the addition is 1 ten; if to the tens, the value is one hundred, because 10 units make one ten, and 10 tens, one hundred, &c. Now by the rule, if 10 be borrowed, 1 must be carried to the

next *lower* figure; by which operation, one more will be taken from the figure in the minuend; and this one more, which is thus removed, is just equal in value to the 10 that was added, for it is taken from a figure one degree farther to the left. But this subject will be more clearly comprehended, when illustrated by example. Take the following sum:

In this example, it is evident that if 6 be taken from 8, 2 will remain; and if the 1 ten be taken from 2 tens, 1 ten will remain. But how is 5 in the place of hundreds to be taken from the 4 above it? Evidently by the third section of the rule; that is, 10 is added to the 4 in the minuend, by which addition, it will become 14 hundred, from which if 5 hundred be taken, 9 hundred will remain, which is the third figure in the remainder. But by this operation the minuend has been increased 10 hundred; if therefore I add 1 to the 2 thousand, it will become 3 thousand, and consequently, when subtracted from the figure 5 above it, will take one thousand more from the minuend, so that only 2 will remain. If therefore 10 hundred was in one instance added to the minuend, in the other, 1 thousand, its equal, has been taken from it. The same reasoning is applicable to the 8; 10 is added to the 3, which increases it to 13; the 8 is taken from the 13, and 5 remains. There is then one to carry to the 3, which thus increased is taken from the 6, and 2 remains. The whole remainder therefore is 252912.

Now if this remainder be added to the lower number or subtrahend, the amount will be the minuend; which proves that the operation is correct, thus:

$$\text{Subtrahend} = 3\ 8\ 2\ 5\ 1\ 6$$

$$\text{Remainder} = 2\ 5\ 2\ 9\ 1\ 2$$

$$\text{Minuend} = 6\ 3\ 5\ 4\ 2\ 8$$

|                          |                          |                        |
|--------------------------|--------------------------|------------------------|
| 2.                       | 3.                       | 4.                     |
| From 6 6 6 8 3           | From 8 9 4 6 7 3         | From 9 8 7 6 5 4 3 2 1 |
| Take 2 5 9 6 6           | Take 7 6 8 5 9 6         | Take 1 2 3 4 5 6 7 8 9 |
| 4 0 7 1 7                | 1 2 6 0 7 7              | 8 6 4 1 9 7 5 3 2      |
| 5.                       | 6.                       |                        |
| Min. 1 0 0 0 0 0 0 0 0   | Min. 1 0 7 5 6 0 8 7 5 6 |                        |
| Subtr. 9 9 9 9 9 9 9 9 9 | Subtr. 6 9 8 4 5 3 8 7 4 |                        |
|                          |                          |                        |

$$\begin{array}{r} \text{7.} \\ \text{Min. } 9\ 6\ 3\ 4\ 6\ 5\ 7\ 9\ 4\ 2 \\ \text{Subtr. } 8\ 3\ 2\ 6\ 5\ 4\ 7\ 2\ 8\ 6 \\ \hline \end{array}$$

$$\begin{array}{r} \text{8.} \\ \text{Min. } 3\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ \text{Subtr. } 2\ 9\ 9\ 9\ 9\ 9\ 9\ 9\ 9 \\ \hline \end{array}$$

$$\begin{array}{r} \text{9.} \\ \text{Min. } 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ \text{Subtr. } \phantom{1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0} 9 \\ \hline \end{array}$$

$$\begin{array}{r} \text{10.} \\ \text{Min. } 9\ 0\ 8\ 0\ 7\ 0\ 6\ 0\ 5\ 0\ 4 \\ \text{Subtr. } 8\ 0\ 9\ 0\ 6\ 0\ 7\ 0\ 4\ 0\ 0 \\ \hline \end{array}$$

$$\begin{array}{r} \text{11.} \\ \text{Min. } 9\ 6\ 5\ 8\ 4\ 3\ 1\ 2\ 5 \\ \text{Subtr. } 4\ 2\ 8\ 6\ 4\ 2\ 0\ 8\ 6 \\ \hline \end{array}$$

$$\begin{array}{r} \text{12.} \\ 5\ 6\ 7\ 8\ 9\ 3\ 5\ 7\ 9\ 1\ 3 \\ 4\ 1\ 3\ 5\ 7\ 2\ 4\ 3\ 6\ 4\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} \text{13.} \\ 8\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 4\ 6 \\ 4\ 8\ 8\ 7\ 6\ 5\ 4\ 3\ 2\ 1\ 0\ 9 \\ \hline \end{array}$$

$$\begin{array}{r} \text{14.} \\ 7\ 5\ 3\ 9\ 7\ 5\ 3\ 1\ 6\ 8 \\ 3\ 6\ 4\ 0\ 8\ 5\ 4\ 2\ 7\ 9 \\ \hline \end{array}$$

$$\begin{array}{r} \text{15.} \\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 0\ 1\ 2 \\ 9\ 2\ 5\ 6\ 7\ 8\ 4\ 5\ 0\ 2\ 3 \\ \hline \end{array}$$

$$\begin{array}{r} \text{16.} \\ 5\ 6\ 7\ 8\ 5\ 3\ 5\ 3\ 4\ 7\ 9\ 6\ 3 \\ 4\ 7\ 5\ 6\ 2\ 4\ 6\ 1\ 2\ 5\ 7\ 8\ 7 \\ \hline \end{array}$$

$$\begin{array}{r} \text{17.} \\ 2\ 5\ 3\ 8\ 6\ 7\ 4\ 4\ 4\ 6\ 6 \\ 1\ 8\ 1\ 9\ 2\ 9\ 3\ 9\ 4\ 9\ 5 \\ \hline \end{array}$$

$$\begin{array}{r} \text{18.} \\ 6\ 6\ 6\ 6\ 8\ 8\ 8\ 8\ 4\ 4\ 4 \\ 5\ 9\ 9\ 9\ 9\ 9\ 9\ 9\ 9\ 9\ 9 \\ \hline \end{array}$$

$$\begin{array}{r} \text{19.} \\ 6\ 3\ 4\ 5\ 6\ 9\ 5\ 6\ 4\ 2\ 9 \\ 5\ 8\ 6\ 8\ 6\ 7\ 2\ 1\ 8\ 9\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} \text{20.} \\ 9\ 8\ 8\ 8\ 8\ 8\ 8\ 8\ 8\ 8 \\ 8\ 9\ 9\ 9\ 9\ 9\ 9\ 9\ 9\ 9 \\ \hline \end{array}$$

$$\begin{array}{r} \text{21.} \\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1 \\ 8\ 8\ 8\ 8\ 8\ 8\ 8\ 8\ 8\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} \text{22.} \\ 6\ 6\ 6\ 6\ 6\ 6\ 6\ 6\ 6\ 6 \\ 5\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7\ 7 \\ \hline \end{array}$$

## APPLICATION.

23. A was born in 1679; how old was he in 1777? *Ans.* 98 years.

24. From 1600000 take 900000, and from the remainder take 699999, and how much will remain? *Ans.* 1.



25. A man has two flocks of sheep; in the one there are 693, and in the other, 499; what is the difference in these flocks? *Ans.* 194.

26. A man has in his possession property to the amount of \$15728, and he owes \$7869; how much will remain in his hands, when his debts are paid? *Ans.* \$7859.

27. America was discovered in 1492. How long will it have been discovered in 1846? *Ans.* 354 years.

28. A man being asked how old he was when his eldest son was born, said that his own age was 79 years, and his son's 42 years; what was his age at the birth of his son? *Ans.* 37 years.

29. The amount of A's debts was 2356 dollars; the amount of his property, 5672 dollars. How much had he left after his debts were paid? *Ans.* \$3316.

30. A merchant bought a quantity of cloth for \$572, and sold it for \$526. Did he gain or lose, and how much? *Ans.* Lost \$46.

31. To what number must I add 576 to make the amount 1726? *Ans.* 1150.

32. Bought cotton in one month to the value of \$572896; and sold the same for \$600027. How much did I gain? *Ans.* \$27131.

33. If the sum of two numbers be 2793, and one of those numbers, 1892, what is the other? *Ans.* 901.

34. A merchant bought 742 yards of cloth and sold all but 7 yards. How much did he sell? *Ans.* 735 yards.

35. A man paid 1182 dollars for a house, and sold the same for 1069 dollars. How much did he lose? *Ans.* \$113.

36. A farmer purchased a farm, for which, including the buildings, he paid \$6782; the buildings were worth \$2896; what was the value of the land? *Ans.* \$3886.

37. A person owed a merchant \$999, and paid him all but \$179. How much did he pay him? *Ans.* \$820.

*Sums requiring in their solution, the application of both Addition and Subtraction.*

38. I hold in my possession a note for \$560, on which there is due \$70 interest. On the back are two endorsements; one, \$320, and the other, \$260. What is now due? *Ans.* \$50.

39. There are \$1000 in four different purses; in the first there

are \$96; in the second, \$310; in the third, \$205. How many are there in the fourth? *Ans.* \$389.

40. Four men agreed to contribute for a benevolent object, as follows; the first, \$34; the second, \$50; the third, \$100; and the fourth, \$150. Three of them having paid, the sum amounted to \$234; which subscription was unpaid? *Ans.* The third.

41. A had 172 yards of cloth, of which he sold 57 to B, and 42 to C. How many yards were left? *Ans.* 73.

42. A man having \$3986, paid one debt of \$1997, and another of \$1089. How many dollars had he left? *Ans.* \$900.

43. A man at death left an estate of \$9876: In his will he gave to each of his three sons, \$1800; to his daughter, \$1500; and the remaining part he left to his wife. What was the wife's portion? *Ans.* \$2976.

44. A person commencing business found that he had \$790 in money; in goods, \$1260; he held also three notes of \$150 each. After trading six years, he retired from business, and found that his property amounted to \$6000. How much had he gained by trading? *Ans.* \$3500.

45. Bought four chests of tea, weighing 72, 79, 83, and 87 pounds. From these I sold, to one man, 46, to another, 95, and to a third, 113 pounds. How much tea had I remaining? *Ans.* 67 pounds.

46. A man owed for his farm, \$2100; for house furniture, \$156; for a horse, \$96; for a yoke of oxen, \$120; for a flock of sheep, \$86. In one year he sold from his farm grain to the value of \$462; butter and cheese to the value of \$156; stock to the amount of \$320. How much did he owe at the end of the year? *Ans.* \$1620.

47. A man received \$7000 as a legacy; he was previously worth \$8560; he then commenced traveling, and in 7 years he spent \$9873. How much was he then worth? *Ans.* \$5687.

48. A man being asked how old he was, replied, that he married at 21 years of age, and that in 19 years more he should have been married 60 years. How old was he? *Ans.* 62 years.

49. Bought 1000 pounds of coffee; from this quantity I sold at one time, 376 pounds; and at another, 512 pounds. How much had I remaining? *Ans.* 112 pounds.

50. A man bought two hogsheads of melasses, the one con-

taining 65 gallons, and the other 69 gallons; from the two he sold 112 gallons. How much had he left? *Ans.* 22 gallons.

**QUESTIONS.**—How does this rule compare with Addition? What are we taught in Subtraction? How many numbers are employed in this rule? What are they called? What is the object of the rule? What name is given to what is left after the operation? What is the rule for subtraction? What is to be done when the lower figure is the larger? When is 1 to be carried? Do you ever have more than one to carry in subtraction? *Ans.* We do not. How do you prove the work? How will you show that carrying one as directed, is equivalent to the ten borrowed?

## SIMPLE MULTIPLICATION.

The rule to which the scholar's attention will now be directed, is one by which a number is produced from two given numbers, which shall contain either of these given numbers as many times as there are units in the other; or, it is the repeating of one number as many times as there are units in the other. For example, let 8 and 4 be the numbers; that is, let 8 be repeated 4 times. The result of these four repetitions of 8 is obviously 32. But 32 contains 8 four times, or 4 eight times; or, in other words, 32 contains either number as many times as there are units in the other. The scholar must first learn the following table:

### MULTIPLICATION TABLE.

|                    |                    |                    |                    |
|--------------------|--------------------|--------------------|--------------------|
| $1 \times 1 = 1$   | $2 \times 1 = 2$   | $3 \times 1 = 3$   | $4 \times 1 = 4$   |
| $1 \times 2 = 2$   | $2 \times 2 = 4$   | $3 \times 2 = 6$   | $4 \times 2 = 8$   |
| $1 \times 3 = 3$   | $2 \times 3 = 6$   | $3 \times 3 = 9$   | $4 \times 3 = 12$  |
| $1 \times 4 = 4$   | $2 \times 4 = 8$   | $3 \times 4 = 12$  | $4 \times 4 = 16$  |
| $1 \times 5 = 5$   | $2 \times 5 = 10$  | $3 \times 5 = 15$  | $4 \times 5 = 20$  |
| $1 \times 6 = 6$   | $2 \times 6 = 12$  | $3 \times 6 = 18$  | $4 \times 6 = 24$  |
| $1 \times 7 = 7$   | $2 \times 7 = 14$  | $3 \times 7 = 21$  | $4 \times 7 = 28$  |
| $1 \times 8 = 8$   | $2 \times 8 = 16$  | $3 \times 8 = 24$  | $4 \times 8 = 32$  |
| $1 \times 9 = 9$   | $2 \times 9 = 18$  | $3 \times 9 = 27$  | $4 \times 9 = 36$  |
| $1 \times 10 = 10$ | $2 \times 10 = 20$ | $3 \times 10 = 30$ | $4 \times 10 = 40$ |
| $1 \times 11 = 11$ | $2 \times 11 = 22$ | $3 \times 11 = 33$ | $4 \times 11 = 44$ |
| $1 \times 12 = 12$ | $2 \times 12 = 24$ | $3 \times 12 = 36$ | $4 \times 12 = 48$ |

|                    |                    |                    |                    |
|--------------------|--------------------|--------------------|--------------------|
| $5 \times 1 = 5$   | $6 \times 1 = 6$   | $7 \times 1 = 7$   | $8 \times 1 = 8$   |
| $5 \times 2 = 10$  | $6 \times 2 = 12$  | $7 \times 2 = 14$  | $8 \times 2 = 16$  |
| $5 \times 3 = 15$  | $6 \times 3 = 18$  | $7 \times 3 = 21$  | $8 \times 3 = 24$  |
| $5 \times 4 = 20$  | $6 \times 4 = 24$  | $7 \times 4 = 28$  | $8 \times 4 = 32$  |
| $5 \times 5 = 25$  | $6 \times 5 = 30$  | $7 \times 5 = 35$  | $8 \times 5 = 40$  |
| $5 \times 6 = 30$  | $6 \times 6 = 36$  | $7 \times 6 = 42$  | $8 \times 6 = 48$  |
| $5 \times 7 = 35$  | $6 \times 7 = 42$  | $7 \times 7 = 49$  | $8 \times 7 = 56$  |
| $5 \times 8 = 40$  | $6 \times 8 = 48$  | $7 \times 8 = 56$  | $8 \times 8 = 64$  |
| $5 \times 9 = 45$  | $6 \times 9 = 54$  | $7 \times 9 = 63$  | $8 \times 9 = 72$  |
| $5 \times 10 = 50$ | $6 \times 10 = 60$ | $7 \times 10 = 70$ | $8 \times 10 = 80$ |
| $5 \times 11 = 55$ | $6 \times 11 = 66$ | $7 \times 11 = 77$ | $8 \times 11 = 88$ |
| $5 \times 12 = 60$ | $6 \times 12 = 72$ | $7 \times 12 = 84$ | $8 \times 12 = 96$ |

|                     |                      |                      |                      |
|---------------------|----------------------|----------------------|----------------------|
| $9 \times 1 = 9$    | $10 \times 1 = 10$   | $11 \times 1 = 11$   | $12 \times 1 = 12$   |
| $9 \times 2 = 18$   | $10 \times 2 = 20$   | $11 \times 2 = 22$   | $12 \times 2 = 24$   |
| $9 \times 3 = 27$   | $10 \times 3 = 30$   | $11 \times 3 = 33$   | $12 \times 3 = 36$   |
| $9 \times 4 = 36$   | $10 \times 4 = 40$   | $11 \times 4 = 44$   | $12 \times 4 = 48$   |
| $9 \times 5 = 45$   | $10 \times 5 = 50$   | $11 \times 5 = 55$   | $12 \times 5 = 60$   |
| $9 \times 6 = 54$   | $10 \times 6 = 60$   | $11 \times 6 = 66$   | $12 \times 6 = 72$   |
| $9 \times 7 = 63$   | $10 \times 7 = 70$   | $11 \times 7 = 77$   | $12 \times 7 = 84$   |
| $9 \times 8 = 72$   | $10 \times 8 = 80$   | $11 \times 8 = 88$   | $12 \times 8 = 96$   |
| $9 \times 9 = 81$   | $10 \times 9 = 90$   | $11 \times 9 = 99$   | $12 \times 9 = 108$  |
| $9 \times 10 = 90$  | $10 \times 10 = 100$ | $11 \times 10 = 110$ | $12 \times 10 = 120$ |
| $9 \times 11 = 99$  | $10 \times 11 = 110$ | $11 \times 11 = 121$ | $12 \times 11 = 132$ |
| $9 \times 12 = 108$ | $10 \times 12 = 120$ | $11 \times 12 = 132$ | $12 \times 12 = 144$ |

## MENTAL EXERCISES IN MULTIPLICATION.

Bought 2 apples for 3 cents apiece; what did they cost?  $2 \times 3 =$  how many? Gave to each of 3 boys, 4 plums; how many did I give away?  $3 \times 4 =$  how many? What cost 4 oranges at 5 cents apiece?  $4 \times 5 =$  how many? What cost 6 oranges at 5 cents apiece?  $6 \times 5 =$  how many? What cost 7 quarts of cherries at 6 cents a quart?  $7 \times 6 =$  how many? 8 quarts of molasses at 7 cents per quart?  $8 \times 7 =$  how many? 9 picture books at 8 cents apiece?  $9 \times 8 =$  how many? What cost 10 pints of wine at 9 cents a pint?  $10 \times 9 =$  how many? 11 yards of muslin at 10 cents a yard?  $11 \times 10 =$  how many? 12 yards at 11 cents a yard? What cost 12 loads of straw at 5 dollars a load?  $12 \times 5 =$  how many? at 9 dollars a load? 7 dollars? 6 dollars? 8 dollars? 10 dollars? 12 dollars? 11 dollars? What will it cost to ride 6 miles at 5 cents a mile? What to ride 7 miles? 9 miles? 4 miles? 5 miles? 8 miles? 10 miles? 11 miles? 12 miles? Six men paid a poor man 6

dollars each; how much did the poor man receive? How much would he have received had they paid 9 dollars each? 7 dollars each? 5 dollars each? 10 dollars each? 12 dollars each? 7 men purchased a horse in company and paid 9 dollars each; what did they pay for the horse? What would the horse have cost them, had they paid only 7 dollars each? If they had paid 8 each? 11? 10? 12? 6? If a man travel 5 miles in 1 hour, how far will he travel in 7 hours? in 5 hours? in 9 hours? in 6 hours? in 8 hours? in 10 hours? in 11 hours? What is the product of 6 multiplied by 11? by 5? by 9? by 7? by 8? by 6? by 10? by 11? by 12? What is the product of 9 multiplied by 3? by 12? by 4? by 11? by 5? by 10? by 6? by 9? by 7? by 8? Of 12 multiplied by 7? by 2? by 12? by 3? by 11? by 4? by 10? by 5? by 9? by 6? by 8? Of 11 multiplied by 11? by 7? by 8? by 6? by 5? by 9? by 10? by 12?

If a man earn 9 dollars a month, what will he earn in 8 months? If the board of a man and his wife be 7 dollars per week, what will it amount to in 11 weeks? If one load of hay cost 11 dollars, what will 12 loads cost? If a yard of ribbon cost 8 cents, what will 11 yards cost?

It has already been said that two numbers are required for the operation, viz. the *multiplicand*, or number to be multiplied or repeated; and the *multiplier*, or number showing how many times the multiplicand is to be taken or repeated. The multiplier and multiplicand, when spoken of together, are called *factors*. The number obtained by the operation is called the *product*. A short illustration will show this rule to be an abbreviation of addition. Suppose it be required to multiply 5 by 4. If the scholar turn to his table he will find the product of these two numbers is 20. The same result is obtained if four 5's be added together; thus,  $5+5+5+5=20$ . It will then be perceived that if one of the two numbers which are to be multiplied together, be written down as many times as there are units in the other, and these several numbers be then added, we obtain the same result as when these two numbers are multiplied together. Multiplication is therefore a short method of performing addition.

It is highly important that the scholar should obtain accurate views of the value of the product arising from the multiplication of any two numbers. There will be no difficulty in this respect, when it is required to multiply unit figures only, or when a

unit figure only is given as a multiplier; for then the product of each figure will be of the same denomination as the figure itself. But when the two factors consist each of several figures, so that tens are to be multiplied by tens, and hundreds by hundreds, the scholar will not so readily comprehend the nature of the operation. He must however remember, that when his multiplying figure is tens, it will raise the value of the product of each figure in the multiplicand one degree; when it is hundreds, it will raise the value of each two degrees; and when thousands, three degrees, &c. Let the scholar carefully notice what is here stated. If tens in the multiplier be multiplied into units in the multiplicand, the product is tens; if into tens, the product is hundreds. If hundreds in the multiplier be multiplied into units in the multiplicand, the product is hundreds; if into tens, the product is thousands; and if into hundreds, the product is tens of thousands, &c. This explanation will enable the scholar to understand the rule under Case 3d, relative to writing down the several products. He will readily perceive it to be nothing more or less than writing units under units, and tens under tens, &c.

CASE 1st.—WHEN THE MULTIPLIER DOES NOT EXCEED 12.

RULE.—*Commence at the right hand and multiply each figure in the multiplicand by the multiplier, carrying and setting down as in the preceding rules.*

Ex. 1. Multiply 452 by 3.

OPERATION.

$$\begin{array}{r} 452 \\ 3 \end{array}$$

Prod. 1 3 5 6    I first say, 3 times 2 are 6, which, being less than 10, I set down; next, 3 times 5 are 15, which is 5 units and 1 ten; the units I set down and carry the ten; thus, 3 times 4 are 12 and one to carry are 13. I thus find the whole product to be 1356. This number must consequently contain the multiplicand 3 times, and may be obtained by adding together three 452's; thus,  $\begin{array}{r} 452 \\ 452 \\ 452 \end{array}$  The same result is therefore obtained by multiplication as by addition, but more expeditiously. The operation is proved by dividing the product by the multiplier, which, if the work be correct, will give  $\begin{array}{r} 1356 \\ 3 \end{array}$  the multiplicand. The scholar is not, however, supposed yet to

## SIMPLE MULTIPLICATION.

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understand division, and will not be required to prove his work, till more advanced.

2. Multiply 6432 by 4.

OPERATION.

6 4 3 2

4

2 5 7 2 8 = Prod.

3. Multiply 123456 by 6.

OPERATION.

1 2 3 4 5 6

6

7 4 0 7 3 6 = Prod.

4. Multiply 234567 by 8.

OPERATION.

2 3 4 5 6 7

8

1 8 7 6 5 3 6 = Prod.

5. Multiply 345678 by 10. *Prod.* 3456780.  
 6. Multiply 456789 by 12. *Prod.* 5481468.  
 7. Multiply 729468 by 5. *Prod.* 3647340.  
 8. Multiply 295105538 by 7. *Prod.* 2065738766.  
 9. Multiply 4285637 by 9. *Prod.* 38570733.  
 10. Multiply 462838 by 11. *Prod.* 5091218.  
 11. Multiply 99887766 by 9. *Prod.* 898989894.  
 12. Multiply 765987879 by 7. *Prod.* 5361915153.  
 13. Multiply 9864579 by 8. *Prod.* 78916632.  
 14. Multiply 7799886655 by 12. *Prod.* 93598639860.

## APPLICATION.

15. Bought 72 yards of cloth for three dollars per yard. What did it cost? *Ans.* \$216.

16. Sold 137 sheep at 5 dollars per head. How much did I receive? *Ans.* \$685.

17. Employed 196 men one week, for \$8 per week. How much did I pay them all? *Ans.* \$1568.

18. How many miles will a man travel in 297 days, if he travel 12 miles a day? *Ans.* 3564.

19. If I take 11 steps in one minute, how many steps shall I take in 2 hours 56 minutes, or 176 minutes? *Ans.* 1936 steps.

20. If a horse trot 7 miles in one hour, how far will he trot in 76 hours? *Ans.* 532 miles.

**CASE 2d.—WHEN THE MULTIPLIER IS A COMPOSITE NUMBER.**

**NOTE.**—A composite number is one which can be produced by the multiplication of two or more small numbers; thus, 6 and 3 are the component parts of 18; because  $6 \times 3 = 18$ .

**RULE.**—*Multiply first by one of the component parts of the multiplier, and this product by the other component part; the last product will be the one sought.*

**Ex. 1.** Multiply 4568 by 24. The component parts of 24 are 6 and 4, therefore,

$$\begin{array}{r}
 4568 \\
 \times 6 \\
 \hline
 27408 = \text{six times the multiplicand.} \\
 \times 4 \\
 \hline
 109632 = \text{24 times the multiplicand.}
 \end{array}$$

**Ex. 2.** Multiply 459684 by 36.  $36 = 4 \times 9$ , therefore,

$$\begin{array}{r}
 459684 \\
 \times 9 \\
 \hline
 4137156 = \text{9 times the multiplicand.} \\
 \times 4 \\
 \hline
 16548524 = \text{36 times the multiplicand.}
 \end{array}$$

It is immaterial in what order the component parts are taken.

- |                              |                           |
|------------------------------|---------------------------|
| 3. Multiply 5634286 by 18.   | <i>Prod.</i> 101417148.   |
| 4. Multiply 4327648 by 27.   | <i>Prod.</i> 116846496.   |
| 5. Multiply 7295678 by 36.   | <i>Prod.</i> 262644408.   |
| 6. Multiply 4639546 by 48.   | <i>Prod.</i> 222698208.   |
| 7. Multiply 3695475 by 42.   | <i>Prod.</i> 155209950.   |
| 8. Multiply 54639578 by 60.  | <i>Prod.</i> 3278374680.  |
| 9. Multiply 578016937 by 96. | <i>Prod.</i> 55489625952. |
| 10. Multiply 79875643 by 63. | <i>Prod.</i> 5032165509.  |

**APPLICATION.**

11. Bought 75 tons of hay at \$15 per ton. What did the whole cost? *Ans.* \$1125.
12. How many hours are there in 76 days? *Ans.* 1824.
13. How many minutes are there in 49 hours? *Ans.* 2940.



14. How many days are there in 21 years? *Ans.* 7665.  
 15. What cost 172 acres of land at \$36 per acre? *Ans.* 6192.  
 16. What will 876 pounds of coffee cost at 28 cents per pound. *Ans.* 24528 cts.

**CASE 3d.—WHEN THE MULTIPLIER EXCEEDS 12, AND IS NOT A COMPOSITE NUMBER.**

**RULE.**—Multiply each figure in the multiplicand by each figure in the multiplier separately, commencing with the right hand figure of each, and set down the first figure of each product directly under the multiplying figure. After each figure in the multiplier has been taken, add together the several products; the amount will be the required product.

**Ex. 1.** 342635 by 125.

**OPERATION.**

3 4 2 6 3 5  
 1 2 5

1 7 1 3 1 7 5=Product of 5 units.

6 8 5 2 7 0=Product of 2 tens removed 1 place to the left.

3 4 2 6 3 5=Product of 1 hundred, two places to the left.

4 2 8 2 9 3 7 5=Product of 125.

**Ex. 2.** Multiply 167498 by 231.

**OPERATION.**

1 6 7 4 9 8  
 2 3 1

1 6 7 4 9 8=Product of 1 unit.

5 0 2 4 9 4=Prod. of 3 tens removed one place to the left.

3 3 4 9 9 6=Prod. of 2 hundreds, two places to the left.

3 8 6 9 2 0 3 8=Prod. of 231.

**3. Multiply 36598674 by 432.**

**PERFORMED.**

3 6 5 9 8 6 7 4  
 4 3 2

7 3 1 9 7 3 4 8

1 0 9 7 9 6 0 2 2

1 4 6 3 9 4 6 9 6

1 5 8 1 0 6 2 7 1 6 8=Prod.

4. Multiply 46354897816 by 56848. *Prod.* 26349514565-54888.

5. Multiply 378199886432 by 42395. *Prod.* 1603378418-5284640.

6. Multiply 85698436946 by 46743. *Prod.* 400580203816-6878.

7. Multiply 6739542 by 346. *Prod.* 2331881532.

8. Multiply 72926495 by 4567. *Prod.* 333055302665.

9. Multiply 89764267 by 999. *Prod.* 89674502733.

10. Multiply 46371674 by 49684. *Prod.* 2303930251016.

11. Multiply 8429638 by 7294. *Prod.* 61485779572.

12. Multiply 7364951 by 888. *Prod.* 6540076488.

13. There are 69 pieces of cloth containing each 112 yards; how many yards are there in all? *Ans.* 7728.

14. Suppose a man travel by steam 21900 miles in a year; how far will he travel in 67 years? *Ans.* 1467300 miles.

15. In a volume of 675 pages, each page containing 156 lines, and each line 136 letters; how many letters? *Ans.* 14320800.

16. How many hills are there in a field of corn, containing 149 rows, with 96 hills in a row? *Ans.* 14304.

17. On the preceding supposition, how many ears of corn are there in the field, allowing the average to be 9 to a hill? and how many kernels of corn, allowing 300 to an ear? *Ans.* 128736 ears of corn, and 38620800 kernels.

**CASE 4th.**—WHEN THERE ARE CYPHERS ON THE RIGHT HAND OF THE MULTIPLIER, OR MULTIPLICAND, OR BOTH.

**RULE.**—*Omit the cyphers and multiply by the significant figures only, and annex to the right hand of the product as many cyphers as were omitted.*

**Ex. 1.** Multiply 2100 by 70.

PERFORMED.

2100

70

————— I multiply the 21 by the 7 only,  
and then annex 3 cyphers to 147,

147000 the product.

2. Multiply 47600 by 150. *Prod.* 7140000.

3. Multiply 9560 by 1200. *Prod.* 11472000.

4. Multiply 462000 by 190. *Prod.* 87780000.  
 5. Multiply 760 by 1000. *Prod.* 760000.

**CASE 5th.**—WHEN THE MULTIPLIER IS ANY NUMBER BETWEEN 11 AND 19 INCLUSIVE.

**RULE.**—Multiply by the right hand figure only, and place the product under the multiplicand one place to the right; then add it to the multiplicand. The sum will be the true product.

**Ex. 1.** Multiply 468 by 15.

PERFORMED.

$$468 \times 5$$

$$2340 = \text{Product of } 468 \text{ multiplied by } 5.$$

$$7020 = \text{Product of } 468 \text{ multiplied by } 15.$$

The reason of this rule is plain. Were we to multiply in the ordinary mode, the only difference would be, that the number 2340 would stand above 468, instead of below it.

**Ex. 2.** Multiply 37464 by 17.

PERFORMED.

$$37464 \times 7$$

$$62248$$

$$636888 = \text{Product.}$$

3. Multiply 65328 by 13. *Prod.* 849264.  
 4. Multiply 23456789 by 14. *Prod.* 328395046.  
 5. Multiply 65432 by 15. *Prod.* 981480.  
 6. Multiply 123456 by 16. *Prod.* 1975296.  
 7. Multiply 437426 by 17. *Prod.* 7436242.  
 8. Multiply 653842 by 18. *Prod.* 11769156.  
 9. Multiply 603040 by 19. *Prod.* 11457760.  
 10. Multiply 999999 by 11. *Prod.* 10999989.

**CASE 6th.**—WHEN THE MULTIPLIER IS EITHER 21, 31, 41, 51, 61, 71, 81, OR 91.

**RULE.**—Multiply by the LEFT hand figure only, and place the product under the multiplicand one place to the left.

**Ex. 1.** Multiply 634982 by 21.

## PERFORMED.

$$634982 \times 2$$

1269964 = Product of 2 tens one place to the left.

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$$13334622 = \text{Product of } 21.$$

2. Multiply 9382716 by 31. *Prod.* 290864196.
3. Multiply 1234567 by 41. *Prod.* 50617247.
4. Multiply 4364369 by 51. *Prod.* 222582819.
5. Multiply 6937845 by 61. *Prod.* 423208545.
6. Multiply 364812 by 71. *Prod.* 25901652.
7. Multiply 482436 by 81. *Prod.* 39077316.
8. Multiply 2468 by 91. *Prod.* 224588.

*Note.*—When in either of the two preceding cases, cyphers intervene between the figures, the same mode of operation may be adopted, if care be taken to give each figure its true place.

Ex. 1. Multiply 6456 by 105.

## PERFORMED.

$$6456 \times 5$$

32280 = Product of the 5 placed two degrees to the right.

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$$677880 = \text{Product of } 105.$$

2. Multiply 37562 by 601.

## PERFORMED.

$$37562 \times 6$$

225372 = Product of 6 hundred placed two degrees to the left.

---


$$22574762 = \text{Product of } 601.$$

3. Multiply 695378 by 5001. *Prod.* 3477585378.

4. Multiply 2579678 by 1007. *Prod.* 2597735746.

## APPLICATION.

1. Bought 52 horses at \$75 each; what did they cost?  
*Ans.* \$3900.

2. What cost 84 tons of hay at \$15 per ton? *Ans.* \$1260.

3. If a man can travel 43 miles in one day, how far can he travel in 60 days? *Ans.* 2580 miles.

4. There are 144 square inches in one square foot. How many square inches are there in 67 square feet? *Ans.* 9648.

5. If there be 18 panes of glass in one window, how many are there in a house which has 56 such windows? *Ans.* 1008.

6. Bought 342 bales of linen, each containing 56 pieces of 25 yards each. How many yards did I buy? *Ans.* 478800.

7. There is an orchard consisting of 126 rows of trees, and in each row there are 109 trees. How many apples are there in the orchard, allowing an average of 1007 on a tree? *Ans.* 12830138.

8. A certain State contains 50 Counties; each County, 35 towns; each town, 300 houses, and each house, 8 persons. What is the population of the State? *Ans.* 4200000.

**QUESTIONS.**—What is the nature of Multiplication? How many numbers are employed in the operation? What are they called, and what is peculiar to each? What is the number obtained called? Of what rule is multiplication an abbreviation? Illustrate. What are the multiplicand and multiplier called when spoken of together? What is the value of each figure in the product when you multiply by a unit figure only? Units multiplied by units give what? Units by tens? Units by hundreds? When the multiplying figure is tens, what effect will it have on the value of the product of each figure in the multiplicand? and what will be the effect if the multiplying figure be hundreds? Give farther illustration of the value of the product figures. What is case 1st? What is the rule? Case 2d? The rule? Case 3d? The rule? What is a composite number? What are the component parts of a number? Case 4th? The rule? Case 5th? The rule? Case 6th? The rule?

## SIMPLE DIVISION.

We now come to the reverse of the preceding rule. There we had two factors given to find their product. Here we have given the product, or what corresponds to it, and one of the factors, and are required to obtain the other factor. Multiplication; as was shown, could be performed by repeated *additions*; Division may be performed by repeated *subtractions*. Suppose it be required to ascertain how many times 4 is contained in 12. It may be done by taking 4 from 12, till nothing remains, or till a number less than 4 remains. Thus,

$$\begin{array}{r}
 12 \\
 4 \\
 \hline
 8 = 12 - 4. \\
 4 \\
 \hline
 4 = 12 - 4 + 4. \\
 4 \\
 \hline
 0 = 12 - 4 + 4 + 4.
 \end{array}$$

The operation shows three 4's may be taken from 12. 4 is therefore contained in 12 three times. This is, however, a slow mode of operation. A more expeditious one must be sought; and, preparatory for it, the scholar is required to learn the following table:

## DIVISION TABLE.

|                  |                  |                  |                  |
|------------------|------------------|------------------|------------------|
| $2 \div 2 = 1$   | $3 \div 3 = 1$   | $4 \div 4 = 1$   | $5 \div 5 = 1$   |
| $4 \div 2 = 2$   | $6 \div 3 = 2$   | $8 \div 4 = 2$   | $10 \div 5 = 2$  |
| $6 \div 2 = 3$   | $9 \div 3 = 3$   | $12 \div 4 = 3$  | $15 \div 5 = 3$  |
| $8 \div 2 = 4$   | $12 \div 3 = 4$  | $16 \div 4 = 4$  | $20 \div 5 = 4$  |
| $10 \div 2 = 5$  | $15 \div 3 = 5$  | $20 \div 4 = 5$  | $25 \div 5 = 5$  |
| $12 \div 2 = 6$  | $18 \div 3 = 6$  | $24 \div 4 = 6$  | $30 \div 5 = 6$  |
| $14 \div 2 = 7$  | $21 \div 3 = 7$  | $28 \div 4 = 7$  | $35 \div 5 = 7$  |
| $16 \div 2 = 8$  | $24 \div 3 = 8$  | $32 \div 4 = 8$  | $40 \div 5 = 8$  |
| $18 \div 2 = 9$  | $27 \div 3 = 9$  | $36 \div 4 = 9$  | $45 \div 5 = 9$  |
| $20 \div 2 = 10$ | $30 \div 3 = 10$ | $40 \div 4 = 10$ | $50 \div 5 = 10$ |
| $22 \div 2 = 11$ | $33 \div 3 = 11$ | $44 \div 4 = 11$ | $55 \div 5 = 11$ |
| $24 \div 2 = 12$ | $36 \div 3 = 12$ | $48 \div 4 = 12$ | $60 \div 5 = 12$ |

|                  |                  |                  |                   |
|------------------|------------------|------------------|-------------------|
| $6 \div 6 = 1$   | $7 \div 7 = 1$   | $8 \div 8 = 1$   | $9 \div 9 = 1$    |
| $12 \div 6 = 2$  | $14 \div 7 = 2$  | $16 \div 8 = 2$  | $18 \div 9 = 2$   |
| $18 \div 6 = 3$  | $21 \div 7 = 3$  | $24 \div 8 = 3$  | $27 \div 9 = 3$   |
| $24 \div 6 = 4$  | $28 \div 7 = 4$  | $32 \div 8 = 4$  | $36 \div 9 = 4$   |
| $30 \div 6 = 5$  | $35 \div 7 = 5$  | $40 \div 8 = 5$  | $45 \div 9 = 5$   |
| $36 \div 6 = 6$  | $42 \div 7 = 6$  | $48 \div 8 = 6$  | $54 \div 9 = 6$   |
| $42 \div 6 = 7$  | $49 \div 7 = 7$  | $56 \div 8 = 7$  | $63 \div 9 = 7$   |
| $48 \div 6 = 8$  | $56 \div 7 = 8$  | $64 \div 8 = 8$  | $72 \div 9 = 8$   |
| $54 \div 6 = 9$  | $63 \div 7 = 9$  | $72 \div 8 = 9$  | $81 \div 9 = 9$   |
| $60 \div 6 = 10$ | $70 \div 7 = 10$ | $80 \div 8 = 10$ | $90 \div 9 = 10$  |
| $66 \div 6 = 11$ | $77 \div 7 = 11$ | $88 \div 8 = 11$ | $99 \div 9 = 11$  |
| $72 \div 6 = 12$ | $84 \div 7 = 12$ | $96 \div 8 = 12$ | $108 \div 9 = 12$ |

|                    |                    |                    |
|--------------------|--------------------|--------------------|
| $10 \div 10 = 1$   | $11 \div 11 = 1$   | $12 \div 12 = 1$   |
| $20 \div 10 = 2$   | $22 \div 11 = 2$   | $24 \div 12 = 2$   |
| $30 \div 10 = 3$   | $33 \div 11 = 3$   | $36 \div 12 = 3$   |
| $40 \div 10 = 4$   | $44 \div 11 = 4$   | $48 \div 12 = 4$   |
| $50 \div 10 = 5$   | $55 \div 11 = 5$   | $60 \div 12 = 5$   |
| $60 \div 10 = 6$   | $66 \div 11 = 6$   | $72 \div 12 = 6$   |
| $70 \div 10 = 7$   | $77 \div 11 = 7$   | $84 \div 12 = 7$   |
| $80 \div 10 = 8$   | $88 \div 11 = 8$   | $96 \div 12 = 8$   |
| $90 \div 10 = 9$   | $99 \div 11 = 9$   | $108 \div 12 = 9$  |
| $100 \div 10 = 10$ | $110 \div 11 = 10$ | $120 \div 12 = 10$ |
| $110 \div 10 = 11$ | $121 \div 11 = 11$ | $132 \div 12 = 11$ |
| $120 \div 10 = 12$ | $132 \div 11 = 12$ | $144 \div 12 = 12$ |

## MENTAL EXERCISES IN DIVISION.

Bought 4 apples for 8 cents; what was the price of one?  $8 \div 4 =$  how many? Bought 3 oranges for 12 cents; what was the price of one?  $12 \div 3 =$  how many? At another time bought 5 for 15 cents; what was the price?  $15 \div 5 =$  how many? Paid 24 cents for 6 peaches; what did one cost?  $24 \div 6 =$  how many? What is the cost of one orange when 8 cost 24 cents? when 4 cost 24 cents? when 3 cost 24 cents?  $24 \div 8 =$  how many?  $24 \div 4 =$  how many?  $24 \div 3 =$  how many? Paid 35 cents for 7 melons; what was the value of one? Paid the same money for 5; what was the value of one?  $35 \div 7 =$  how many?  $35 \div 5 =$  how many? Bought 8 silks for 16 cents; what was one worth? What is the value of one when I pay 24 cents for 8? when I pay 32 cents? 40 cents? 48 cents? 56 cents?  $16 \div 8 =$  how many?  $24 \div 8 =$  how many?  $32 \div 8 =$  how many?  $40 \div 8 =$  how many?  $48 \div 8 =$  how many?  $56 \div 8 =$  how many?  $64 \div 8 =$  how many?  $72 \div 8 =$  how many? Paid 18 shillings for 9 yards of calico; what was the price per yard? What is the price per yard when the same quantity costs 27 shillings? 45 shillings? 36 shillings? 72 shillings? 63 shillings?  $18 \div 9 =$  how many?  $27 \div 9 =$  how many?  $45 \div 9 =$  how many?  $36 \div 9 =$  how many?  $72 \div 9 =$  how many?  $63 \div 9 =$  how many? What is the cost of one lemon, if 10 cost 30 cents? 70 cents? 60 cents? 50 cents? 80 cents? 40 cents? 90 cents?  $30 \div 10 =$  how many? &c. What is the cost of one sheep, when 8 cost 24 dollars? What, if 8 cost 64 dollars? 88 dollars? 72 dollars? 48 dollars? 80 dollars? 40 dollars? 32 dollars?  $24 \div 8 =$  how many?  $64 \div 8 =$  how many? &c.

What is the value of one bushel of wheat, when 12 bushels are worth 24 dollars? What, when the 12 are worth 36 dollars? 72 dollars? 96 dollars? 48 dollars? 108 dollars? 60 dollars? 132 dollars? 84 dollars? 120 dollars? 144 dollars?  $24 \div 12 =$  how many?  $36 \div 12 =$  how many? &c. Bought 8 pounds of raisins for 72 cents; what was the price per pound? If 9 pounds of rice cost 81 cents, what will one pound cost? A man traveled 96 miles in 8 days; how far was that per day? A horse ran 54 miles in 6 hours; how far was that per hour? How many pounds of sugar at 9 pence per lb. can be bought for 54 pence? for 63 pence? for 81 pence? for 108 pence? for 99 pence? for 72 pence? for 45 pence? for 36 pence?

The scholar must now be prepared to apply what he has learnt from the preceding table and questions, to division on a more extensive scale. He will have noticed that for each operation *two numbers* are given; viz. a number to be divided, which is called the *dividend*; and a number by which to divide, called the *divisor*. The number obtained is called the *quotient*; a word which signifies, *how many*; because this number always determines how many times the divisor is contained in the dividend. After the operation is performed there is frequently a number left. This is called the *remainder*, and is always less than the divisor. When the division is performed, if there be no remainder, the quotient multiplied by the divisor will always produce the dividend; and if there be a remainder, the dividend will be produced by multiplying as before, and adding the remainder to the product. Hence division is proved by multiplication. The scholar will readily perceive that these two rules are the reverse of each other.

The operations in division will be illustrated under two general heads; viz. Short Division, and Long Division.

### I. SHORT DIVISION.

When the divisor does not exceed 12, the process is abbreviated by keeping the computation in the mind, and writing down only the quotient figures.

**RULE.**—1st. Write down the dividend, and place the divisor on the left, with a curve line drawn between them.

2d. Take as many figures on the left of the dividend, as will contain the divisor once or more, and write the figure expressing the number of times, directly under those divided.

3d. If in dividing there be a remainder, imagine the next figure in the dividend to be placed on the right hand of it. This will form a new number, which may be divided as before. Continue the same process till all the figures of the dividend have been disposed of, and the number obtained will be the quotient required.

4th. If in taking any figure of the dividend the number produced be not sufficient to contain the divisor once, a cypher must be placed in the quotient, and another figure of the dividend taken.



Ex. 1. Divide 496 by 4.

PERFORMED.

$$\begin{array}{r} 4 \overline{) 496} \\ \hline \end{array}$$

124 = quotient.

I first say 4 is in 4 once, and place the 1 as directed by the rule. I next say 4 is in 9 twice and 1 remains; I write down the 2 as directed, and on the right hand of the 1 I place the 6, and thus obtain 16. Lastly, I say, 4 is in 16 four times. This operation gives 124 as the number of times which 496 contains 4. Now the scholar will readily perceive, that this is reversing a process of multiplication. If he multiply 124 by 4, he will obtain 16 units, 8 tens, and 4 hundreds; and these are precisely the numbers divided. But the 16 units are equal to 1 ten and 6 units; the whole is, therefore, equal to 4 hundreds, 9 tens, and 6 units; or to 496.

This multiplication is the proof of the work.

Ex. 2. Divide 1512 by 7.

PERFORMED.

$$\begin{array}{r} 7 \overline{) 1512} \\ \hline \end{array}$$

216 = quotient.

The numbers here, as they are severally divided, are 15, 11, and 42. If 216 be multiplied by 7, it will produce the same numbers.

3. Divide 5463 by 3.

PERFORMED.

$$\begin{array}{r} 3 \overline{) 5463} \\ \hline \end{array}$$

1821

4. Divide 1256 by 2.

PERFORMED.

$$\begin{array}{r} 2 \overline{) 1256} \\ \hline \end{array}$$

628

5. Divide 63548 by 4. Quo. 15887.

6. Divide 256788 by 8. Quo. 32098 and 4 remains.

7. Divide 65342167 by 4 and by 5, and add the quotients.

Ans. 29403974 and 5 remains.

8. Divide 735649 by 5 and by 7, and add the quotients.  
*Ans.* 252221 and 9 remains.

9. Divide 456789 by 6 and by 8, and add the quotients.  
*Ans.* 133229 and 8 remains.

10. Divide 68890 by 7 and by 9, and add the quotients.  
*Ans.* 17495 and 7 remains.

11. Divide 78901 by 8 and by 10, and add the quotients.  
*Ans.* 17752 and 6 remains.

12. Divide 89012 by 9 and by 11, and add the quotients.  
*Ans.* 17982 and 2 remains.

13. Divide 90123456 by 10 and 12, and add the quotients.  
*Ans.* 16522633 and 6 remains.

14. Nine persons drew a prize of \$198; what was each one's share? *Ans.* \$22.

15. Paid \$750 for 30 cows; what was the average price?  
*Ans.* \$25.

16. A person dying leaves an estate of \$4500 to 9 children; what will be each one's share? *Ans.* \$500.

17. A man traveled 224 miles in 7 days; what was his daily progress? *Ans.* 32 miles.

18. If 12 ounces make a pound of silver; how many pounds are there in 2040 ounces? *Ans.* 170.

19. How many times may 12 be subtracted from 1416. *Ans.* 118.

20. Four persons boarded at a public house till the bill of their board was \$184; what was the average bill? *Ans.* 46.

When the divisor is more than 12 and is a composite number, the same mode of operation can be adopted.

**RULE 2d.**—*Divide first by one of the component parts, and the quotient arising from this division, by the other.*

The only difficulty which will here present itself, will be to ascertain the true remainder. The scholar needs only to remember, that if a remainder occur after the first division only, that is the true remainder. If after the second division only, the true remainder is obtained by multiplying this remainder by the first divisor. If there be a remainder after each division, the true remainder is found by multiplying the last remainder by the first divisor, and adding the first remainder.

**Ex. 1.** Divide 864 by 18. The component parts of 18 are 6 and 3, therefore,

$$6 \overline{) 864}$$

$$3 \overline{) 144} = \text{Quotient of } 864 \text{ divided by } 6.$$

4 8 = Quotient of 144 divided by 3. This is the true quotient of  $864 \div 18$ .

Ex. 2. Divide 162641 by 72. The component parts are 9 and 8, therefore,

$$9 \overline{) 162641}$$

$$8 \overline{) 18071} \text{ and 2 remainder.}$$

2 2 5 8 and 7 remainder. Therefore,  $7 \times 9 + 2 =$  the true remainder, viz. 65.

Ex. 3. Divide 793 by 18. The component parts are 6 and 3.

$$6 \overline{) 793}$$

$$3 \overline{) 132} \text{ and 1 remains.}$$

4 4 and nothing remains. Therefore, by rule 2d, 1 is the whole remainder.

Ex. 4. Divide 792 by 16. Component parts of 16 are 4 and 4.

$$4 \overline{) 792}$$

$$4 \overline{) 198} \text{ and nothing remains.}$$

4 9 and 2 remains ; therefore, by rule 2d,  $4 \times 2 = 8$ , the true remainder.

5. Divide 2592 by 63. Quo. 41, and 9 remains.
6. Divide 7776 by 108. Quo. 72.
7. Divide 6750 by 15. Quo. 450.
8. Divide 437639 by 42. Quo. 10419, and 41 remains.
9. Divide 738246 by 27. Quo. 27342, and 12 remains.
10. Divide 60400 by 25. Quo. 2416.
11. Divide 45678 by 16. Quo. 2854, and 14 remains.
12. Divide 4688 by 48. Quo. 97, and 32 remains.
13. Divide 347628 by 84. Quo. 4138, and 36 remains.

## II. LONG DIVISION.

WHEN THE DIVISOR EXCEEDS 12, AND IS NOT A COMPOSITE NUMBER.

**RULE.**—1st. Write down the dividend, and, drawing a curve line both on the right and left of it, place the divisor on the left.

2d. Find how many times the divisor is contained in the fewest figures that will contain it, taken from the left of the dividend; and place the figure expressing the number of times on the right of the dividend, as the first quotient figure.

3d. Multiply the divisor by this quotient figure, and place the product under the figures divided.

4th. Subtract, and to the remainder bring down the next figure of the dividend.

5th. Divide again and place the result as the second figure in the quotient.

6th. Continue the process of multiplying, subtracting, bringing down, &c. till all the figures have been divided.

7th. If after all the figures have been divided, there be still a remainder, place it as the numerator, and the divisor as the denominator of a fraction, on the right hand of the quotient.

*Note 1st.*—Whenever a figure has been placed on the right of the remainder, and the number produced will not contain the divisor, a cypher must be placed in the quotient.

*Note 2d.*—If the remainder, after subtracting, be greater than the divisor, the quotient figure is too small. If the number obtained by multiplying the divisor by the quotient figure, be greater than the number divided, that quotient figure is too large.

Example 1. Divide 15359 by 29.

PERFORMED.

29) 15359 ( 529  $\frac{11}{29}$   
 145

85

58

279

261

18 Rem.

**Explanation.**—I in the first place notice that at least three figures are required to contain the divisor, and that in this number, 153, the divisor is contained 5 times. 5 is then my first quotient figure. I then proceed to multiply and subtract as the rule directs, and obtain a remainder of 8. To this, I bring down the next figure of the dividend, and obtain 85. I proceed to divide, multiply, and subtract, as before, and obtain a remainder of 27, and a quotient figure 2. Again, I bring down, divide, multiply, &c. and obtain the quotient figure, 9, and a remainder of 18, which being placed according to the 7th article of the rule, gives the fraction  $\frac{18}{29}$ .

**Note 3d.**—It will be observed that each figure in the quotient is obtained by *four successive operations*, and that these operations uniformly succeed each other in the same order. In the first place, we are required to *divide*; in the second, to *multiply* the divisor by the quotient figure; in the third, to *subtract* the product of this multiplication from the figures divided; and, lastly, to the remainder thus obtained, to *bring down* another figure of the dividend.

**Ex. 2.** Divide 6283459 by 29.

PERFORMED.

29) 6283459 (216671 = Quotient.

58

48

29

193

174

194

174

205

203

29

29

00

3. Divide 7461389 by 95.

$$\begin{array}{r}
 \text{PERFORMED.} \\
 95 \overline{) 7461389} \quad (78540 \frac{11}{2} \\
 \underline{665} \phantom{00} \\
 811 \phantom{00} \\
 \underline{760} \phantom{00} \\
 513 \phantom{00} \\
 \underline{475} \phantom{00} \\
 388 \phantom{00} \\
 \underline{380} \phantom{00} \\
 89
 \end{array}$$

4. Divide 1893312 by 2076. Quo. 912.
5. Divide 455678 by 78. Quo. 5842, and 2 rem.
6. Divide 6495685 by 85. Quo. 76419, and 70 rem.
7. Divide 9424789962 by 978. Quo. 9636799, rem. 540.
8. Divide 2686211248 by 296. Quo. 9075038.
9. Divide 84764367 by 431. Quo. 196669, rem. 28.
10. Divide 4683579 by 234. Quo. 20015, rem. 69.
11. Divide 3579864 by 135. Quo. 26517, rem. 69.
12. Divide 1748 by 18. Quo. 97, and 2 rem.

*Note 4th.*—When the divisor is 10, 100, 1000, 10000, &c. point off as many figures from the right of the dividend, as there are cyphers in the divisor; the figures on the left of the point will be the quotient, and those on the right, the remainder.

- Ex. 1. Divide 19375468 by 10000. Quo. 1937, rem. 5468.
2. Divide 99885566 by 100000. Quo. 998, rem. 85566.
3. Divide 47429 by 10. Quo. 4742, rem. 9.
4. Divide 463581 by 100. Quo. 4635, rem. 81.
5. Divide 618293 by 1000. Quo. 618, rem. 293.

*Note 5th.*—When the divisor consists of a number of figures with cyphers annexed, the cyphers may be cut off from the divisor, and an equal number of figures from the right of the dividend, and the remainder of the dividend divided by the significant figures of the divisor. After the division, the figures cut off from the dividend are to be placed at the right of the remainder.

**Ex. 1.** Divide 38418235700 by 98700. Quo. 368079, rem. 8400.

2. Divide 11579112 by 890000. Quo. 13, rem. 9112.

3. Divide 8317642500 by 814600. Quo. 10210, rem. 576500.

## APPLICATION.

1. If 246 men incur an expense of \$175152, what is each man's share? Ans. \$712.

2. A gentleman left an estate of \$65468 to 6 sons; what was each one's share? Ans. \$10911 $\frac{1}{3}$ .

3. Twelve men own a bridge, for which they annually receive \$2352, in toll; what is each man's share? Ans. \$196.

4. Suppose 7776 peach trees to be planted in 108 rows; how many trees are there in a row? Ans. 72.

5. If light comes from the sun to the earth in 8 minutes, how far does it travel in one minute, the distance being 95000000 miles? Ans. 11875000 miles.

6. If a man travel 9125 miles in a year, what is his average daily progress? Ans. 25 miles.

7. If a horse run 288 miles in 36 hours, how far does he run in one hour? Ans. 8 miles.

8. In 437850 yards of cloth, how many rolls of 75 yards each? Ans. 5838.

## APPLICATION OF THE PRECEDING RULES.

1. A farmer sold 3 yoke of oxen at \$96 each; 12 cows at \$24 each; 83 sheep at \$3 a head; 239 bushels of wheat at \$2 per bushel, and distributed the avails equally among his 7 sons. What was each one's share? Ans. \$186 $\frac{1}{2}$ .

2. A man to whom was entrusted the settlement of an estate, found that the whole value of the estate was \$95688. There were five claims against the estate; viz. one of \$8672; another of \$3421; a third of \$10637; a fourth of \$356; and a fifth of \$1673. After the payment of these several claims, the balance was to be divided equally among 9 heirs. What was the share of each? Ans. \$7881.

3. A farmer had 16 calves worth 5 dollars per head; 45 sheep worth \$3 per head; and 75 bushels of grain worth \$2 per bushel; he gave the whole for a horse worth \$136; a carriage worth \$195, and a harness worth \$63. Did he gain or lose, and how much? Ans. Gained \$29.

4. A merchant received by boat, 9696 bushels of salt, and

hired it carted 16 miles, at a dollar a load of 24 bushels. How much did the cartage cost? *Ans.* \$404.

5. When the dividend is 290864196 and the quotient 9382-716, what is the divisor? *Ans.* 31.

6. A man purchased a farm for which he paid \$18000. He sold 60 acres for 50 dollars an acre, and then the remainder stood him at \$75 per acre. How much land did he purchase? *Ans.* 260 acres.

**QUESTIONS.**—How does division differ from multiplication? How may multiplication be performed? How may division be performed? How many numbers are given in division? What are they, and what are they called? What is the number obtained by the operation called? What does it signify? What is the remainder? How does it always compare with the divisor? How is division proved? What is short division? What is the rule for it? What is the rule when the divisor is more than 12 and a composite number? How is the true remainder obtained, when we divide by the component parts of the divisor? What is long division? What is the rule? How do you divide by 10, 100, 1000, &c.? When the divisor consists of a figure greater than 1, with cyphers annexed, how do you divide?

## FEDERAL MONEY.

Federal Money is the currency of the United States. Its denominations are Mills, Cents, Dimes, Dollars, and Eagles, increasing like simple numbers in a ten-fold ratio, as represented in the following table :

TABLE OF FEDERAL MONEY.

|                      |                                 |
|----------------------|---------------------------------|
| 10 Mills (marked m.) | make one cent, marked c. or ct. |
| 10 Cents             | “ one dime, “ d.                |
| 10 Dimes             | “ one dollar, “ \$ or dol.      |
| 10 Dollars           | “ one eagle, “ E.               |

The coins of the United States are of three kinds; viz. gold, silver, and copper coins.

The gold coins are,

|               |         |
|---------------|---------|
| The Eagle     | = \$10. |
| Half-Eagle    | = \$5.  |
| Quarter-Eagle | = \$2½. |

The silver coins are,

|                |            |
|----------------|------------|
| The dollar     | = 100 cts. |
| Half-dollar    | = 50 cts.  |
| Quarter-dollar | = 25 cts.  |
| Dime           | = 10 cts.  |
| Half-dime      | = 5 cts.   |



The  $6\frac{1}{2}$  cent and  $12\frac{1}{2}$  cent pieces, &c. are not American coinage.

The copper coins are, the cent and the half-cent. Half-cents are seldom used. The gold and silver coins are not composed of pure metal; but are alloys,—that is, compounds of these metals with the baser metals. The purity of a metal is expressed by the word, *carat*; which word is used to express a twenty-fourth part of a given quantity. If, for example, a quantity of gold be said to be 18 carats fine, the meaning is, that 18 equal portions of the whole are gold, and 6 equal portions are of a less valuable metal.

By recurring to the preceding table, it will be seen that the denominations of federal money may be added, subtracted, multiplied, and divided by the same rules as the simple numbers, as they increase in the same ratio. The scholar should however remember that the dollar is the unit money; and that dimes, cents, and mills are decimals; or tenths, hundredths, and thousandths of the dollar, or unit denomination.

It is therefore always important to know which of a number of figures is the dollar, or unit figure. This is shown by the decimal point (.) or period, which always stands between the dollars and dimes; thus, 63.78, read, sixty-three dollars and seventy-eight cents.

The first figure on the left of the point is dollars, and the second eagles. They are however called dollars indiscriminately; as in the above number, the 6 is eagles, and the 3, dollars; but usually read, 63 dollars. Likewise, the 78 is usually read, 78 cents, instead of 7 dimes and 8 cents. Both methods express the same value.

To reduce the higher denominations to the lower, it is necessary to bear in mind, 1st. That cents are converted into mills by annexing one cypher; thus, 8 cents=80 mills. 2d. That dollars may be changed into cents by annexing two cyphers; thus, 3 dollars=300 cents; and into mills by annexing three cyphers; thus, \$3=3000 mills. 3d. The reverse operation will convert mills into cents and cents into dollars.

Ex. 1. How many mills in 47 cents? *Ans.* 470.

2. How many mills in 69 cents? *Ans.* 690.

3. How many mills in 156 cents? *Ans.* 1560.

4. In 78 dollars, how many cents? *Ans.* 7800. How many mills? *Ans.* 78000.

5. In \$637, how many cents? *Ans.* 63700. How many mills? *Ans.* 637000.

6. In 450 mills, how many cents? *Ans.* 45.  
 7. In 470 mills, how many cents? *Ans.* 47.  
 8. In 6700 mills, how many cents? *Ans.* 670.  
 9. In 6700 mills, how many dollars? *Ans.* 67.  
 10. Change \$6 into cents. 11. Change 42 cents into mills.  
 12. Change \$95 dollars into mills. 13. Change 460 mills into cents.  
 14. 28000 mills into dollars. 15. 439 mills into cents.  
 16. 9876 mills into dollars.

### ADDITION OF FEDERAL MONEY.

**RULE.**—*Set the numbers one under another, so that dollars shall stand under dollars, dimes under dimes, cents under cents, and mills under mills. Then add up the several columns as in Simple Addition, and place the decimal point, in the amount, directly under those in the numbers added.*

If the above rule be followed in writing down the several numbers, the separating points will stand directly under each other.

**Ex. 1.** What is the sum of 136 dollars 21 cents; 75 dollars 13 cents; 7 dollars 78 cents; 66 dollars 19 cents; and 196 dollars 72 cents? *Ans.* 482.03.

#### PERFORMED.

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 1 | 3 | 6 | . | 2 | 1 |
|   | 7 | 5 | . | 1 | 3 |
|   |   | 7 | . | 7 | 8 |
|   |   | 6 | . | 6 | 1 |
|   |   | 1 | . | 9 | 6 |
|   |   | 1 | . | 9 | 6 |

\$ 4 8 2 . 0 3 Amount.

2. Add together \$432.73; \$297.38; \$172.66; and \$333.62. *Amount,* \$1236.39.

3. What is the sum of \$1.55; \$0.72; \$340.89; \$0.01; \$1460.99? *Ans.* \$1804.16.

4. What is the sum of \$72.01; \$1; \$0.01; \$0.10; \$40.70; \$560.88? *Ans.* \$674.70.

5. What is the sum of \$101.01; \$20.15; \$42.89; \$79.81; \$41.41; \$51.51; \$38.41? *Ans.* \$375.19.

6. What is the sum of \$16.64; \$20.84; \$462.573; \$29.922; \$56.32; \$84.48? *Ans.* \$670.775.

7. A farmer bought a cow for \$23.75; a yoke of oxen for

\$96.78; a horse for \$69.83; and a pig for \$1.625; what did they all cost him? *Ans.* \$191.985.

8. A grocer paid for a box of cheese, \$37.21; for candles, \$8.32; for a cask of wine, \$7.38; for a box of raisins, \$3.625; what was the whole cost? *Ans.* \$56.535.

9. Bought 5 gallons of molasses for \$1.80; three pounds of tea for \$1.125; 3 yards of broadcloth for \$9.82; 11 yards of cotton cloth for \$1.83; 6 yards of linen for \$3.82; what was the amount of my bill? *Ans.* \$18.395.

10. Paid for building my house, \$2169.72; for my barn, \$972.87; for my out-houses, \$1272.69; for digging my well, \$56.38; what is the amount of my expenses? *Ans.* \$4471.66.

11. My expenses for a journey were as follows: for stage fare, \$66.89; for meals, \$18.50; for lodging, \$6.58; for carrying baggage, \$2.57; for cleansing boots, \$1.36; for washing, \$2.66; what is the amount? *Ans.* \$98.56.

12. My hat cost me \$4.75; my coat, \$18.96; my pantaloons, \$9.74; my vest, \$5.82; and my boots, \$6.54; what did the whole cost? *Ans.* \$45.81.

### SUBTRACTION OF FEDERAL MONEY.

*RULE.—Place the numbers as in Simple Subtraction, and after subtracting, place the separating point as directed in Addition of Federal Money.*

Ex. 1. From \$463.42, take \$399.99. *Rem.* \$63.43.

2. From \$179.364, take \$88.449. *Rem.* \$90.915.

3. From \$125, take \$9.09. *Rem.* \$115.91.

4. From \$642.99, take \$99.99. *Rem.* \$543.

5. From \$127.01, take \$41.10. *Rem.* \$85.91.

6. From \$200, take \$0.90. *Rem.* \$199.10.

7. From \$2, take \$0.05. *Rem.* \$1.95.

8. From \$99, take \$0.99. *Rem.* \$98.01.

9. I have \$473, and my brother twice as much, wanting 90 cents. How much has my brother? *Ans.* \$945.10.

10. Having in my possession \$1600, I paid \$516.95 for a span of fine horses; \$156.55 for a carriage; \$221.19 for a gold watch; and spent \$450.71 in traveling. How much had I left? *Ans.* \$254.60.

11. If a man has an income of \$4500, and spends \$2461.85, how much does he lay up? *Ans.* \$2038.15.

12. If one man lays up \$339.86 in a year, and another

\$299.99, how much does one lay up more than the other?  
*Ans.* \$39.87.

13. Bought a farm for \$3946, and sold a part for \$1426.82;  
 what did the remaining part cost me? *Ans.* \$2519.18.

### MULTIPLICATION OF FEDERAL MONEY.

**RULE.**—*Multiply, as in simple numbers. The product will always be in the lowest denomination given, which may be reduced to dollars, cents, and mills, by the preceding rules.*

Ex. 1. What will 36 yards of cloth cost at \$4 50 per yard?  
*Ans.* \$162.00.

$$\begin{array}{r}
 \text{PERFORMED.} \\
 4.50 \\
 36 \\
 \hline
 2700 \\
 1350 \\
 \hline
 16200
 \end{array}$$

The given price being 450 cents, the whole price is 16200 cents, which equals \$162.00.

2. What will 29 pairs of shoes cost at \$1.50 per pair? *Ans.* \$43.50.

3. What will 35 pounds of beef cost at 8 cents per pound?  
*Ans.* \$2.80.

4. Bought 280 reams of paper at \$2.35 per ream; what was the whole cost? *Ans.* \$658.00.

5. What cost 600 pounds of lard, at 15 cents per pound?  
*Ans.* \$90.00.

6. Bought 15 tons of hay at \$16.42 per ton; what was the whole cost? *Ans.* \$246.30.

7. What cost 349 acres of land, at \$15.49 per acre? *Ans.* \$5406.01.

8. Bought 18 yoke of oxen for \$72.50 per yoke; what was the whole cost? *Ans.* \$1305.00.

9. Bought 32 pounds of butter at 20 cents per pound; 45 pounds of loaf sugar at 18 cents per pound; 56 pounds of coffee at 15 cents per pound; 26 pounds of tea at \$1.75 per pound; 21 cwt. of raisins at \$6.75 per cwt.; 42 barrels of flour at \$7.50 per barrel; and 29 pairs of boots at \$4.50 per pair. What did the whole cost me? *Ans.* \$655.65.

## DIVISION OF FEDERAL MONEY.

Division of Federal Money is employed whenever the cost of a number of articles, as yards, pounds, &c., is given, and the price of one required.

*RULE.—Divide the cost by the number of articles, and point off as many figures from the quotient for cents and mills, as there are in the given sum. If dollars only be given, and cyphers are added to complete the division, these cyphers must be regarded as cents and mills.*

Ex. 1. If 9 pounds of butter cost \$225, what is the value of one pound? *Ans.* \$0.25.

2. Sold 69 bushels of wheat for \$625; what was the price per bushel? *Ans.* \$9.057.

3. Paid \$75.00 for 500 lbs. of butter; what was the price per pound? *Ans.* \$0.15.

4. Paid \$311.70 for 15 tons of hay; what was the price per ton? *Ans.* \$20.78.

5. Paid \$658 for 280 reams of paper; what did I pay per ream? *Ans.* \$2.35.

6. Paid \$505.44 for 144 lbs. of tea; what was the price of one pound? *Ans.* \$3.51.

7. Paid \$375 for 50 firkins of butter; what was the price per firkin? *Ans.* \$7.50.

8. Paid \$43.79 for 29 pairs of boots; what was the price per pair? *Ans.* \$1.51.

9. Paid \$2.80 for 35 lbs. of beef; what was the price per pound? *Ans.* .08.

## APPLICATION OF THE PRECEDING RULES.

1. A man dying, left an estate of \$12000, which was divided equally among 7 children after his wife had received her third. What was the portion of the wife, and what did each child receive? *Ans.* \$4000, wife's portion; \$1142.857, each child's portion.

2. A man settling with his grocer, finds himself charged with 15 lbs. of tea at 75 cts. per pound; 42 pounds of brown sugar at 11 cts. a pound; 3 barrels of flour at \$7.50 per barrel; 8 gallons of lamp oil at \$1.25 per gallon; and 45 pounds of ham

at 15 cts. per pound. He is also credited \$11.62. How much does he owe his grocer? *Ans.* \$43.50.

3. A man sells a horse for \$84; 5 cows for \$25 each; and agrees to take 80 sheep in pay. How much do the sheep cost him per head? *Ans.* \$2.61 $\frac{1}{4}$ .

4. A person agrees to furnish a grocer with 56 bushels of rye at 50 cts. a bushel, and to take his pay in coffee at 15 cts. a pound. How many pounds of coffee will he receive? *Ans.* 186 $\frac{1}{2}$ .

5. If I pay \$21311 for 844 acres of land, what do I pay per acre? *Ans.* \$25.25.

6. Bought 350 yards of cloth at \$3.50 per yard. Of this I sold 200 at \$5.00 per yard. How much money did I pay out; how much did I receive; how many yards had I left; and how much did it cost me per yard? *Ans.* I paid out \$1225; I received \$1000; 150 yards were left; and what remained cost me \$1.50 per yard.

7. A man sold his farm for \$8456, and his stock for \$1560; at the same time he had on deposit in the bank, \$872.97. He then purchased a house in the city, for which he paid \$3845; he also purchased a horse and carriage for \$392.53, and paid up his old debts to the amount of \$1787. How much money had he left? *Ans.* \$4864.44.

8. Suppose the man in the preceding sum had laid out the balance of his money in wagons, for which he paid \$72 each; and that he took these wagons to the south and sold them for \$97 each. How many wagons did he purchase, and how much did he gain by the transaction, allowing his expenses to have been \$297.83? *Ans.* He purchased 67 wagons, and had \$40.44 left; and he gained \$1377.17.

9. On the first of Jan. a spendthrift was in possession of \$3860.90: after 30 days he had only \$1680 remaining; how much had he spent during the whole time; and how much daily? *Ans.* Whole sum, \$2180.90; daily, \$72.696.

10. A man bought 20 pounds of coffee, for 15 cents a pound; and 18 pounds of sugar, at 12 cts. a pound. He paid 96 cents in cash, and the balance in butter at 20 cents a pound; how much butter did it take? *Ans.* 21 pounds.

11. How much tea worth 56 cents a pound must be given for 16 sacks of salt, worth \$2.87 per sack? *Ans.* 82 pounds.

QUESTIONS.—What is Federal Money? What are its denominations? How do they increase in value? Repeat the table. What are the coins of the United States? What are the gold coins? What

are their values? What are the silver coins? What are their values? Are the four-pence half-penny and the nine-penny pieces, American coin? What are the copper coins? Are the gold and silver coins pure metal? What is their composition? By what word is the purity of a metal expressed? What does that word express? If a quantity of gold be said to be 18 carats fine, what is meant? Suppose it be said to be 22 carats fine, what is meant? How may the denominations of Federal Money be added? Which is the unit money? What are dimes, cents, and mills? How is the dollar or unit figure always shown? How does the period always stand? What is the first figure on the left of the point? The second? How are they usually read? How are the figures on the right of the point read? How are cents converted into mills? How are dollars converted into cents? What is the rule for the addition of Federal Money? How is the decimal point placed? What is the rule for the subtraction of Federal Money? How is the point placed? What is the rule for the multiplication of Federal Money? How many figures are cut off from the right of the product? When is division of Federal Money employed? What is the rule? How many figures do you point off in the quotient? If dollars only are given and cyphers are added, how are they to be regarded?

## CANCELING.

In all arithmetical operations, it is important not only to know *how* to solve a proposition, but also, how it may be done most *expeditiously*. The object of this rule is to acquaint the scholar with a principle by which *peculiar expedition* is attained in the solution of such sums as involve in their operation both *multiplication* and *division*. This principle is founded on the following facts:

First. The value of any quotient depends on the *ratio*, or *relative size of the numbers divided*; that is, if the dividend be three times as large as the divisor, the value of the quotient is 3; and if it be four times as large, the value is 4, &c.

Second. If two or more numbers are to be multiplied together, and their product divided by any other number, the true result is obtained by first dividing one of these numbers by the dividing number, and then multiplying the quotient by the remaining number or numbers. Thus, if it be required to multiply 8 by 4 and to divide the product by 2, first divide 8 by 2 and multiply the quotient by 4; thus,  $8 \div 2 = 4$ , and  $4 \times 4 = 16$ .

The advantage of this process will be more obvious if we take large numbers. Suppose we wish to multiply 288 by 16, and to divide the product by 144. The usual process would be thus :

$$\begin{array}{r}
 288 \\
 16 \\
 \hline
 1728 \\
 288 \\
 \hline
 4608 = \text{product.}
 \end{array}
 \qquad
 \begin{array}{r}
 144) 4608 (32 \text{ quotient.} \\
 \underline{432} \\
 288 \\
 \underline{288} \\
 000
 \end{array}$$

But by first dividing, the operation is much abbreviated ; thus :

$$\begin{array}{r}
 144) 288 (2 \\
 \underline{288} \\
 000 \text{ and } 16 \times 2 = 32.
 \end{array}$$

By the usual method, 37 figures are required ; by the other, only 18. There is still another advantage. The scholar can see at a glance that 144 is contained in 288, twice ; and that twice 16 is 32 ; so that an operation, which is long and protracted, is often reduced nearly or quite to a mental operation.

Third. When any large number is to be divided by the product of two or more smaller numbers, it may be divided by each number separately. This needs no explanation ; it is the same as dividing by the component parts of any number, instead of the number itself.

Fourth. When the operation is of such a nature as to require the *product of several numbers* to be divided by the *product of several other numbers*, these numbers may be divided before multiplication, and their quotients used instead of the numbers themselves. For illustration, suppose the product of 36 and 42 is to be divided by the product of 6 and 7. The usual mode of operation would be as follows, viz. :

$$\begin{array}{r}
 42 \\
 36 \\
 \hline
 252 \\
 126 \\
 \hline
 1512 \text{ dividend.}
 \end{array}$$



$7 \times 6 = 42$  divisor ; therefore,  $42 \overline{) 1512}$  (36, the required quotient.

$$\begin{array}{r} 1512 \\ 42 \overline{) 1512} \\ \underline{126} \phantom{00} \\ 252 \phantom{0} \\ \underline{252} \phantom{0} \\ 0 \end{array}$$

But by the preceding fourth principle,  $36 \div 6 = 6$ , and  $42 \div 7 = 6$ , and  $6 \times 6 = 36$  Ans. In this example the divisors are, as it were, expunged or lost, since they divide without remainder. This will not, however, always be the case. It will frequently be necessary to assume some number, which will divide some two given numbers, without remainder, agreeably to a rule soon to be given.

But for further illustration, suppose it be required to multiply the numbers 36, 12, 27, and 72 together, and to divide the product successively by 24, 18, and 48. Now it is evidently desirable to arrange these numbers so that they may be conveniently compared with each other. We will adopt the following mode :—We will place the numbers whose product is to form a dividend, above a horizontal line ; and those whose product is to form a divisor, below the same line, thus :

$36. 12. 27. 72 = \text{Dividend.}$

$24. 18. 48 = \text{Divisor.}$

Now by the fourth and last principle laid down, I can divide 36 in the dividend and 18 in the divisor by 18, without a remainder, and obtain 2 in the dividend and 1 in the divisor ; thus,  $\frac{2. 12. 27. 72}{24. 1. 48}$

I can also divide 72 in the dividend and 24 in the divisor by 24, and obtain 3 in the dividend and 1 in the divisor—(it will be remembered that the divisors stand below the line)—thus,  $\frac{2. 12. 27. 3}{1. 1. 48}$ . Again, I can divide 12 in the dividend and 48 in the divisor by 12, and obtain 1 in the dividend and 4 in the divisor ; thus,  $\frac{2. 1. 27. 3}{1. 1. 4}$ . Again, I can divide 2 in the dividend and 4 in the divisor by 2, and obtain 1 in the dividend and 2 in the divisor ; thus,  $\frac{1. 1. 27. 3}{1. 1. 2}$ . It is now evident that the division can be carried no farther without remainder. The next step therefore is to divide the product of the numbers remaining above the line by the product of those below it. The product of those above the line is  $27 \times 3 = 81$  ; and of those below the line, 2 ; therefore,  $81 \div 2 = 40\frac{1}{2}$ , the

number required. The same result would have been obtained by multiplying the numbers above the line and dividing their product by the product of those below it, previous to canceling. In the above example, as the numbers have been canceled, they have been omitted, and a new statement made. This is by no means necessary. One statement is sufficient.

It will be noticed that in every instance division is effected without a remainder. Such must always be the case.

The following rule will be found a competent guide for the scholar in all operations of canceling.

**RULE 1st.**—Place all the numbers whose product is to form a dividend, above a horizontal line; and all those whose product is to form a divisor, below the same line.

2d. Notice whether there are cyphers both above and below the line; if so, erase an equal number from each side.

3d. Notice whether the same number stands both above and below the line; if so, erase them both.

4th. Notice again if any number on either side of the line will divide any number on the opposite side, without remainder; if so, divide and erase the two numbers, retaining the quotient figure only on the side of the larger number.

5th. See if any two numbers, one on each side, can be divided by any assumed number, without a remainder; if so, divide them by that number, and retain only their quotients. Proceed in the same manner as far as practicable; then,

6th. Multiply all the numbers remaining above the line for a dividend, and those remaining below, for a divisor.

7th. Divide, and the quotient will be the number required.

**Note 1st.**—If only one number remain on either side of the line, that number is the dividend or divisor, according as it stands above or below the line.

**Note 2d.**—The figure 1 is not to be regarded in the operation, because it avails nothing either to multiply or divide by 1.

In the two following examples, a separate statement will be made for each step of the rule, as successively taken, with a reference to the rule.

**Ex. 2.** Multiply together the numbers 100, 16, 24, and 36, and divide their product by the product of 60, 10, and 16.

Statement,  $\frac{100. 16. 24. 36}{60. 10. 16}$ . By applying the second step of the rule, the statement is reduced to  $\frac{1. 16. 24. 36}{6. 1. 16}$ . By applying the third step, we obtain  $\frac{1. 24. 36}{6. 1}$ . By application of the fourth step, we obtain  $\frac{1. 4. 36}{1. 1}$ . The statement is capable of no farther reduction, for the divisors or lower numbers are all canceled. The product of the numbers remaining above the line is, therefore, the quotient required; viz.  $4 \times 36 = 144$  Ans.

Ex. 3. Divide the product of 96, 18, 110, 5, and 42 by the product of 50, 27, 11, and 28. Statement,  $\frac{96. 18. 110. 5. 42}{50. 27. 11. 28}$ . By second step of the rule, we obtain  $\frac{96. 18. 11. 5. 42}{5. 27. 11. 28}$ . By the third,  $\frac{96. 18. 42}{27. 28}$ . By the fifth,  $\frac{96. 2. 42}{3. 28}$ . 9 is assumed for a divisor. By the fourth,  $\frac{96. 1. 42}{3. 14}$ . By the fourth, again,  $\frac{96. 1. 3}{3. 1}$ ; and by the third,  $\frac{96. 1. 1}{1}$ . 96 is the only number remaining above the line, except the 1, and nothing but 1 remains below the line. 96 is therefore the quotient sought.

To show more completely the mode of solution, the canceled numbers will be retained in the following sum; they will, however, be *crossed*, to show that they are canceled.

Ex. 4. Divide the product of 16, 9, 4, by the product of 3 and 32.

Statement,  $\frac{16. 9. 4}{3. 32}$ . By fourth step of the rule,  $\frac{16. 9. 12}{3. 32}$ .  
 Again, by the same,  $\frac{16. 9. 12}{3. 32}$ . Again, by the same,  $\frac{16. 9. 12}{3. 32}$ .  
 and  $6 \times 3 = 18$ , the required quotient.

The following statements are solved without repetition; that the scholar may obtain accurate views relative to the mode of solution here presented.

5. Divide the product of 21, 12, 42, 100, by the product of 7, 35, and 50.

$$\text{Statement, } \frac{21. 12. 42. 100}{7. 35. 50} \quad \text{Canceled, } \frac{\overset{3}{21}. \overset{6}{12}. \overset{2}{42}. 100}{\underset{5}{7}. 25. 50} \quad \text{Then}$$

$3 \times 6 \times 2 \times 12 = 432$ , and  $432 \div 5 = 86\frac{2}{5}$ . Ans.

6. Divide the product of 99, 49, 15, 20, 32, 13, 16 by the product of 77, 10, 16, 49, 39, and 12.

$$\text{Statement, } \frac{99. 49. 15. 20. 32. 13. 16}{77. 10. 16. 49. 39. 12}$$

$$\text{Canceled, } \frac{\overset{32}{99}. \overset{5}{49}. \overset{2}{15}. \overset{8}{20}. 32. 13. 16}{\underset{7}{77}. 10. 16. 49. 39. 12}$$

then  $3 \times 5 \times 2 \times 8 = 240$ , and  $240 \div 7 = 34\frac{2}{7}$ . Ans.

7. Divide the product of 164, 88, 4, 28, 3, 2, by the product of 328, 36, 21, 12, 32.

$$\text{Statement, } \frac{164. 88. 4. 28. 3. 2}{328. 36. 21. 12. 32} \quad \text{Canceled, } \frac{\overset{11}{164}. \overset{4}{88}. \overset{1}{4}. \overset{28}{28}. \overset{3}{3}. \overset{2}{2}}{\underset{2}{328}. \underset{9}{36}. \underset{3}{21}. \underset{4}{12}. \underset{4}{32}}$$

therefore, 11 divided by the product of  $9 \times 3 \times 4 = 108$ , is the Ans.; and may be thus expressed,  $\frac{11}{108}$ . It will be observed, that, in the upper numbers, the product of 3 and 4 cancels 12 in the lower numbers; and generally, when the product of any two or more numbers on one side equals any number on the opposite side, they may all be erased at the same time. By a careful examination of the preceding seven examples, the scholar will be prepared to solve the remaining examples without further elucidation. He will, however, remember that the only object in view is to prepare him to make a successful application of the principle of canceling, to the solution of useful problems.

*Note 3d.*—Whenever the product of the remaining numbers above the line is less than the product of those below, the answer will be in the form of a fraction, as in the last example. The scholar, when he has learned respecting the nature of fractions, will find that a number standing directly over another, with a line between them, always indicates division; that is, the number above the line is divided by that below it.

8. Divide the product of 27, 111, 32, 40, 42, by the product of 12, 4, 24, 12, 30. Quo.  $388\frac{1}{12}$ .

9. Divide the product of 132, 8, 49, 4, 84, 81, 42, by the product of 16, 28, 7, 5, 6, 45, and 35. Quo.  $399\frac{1}{11}$ .

10. Divide the product of 32, 18, 72, 81, 7, 56, 63, by the product of 24, 36, 162, 63, 2, 32. *Quo.* 147.

11. Divide the product of 96, 54, 108, 72, 56, 18, and 21, by the product of 27, 81, 324, 28, 72, 3. *Quo.* 199 $\frac{1}{2}$ .

12. Divide the product of 16, 27, 42, 44, 55, and 66, by the product of 48, 88, 22, 33, and 49: *Quo.* 194.

13. Divide 6, 8, 10, 5, 20, 25, 36, 48, and 60, by the product of 9, 12, 15, 28, 36, 42, and 9. *Quo.* 201 $\frac{1}{3}$ .

14. If 12 horses eat 15 bushels of oats, how many would 36 horses eat in the same time? In solving this sum, if I divide 15 bushels by 12, it will evidently give me what one horse will eat, viz.  $1\frac{1}{4}$  bushels; and if I multiply this by 36, I shall obtain the quantity that 36 horses will eat. This sum may then be easily solved by the principle here introduced.

The statement would be thus :  $\frac{15.36}{12}$

Solved,  $\frac{15.36}{12}$ , and  $15 \times 3 = 45$  bushels.

But we will not anticipate: enough has already been done to show that the principle is a practical one. We will leave its application, for future consideration.

**QUESTIONS.**—What is the object of the rule of canceling? What problems may be solved on this principle? What is the first of the facts on which this principle is founded? Illustrate. What is the second? Illustrate. What is another advantage? What is the third fact? What is the fourth? Illustrate. What is the first step of the rule? What is the second step? The third? The fourth? The fifth? The sixth? The seventh, and last? What is note first? What note second? When the product of the remaining numbers above the line is less than the product of those below, in what form will the answer be? What does one number standing directly over another, with a line between them, always indicate?

## COMPOUND NUMBERS.

We have thus far been operating with numbers, of the same denomination, and increasing in the constant ratio of 10.

There is, however, another class of numbers composed of several denominations, increasing in no uniform ratio, and requiring to be separately denoted or expressed. These are called Compound Numbers. Under this head are included all those denominations employed to express measures of any defi-

nite kind; such as length, breadth, solidity, weight, time, money, capacity, &c.

The following are the tables of these denominations. They require to be made very familiar, as they show how many units of each lower denomination, are equal to a unit of the next higher denomination.

### I. ENGLISH MONEY.

The denominations of English Money are pounds, shillings, pence, and farthings.

TABLE.

|                                                  |               |      |                             |
|--------------------------------------------------|---------------|------|-----------------------------|
| 4 farthings (marked qr.) make 1 penny, marked d. |               |      | <sup>d.</sup><br>1= 4.      |
| 12 pence                                         | " 1 shilling, | " s. | <sup>s.</sup><br>1= 12= 48. |
| 20 shillings                                     | " 1 pound,    | " £. | 1=20=240=960.               |

### II. TROY WEIGHT.

By this weight, the precious metals, such as gold and silver, also jewels and precious stones, are weighed. The following are the denominations :

TABLE.

|                                                 |       |  |                                  |
|-------------------------------------------------|-------|--|----------------------------------|
| 24 grains (gr.) make 1 pennyweight, marked pwt. |       |  | <sup>pwt.</sup><br>1= 24.        |
| 20 pennyweights " 1 ounce,                      | " oz. |  | <sup>oz.</sup><br>1= 20= 480.    |
| 12 ounces " 1 pound,                            | " lb. |  | <sup>lb.</sup><br>1=12=240=5760. |

### III. AVOIRDUPOIS WEIGHT.

By this weight, all coarse materials, such as hay and grain, and also the baser metals, such as copper, are weighed.

TABLE.

|                                 |  |  |                                             |
|---------------------------------|--|--|---------------------------------------------|
| 16 drams (dr.) make 1 ounce—oz. |  |  | <sup>oz.</sup><br>1= 16.                    |
| 16 ounces " 1 pound—lb.         |  |  | <sup>lb.</sup><br>1= 16= 256.               |
| 28 pounds " 1 quarter—qr.       |  |  | <sup>qr.</sup><br>1= 28= 448= 7168.         |
| 4 quarters " 1 hun. wght.—cwt.  |  |  | <sup>cwt.</sup><br>1= 4= 112= 1792= 28672.  |
| 20 hundred " 1 ton—T.           |  |  | <sup>T.</sup><br>1=20=80=2240=35840=573440. |

### IV. APOTHECARIES' WEIGHT.

This weight is used by apothecaries and physicians in mixing and preparing medicines.

TABLE.

|                 |                            |                 |                 |
|-----------------|----------------------------|-----------------|-----------------|
| 20 grains (gr.) | make 1 scruple— $\odot$ .  | $\odot$ .       | $gr.$           |
|                 |                            | 1=              | 20.             |
| 3 scruples      | " 1 dram—3.                | 3.              |                 |
|                 |                            | 1=              | 3= 60.          |
| 8 drams         | " 1 ounce— $\frac{3}{4}$ . | $\frac{3}{4}$ . |                 |
|                 |                            | 1=              | 8= 24= 480.     |
| 12 ounces       | " 1 pound— $\text{lb}$ .   | $\text{lb}$ .   |                 |
|                 |                            | 1=              | 12=96=288=5760. |

## V. CLOTH MEASURE.

This is the measure used for measuring all kinds of cloth.

TABLE.

|                              |                        |        |                   |
|------------------------------|------------------------|--------|-------------------|
| 2 $\frac{1}{2}$ inches (in.) | make 1 nail—na.        | na.    | in.               |
|                              |                        | 1=     | 2 $\frac{1}{2}$ . |
| 4 nails                      | " 1 quarter yard—qr.   | qr.    |                   |
|                              |                        | 1=     | 4= 9.             |
| 4 quarters                   | " 1 yard—yd.           | yd.    |                   |
|                              |                        | 1=     | 4=16=36.          |
| 3 quarters                   | " 1 Ell Flemish—E. Fl. | E. Fl. |                   |
|                              |                        | 1=     | 3=12=27.          |
| 5 quarters                   | " 1 Ell English—E. E.  | E. E.  |                   |
|                              |                        | 1=     | 5=20=45.          |
| 6 quarters                   | " 1 Ell French—E. Fr.  | E. Fr. |                   |
|                              |                        | 1=     | 6=24=54.          |

## VI. WINE MEASURE.

This measure is used for measuring all liquors, with the exception of beer and ale. The gallon contains 231 cubic inches.

TABLE.

|                          |                   |      |                                 |
|--------------------------|-------------------|------|---------------------------------|
| 4 gills (gi.)            | make 1 pint—pt.   | pt.  | gi.                             |
|                          |                   | 1=   | 4.                              |
| 2 pints                  | " 1 quart—qt.     | qt.  |                                 |
|                          |                   | 1=   | 2= 8.                           |
| 4 quarts                 | " 1 gallon—gal.   | gal. |                                 |
|                          |                   | 1=   | 4= 8= 32.                       |
| 31 $\frac{1}{2}$ gallons | " 1 barrel—bar.   | bar. |                                 |
|                          |                   | 1=   | 31 $\frac{1}{2}$ =126=252=1008. |
| 63 gallons               | " 1 hogshead—hhd. | hhd. |                                 |
|                          |                   | 1=   | 2=63=252=504=2016.              |
| 2 hogsheads              | " 1 pipe—P.       |      |                                 |
| 2 pipes                  | " 1 tun—T.        |      |                                 |

## VII. ALE OR BEER MEASURE.

Besides ale and beer, milk is measured by this measure. The gallon contains 282 cubic inches.

TABLE.

|                                    |  |                                 |                    |
|------------------------------------|--|---------------------------------|--------------------|
| 2 pints (pt.) make 1 quart—qt.     |  |                                 | <i>qt.</i><br>1= 2 |
| 4 quarts     "     1 gallon—gal.   |  | <i>gal.</i><br>1= 4= 8.         |                    |
| 36 gallons   "     1 barrel—bar.   |  | <i>bar.</i><br>1=36=144=288.    |                    |
| 54 gallons   "     1 hogshead—hhd. |  | <i>hhd.</i><br>1=1½=54=216=432. |                    |

## VIII. DRY MEASURE.

This measure is used for measuring all kinds of grain, fruit, salt, coal, &c.

TABLE.

|                                   |  |                                        |                     |
|-----------------------------------|--|----------------------------------------|---------------------|
| 4 gills (gi.) make 1 pint—pt.     |  |                                        | <i>pt.</i><br>1= 4. |
| 2 pints       "     1 quart—qt.   |  | <i>qt.</i><br>1= 2= 8.                 |                     |
| 8 quarts     "     1 peck—pk.     |  | <i>pk.</i><br>1= 8= 16= 64.            |                     |
| 4 pecks       "     1 bushel—bu.  |  | <i>bu.</i><br>1= 4= 32= 64= 256.       |                     |
| 36 bushels   "     1 chaldron—ch. |  | <i>ch.</i><br>1=36=144=1152=2304=9216. |                     |

## IX. LONG MEASURE.

The following denominations and numbers are used for measuring distance.

TABLE.

|                                        |  |                                                     |                     |
|----------------------------------------|--|-----------------------------------------------------|---------------------|
| 3 barley corns (b. c.) make 1 inch—in. |  |                                                     | <i>in.</i><br>1= 3. |
| 12 inches make 1 foot—ft.              |  | <i>ft.</i><br>1= 12= 36.                            |                     |
| 3 feet make 1 yard—yd.                 |  | <i>yd.</i><br>1= 3= 36= 108.                        |                     |
| 5½ yards or 16½ feet 1 rod—rd.         |  | <i>rd.</i><br>1= 5½= 16½= 198= 594.                 |                     |
| 40 rods 1 furlong—fur.                 |  | <i>fur.</i><br>1= 40= 220= 660= 7920= 23760.        |                     |
| 8 furlongs 1 mile—m.                   |  | <i>m.</i><br>1= 8=320=1760= 5280= 63360= 190080.    |                     |
| 3 miles 1 league—L.                    |  | <i>L.</i><br>1=3=24=960=5280=87120=1045440=3136320. |                     |

60 geographic, or 69½ statute miles, make 1 degree on the earth's surface. 360 degrees make the earth's circumference.



## X. LAND OR SQUARE MEASURE.

TABLE.

|                                                        |                        |                                               |
|--------------------------------------------------------|------------------------|-----------------------------------------------|
| 144 square inches (Sq. in.) make 1 square foot—Sq. ft. | <sup>Sq. ft.</sup> 1 = | <sup>Sq. in.</sup> 144.                       |
| 9 square feet make 1 square yard—Sq. yd.               | <sup>S. Yd.</sup> 1 =  | 9 = 1296.                                     |
| 30 $\frac{1}{4}$ square yards 1 square rod—Sq. rd.     | <sup>rd.</sup> 1 =     | 30 $\frac{1}{4}$ = 272 $\frac{1}{4}$ = 39204. |
| 40 square rods—1 square rood—R.                        | <sup>R.</sup> 1 =      | 40 = 1210 = 10890 = 1568160.                  |
| 4 square roods—1 Acre—A.                               | <sup>A.</sup> 1 =      | 4 = 160 = 4840 = 43560 = 6372640.             |
| 640 square acres—1 square mile.                        |                        |                                               |

Land is usually measured by Gunter's chain, which is 4 rods or 66 feet in length. The whole chain is divided into 100 equal parts, called links. The link is therefore  $\frac{1}{100}$  part of the rod, and is 7 $\frac{1}{4}$  inches in length. 80 chains, or 320 rods, make 1 mile in length. 1 square chain makes 16 square rods, and 10 square chains make 1 acre.

## XI. SOLID MEASURE.

This measure is employed in measuring substances which have three dimensions; viz. length, breadth, and thickness. Timber, stone, &c., are among these substances.

TABLE.

|                                                                                                                       |                     |
|-----------------------------------------------------------------------------------------------------------------------|---------------------|
| 1728 solid inches make . . . . .                                                                                      | 1 solid foot—S. ft. |
| 27 solid feet make . . . . .                                                                                          | 1 solid yard—S. yd. |
| 40 feet of round or 50 feet of hewn timber make 1 ton—T.                                                              |                     |
| 128 solid feet make . . . . .                                                                                         | 1 cord—C.           |
| A pile of wood 8 feet long, 4 feet wide, and 4 feet high, contains just one cord, since $8 \times 4 \times 4 = 128$ . |                     |

## XII. CIRCULAR MOTION.

TABLE.

|                     |               |
|---------------------|---------------|
| 60 seconds (") make | 1 minute—'    |
| 60 minutes (')      | " 1 degree—°. |
| 30 degrees          | " 1 Sign—S.   |
| 12 signs or 360°    | " 1 circle—C. |

## XIII. TIME.

|                                                                          |                  |     |    |      |                                  |
|--------------------------------------------------------------------------|------------------|-----|----|------|----------------------------------|
| 60 seconds (sec.)                                                        | make 1 minute—m. | m.  | 1= | sec. | 60.                              |
| 60 minutes                                                               | " 1 hour—h.      | h.  | 1= | 60=  | 3600.                            |
| 24 hours                                                                 | " 1 day—d.       | d.  | 1= | 24=  | 1440= 86400.                     |
| 7 days                                                                   | make 1 week—w.   | w.  | 1= | 7=   | 168= 10080= 604800.              |
| 4 weeks                                                                  | " 1 month—m.     | m.  | 1= | 4=   | 28= 672= 40320= 2419200.         |
| 13 months 1 day and 6 hrs.,<br>or 365 days 6 hours, 1<br>common year—yr. |                  | yr. | 1= | 13=  | 52= 365= 8760= 525960= 31557600. |

## TABLE OF PARTICULARS.

|                      |                      |
|----------------------|----------------------|
| 12 particular things | make 1 dozen—doz.    |
| 12 dozen             | " 1 gross.           |
| 12 gross             | " 1 great gross.     |
| 20 things            | " 1 score.           |
| 24 sheets            | " 1 quire of paper.  |
| 20 quires            | " 1 ream.            |
| 112 pounds           | " 1 quintal of fish. |

1Uz \_\_\_\_\_

## REDUCTION OF COMPOUND NUMBERS.

The scholar is now requested to turn back to the table of English Money, and from it answer the following questions: How many farthings make a penny? How many make 2 pennies? How many make 4? How many make 6? How many make 7? How many make 8? How many 9? How many 10? How many 11? How many pence make 1 shilling? How many make 2 shillings? How many make 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? How many shillings make 1 pound? How many make 2? 3? 4? 5? &c. Which is worth most, 1 penny or 4 farthings? 2 pence or 8 farthings? Which is worth most, 1 shilling or 12 pence? 2 shillings or 24 pence? 1 pound or 20 shillings? 2 pounds or 40 shillings? If these expressions are equal in value, in what do they differ? Ans. They are different expressions for the same value? In what then does Reduction consist? Ans. In changing numbers from one denomination to another without altering the value. Reduce 6 pence to farthings. By what do

you multiply the 6? Ans. By 4. And why? Because 4 farthings make one penny; and consequently, there must be four times as many farthings as pence? Reduce 4 shillings to pence. How do you reduce shillings to pence? Reduce 3 pounds to shillings. How do you reduce pounds to shillings? In these last examples, were high denominations brought into low, or low into high? Ans. High denominations were brought into low. How was it effected? Ans. By Multiplication. Reduce 12 farthings to pence. By what do you divide? Ans. By 4. Why? Because 4 farthings make one penny. Reduce 36 pence to shillings. By what do you divide? Reduce 40 shillings to pounds? By what do you divide? What change is here made in the denomination? Ans. Low denominations are brought into high. How then are low denominations brought into high? Ans. By Division. After a careful examination of the preceding questions and remarks, the scholar will readily perceive the appropriateness of the following definition.

Reduction is an operation by which a number expressing the value of a quantity in one denomination is changed to another number, expressing the same value in a different denomination.

But it has already been shown that high denominations are brought into low by multiplication, and that low denominations are brought into high by division. The scholar, therefore, needs only the rules by which to guide his operation.

**WHEN A HIGHER DENOMINATION IS TO BE REDUCED TO A LOWER.**

**RULE 1st.**—*Multiply the higher denomination by that number which it takes of the LOWER denomination to make ONE of the higher, remembering to add to the product whatever may be given of this lower denomination. If it be required to reduce the quantity still lower, multiply the number already obtained by the number required to reduce it to the next lower denomination, adding to the product the given number of that denomination, if any. Continue the same operation till you come to the required denomination.*

**WHEN A LOWER DENOMINATION IS TO BE BROUGHT TO A HIGHER.**

**RULE 2d.**—*Divide the lower denomination by that number which is required of that denomination to make one of the next*

*higher. The quotient obtained will be of the higher denomination; and if there be any remainder, it will be of the same denomination as the number divided. Divide the quotient again, (if it be not already reduced to as high a denomination as practicable,) by the same general principle; and continue so to do till you have reached the highest denomination of which the given quantity is susceptible.*

Ex. 1. Reduce 6 £. 17 s. 9 d. 3 qr. to farthings.

PERFORMED.

6. 17. 9. 3.

Multiply by 20

120 = shillings in 6 pounds.

17 = given shillings added.

137 = whole number of shillings.

Multiply by 12

1644 = pence in 137 shillings.

9 = given pence added.

1653 = whole number of pence.

Multiply by 4

6612 = farthings in 1653 pence.

3 = given farthings added.

6615 = whole number of farthings.

Each pound is of the value of 20 shillings; therefore, 6 £. = 120 s. and 6 £. and 17 s. = 137 s. Each shilling is of the same value as 12 pence; therefore, 137 s. = 1644 d. and 1644 d. + 9 d. = 1653 d. Each penny = 4 farthings, therefore, 1653 d. = 6612 qr. and 6612 qr. + 3 qr. = 6615 qr.

The scholar will notice that each denomination below the pounds, has been added by a separate process. This is not necessary; it may be added mentally, as in the following example:

2. Reduce 18 £. 13 s. 11 d. 2 qr. to farthings. The thing to be done is the same as before. The scholar will bear in mind that the object of this example is to show that the lower denominations may be added mentally.

## PERFORMED.

$$\begin{array}{r} 18 \text{ £. } 13 \text{ s. } 11 \text{ d. } 2 \text{ qr.} \\ 20 \\ \hline \end{array}$$

$$\begin{array}{r} 373 = \text{shillings in } 18 \text{ £. } 13 \text{ s. ; the } 13 \text{ s. being} \\ 12 \text{ added mentally.} \\ \hline \end{array}$$

$$\begin{array}{r} 4487 = \text{pence in } 373 \text{ s. } 11 \text{ d. ; the } 11 \text{ d. being} \\ 4 \text{ also added mentally.} \\ \hline \end{array}$$

$$17950 = \text{farthings in } 4487 \text{ d. } 2 \text{ qr. ; the } 2 \text{ qr. add-} \\ \text{ed as before.}$$

In the above example, to the product of 18 multiplied by 20, I add 13; that is, I add 3 to the units and 1 to the tens. And when I multiply by 12, I say, 12 times 3 shillings are 36 pence, and the 11 given pence added make 47 pence. I proceed in the same manner in reducing pence to farthings.

The preceding examples will serve to illustrate Rule 1st. An illustration or two will also be given of Rule 2d, that is, of bringing low denominations into high.

3. Reduce 17950 farthings to pounds, shillings, pence, and farthings.

## PERFORMED.

$$4) 17950$$

$$12) 4487 - 2 \text{ qr.} = 4487 \text{ d. } 2 \text{ qr. obtained by first division.}$$

$$20) 373 - 11 \text{ d.} = 373 \text{ s. } 11 \text{ d. obtained by dividing by 12.}$$

$$18 - 13 \text{ s.} = 18 \text{ pounds obtained by dividing by 20,} \\ \text{and } 13 \text{ s. remain.}$$

Since 4 farthings make 1 penny, it is evident that there are as many pence in 17950 qrs. as there are 4's contained in it. The same reasoning may be applied to the other divisors.

It will be observed, that in this last example we have reversed what was done in the second example. We there had the same *value* given, which we have here; but were required to change it from a higher to a lower denomination, instead of from a lower to a higher, as in the last example.

4. Reduce 44447 farthings to pence, shillings, and pounds.

PERFORMED.

$$\begin{array}{r} 4 \overline{) 44447} \end{array}$$

$$12 \overline{) 11111} - 3 \text{ qr. remain.}$$

$$20 \overline{) 925} - 11 \text{ pence remain.}$$

$$46 - 5 \text{ shillings remain.}$$

The answer, therefore, is 46 £. 5 s. 11 d. 3 qr.

5. Reduce 22685 qr. to pounds, &c.

PERFORMED.

$$\begin{array}{r} 4 \overline{) 22685} \end{array}$$

$$12 \overline{) 5671} - 1 \text{ qr. remains.}$$

$$20 \overline{) 472} - 7 \text{ pence remain.}$$

$$23 - 12 \text{ s. remain.}$$

The answer is, 23 £. 12 s. 7 d. 1 qr.

6. Reduce 7195 pence to pounds, &c.

The scholar will observe that the given number is already pence.

PERFORMED.

$$\begin{array}{r} 12 \overline{) 7195} \end{array}$$

$$20 \overline{) 599} - 7 \text{ d. remain.}$$

$$29 - 19 \text{ s. remain.}$$

The answer then, is, 29 £. 19 s. 7 d.

The scholar must first consider whether the quantity given is to be brought from a higher denomination to a lower, or from a lower denomination to a higher. When this is determined, let him apply the corresponding rule.

## TABLE I.—ENGLISH MONEY.

6. Reduce 23 £. 12 s. 7 d. 1 qr. to farthings. *Ans.* 22685 qr.

N. B. 23 £. 12 s. 7 d. 1 qr. = 22685 qr. So in all operations of mere reduction, the quantity given equals in value the quantity obtained

7. Reduce 71 £. 13 s. 2 d. 3 qr. to farthings. *Ans.* 68795 qr.

8. Reduce 299924 qr. to pounds, shillings, &c. *Ans.* 312 £.  
8 s. 5 d.
9. Reduce 68795 qr. to pounds, shillings, &c. *Ans.* 71 £.  
13 s. 2 d. 3 qr.
10. Reduce 46 £. 5 s. 11 d. 3 qr. to farthings. *Ans.* 44447.
11. Reduce 29 £. 19 s. 7 d. to pence and farthings. *Ans.*  
7195 d.; 28780 qr.
12. Reduce 5974681369 qr. to pounds, &c. *Ans.* 6223626  
£. 8 s. 6 d. 1 qr.
13. Reduce 7195 pence to pounds, &c. *Ans.* 29 £. 19 s. 7 d.
14. Reduce 40320 half-pence to pounds. *Ans.* 84 £.
15. Reduce 125 £. 19 s. 11 d. 3 qr. to farthings. *Ans.*  
120959 qr.
16. Reduce 475 dollars, at 6 shillings each, to pence. *Ans.*  
34200 pence.
17. Reduce 312 £. 8 s. 5 d. to half-pence. *Ans.* 149962  
half-pence.
18. Reduce 121 pistoles, at 22 shillings each, to pence and  
farthings. *Ans.* 31944 d.; 127776 qr.
19. Reduce 34200 pence to dollars, at 6 shillings each.  
*Ans.* 475 dollars.
20. Reduce 359548 qr. to pistoles, at 22 shillings each.  
*Ans.* 340 pistoles, 10 s. 7 d.
21. Reduce 740 dollars, at 8 shillings each, to pence. *Ans.*  
71040 pence.
22. Reduce 79 £. to pence and half-pence. *Ans.* 18960 d.;  
37920 half-pence.

## APPLICATION OF TABLE II.—TROY WEIGHT.

Ex. 1. Reduce 23 lb. 9 oz. 6 pwt. 22 gr. to grains.

PERFORMED.

23. 9. 6. 22.

12 oz. = 1 lb.

---

285 = ounces in 23 lb. 9 oz.

20 pwt. = 1 oz.

---

5706 = pwt. in 285 oz. and 6 pwt.

24 gr. = 1 pwt.

---

22846

11412

---

*Ans.* 136966 = grains in 5706 pwt. and 22 gr.

2. Reduce 8578 grains to pounds, &c.

PERFORMED.

$$\begin{array}{r} 2.4 \overline{) 8578} \\ \hline \end{array}$$

$$20 \overline{) 357} \text{---} 10 \text{ gr. remain.}$$

$$12 \overline{) 17} \text{---} 17 \text{ pwt. remain.}$$

$$1 \text{---} 5 \text{ oz. remain.}$$

*Ans.* 1 lb. 5 oz. 17 pwt. 10 gr.

3. Reduce 62 lb. 7 oz. 14 pwt. 18 gr. to grains. *Ans.* 360834 gr.

4. Reduce 360834 grains to pounds. *Ans.* 62 lb. 7 oz. 14 pwt. 18 gr.

5. Reduce 1 lb. 11 oz. 19 pwt. 23 gr. to grains. *Ans.* 11519 gr.

6. Reduce 11519 grains to pounds, &c. *Ans.* 11 lb. 11 oz. 19 pwt. 23 grains.

APPLICATION OF TABLE III.—AVOIRDUPOIS WEIGHT.

- Ex. 1. Reduce 4 cwt. 3 qr. 26 lb. 10 oz. 12 dr. to drams.

PERFORMED.

$$4. \quad 3. \quad 26. \quad 10. \quad 12.$$

4 Multiply by 4, because 4 quarters = 1 cwt.

$$\begin{array}{r} 19 \\ \hline \end{array}$$

28 Multiply by 28, because 28 pounds = 1 qr.

$$\begin{array}{r} 178 \\ \hline \end{array}$$

$$\begin{array}{r} 38 \\ \hline \end{array}$$

$$\begin{array}{r} 558 \\ \hline \end{array}$$

16 Multiply by 16, because 16 oz. = 1 lb.

$$\begin{array}{r} 3358 \\ \hline \end{array}$$

$$\begin{array}{r} 558 \\ \hline \end{array}$$

$$\begin{array}{r} 8938 \\ \hline \end{array}$$

16 Multiply by 16 again, because 16 dr. = 1 oz.

$$\begin{array}{r} 53640 \\ \hline \end{array}$$

$$\begin{array}{r} 8938 \\ \hline \end{array}$$

$$\begin{array}{r} 143020 \\ \hline \end{array}$$



The same reversed :

$$16 \overline{) 143020}$$

$$16 \overline{) 8938} - 12 \text{ dr. remain.}$$

$$28 \overline{) 558} - 10 \text{ oz.} \quad "$$

$$4 \overline{) 19} - 26 \text{ lb.} \quad "$$

$$4 - 3 \text{ qr.} \quad "$$

*Ans.* 4 cwt. 3 qr. 26 lb. 10 oz. 12 dr.

2. Reduce 7 cwt. 3 qr. 19 lb. to ounces. *Ans.* 14192 ounces.

3. Reduce 14192 ounces to pounds, quarters, &c. *Ans.* 7 cwt. 3 qr. 19 lb.

4. Reduce 29548 ounces to hundreds, quarters, &c. *Ans.* 16 cwt. 1 qr. 26 lb. 12 oz.

5. Reduce 480 drams to pounds, &c. *Ans.* 1 lb. 14 oz.

6. Reduce 12 tons to ounces. *Ans.* 430080 oz.

7. Reduce 430080 ounces to tons. *Ans.* 12 tons.

#### APPLICATION OF TABLE IV.—APOTHECARIES' WEIGHT.

Ex. 1. Reduce 8 lb. 8 3/4. 5 3/4. 2 3/4. 12 gr. to grains.

$$8. \quad 8. \quad 5. \quad 2. \quad 12.$$

$$12 \text{ oz.} = 11 \text{ b.}$$

$$\begin{array}{r} 104 \\ \hline \end{array}$$

$$83 = 1 \frac{3}{4}.$$

$$\begin{array}{r} 837 \\ \hline \end{array}$$

$$3 \frac{3}{4} = 13.$$

$$\begin{array}{r} 2513 \\ \hline \end{array}$$

$$20 \text{ gr.} = 1 \frac{3}{4}.$$

$$50272 \text{ } \textit{Ans.}$$

The same reversed :

$$20 \overline{) 50272}$$

$$3 \overline{) 2513} - 12 \text{ remain.}$$

$$8 \overline{) 837} - 2 \quad "$$

$$12 \overline{) 104} - 5 \quad "$$

$$8 - 8 \quad "$$

3. Reduce 27 lb. 9  $\frac{3}{4}$ . 6  $\frac{3}{4}$ . 2  $\odot$ . to scruples. *Ans.* 8012 scruples.

4. Reduce 477816 grains to pounds, &c. *Ans.* 82 lb. 11  $\frac{3}{4}$ . 3  $\frac{3}{4}$ . 1  $\odot$ . 16 gr.

5. Reduce 6 lb. 7  $\frac{3}{4}$ . 6  $\frac{3}{4}$ . 1  $\odot$ . 12 gr. to grains. *Ans.* 38312 grains.

6. Reduce 6348 scruples to pounds. *Ans.* 22 lb. 4  $\frac{3}{4}$ .

7. Reduce 480 drams to pounds. *Ans.* 5 lb.

8. Reduce 1 lb. 1  $\frac{3}{4}$ . 1  $\frac{3}{4}$ . 1  $\odot$ . 1 gr. to grains. *Ans.* 6321 grains.

#### APPLICATION OF TABLE V.—CLOTH MEASURE.

Ex. 1. Reduce 30 yd. 3 qr. 3 na. to nails.

PERFORMED.

30. 3. 3.

4=1 yd.

---

123

4=1 qr.

---

495

The same reversed: 495 nails=how many qr?

4)495

---

4)123—3 na. remain.

---

30 yd.—3 qr. “

2. Reduce 450 E. E. 3 qr. 2 na. to nails. *Ans.* 9014 nails.

3. Reduce 678 ells Flemish to nails. *Ans.* 8136 nails.

4. Reduce 8136 nails to ells Flemish. *Ans.* 678 ells.

5. Reduce 9014 nails to ells English. *Ans.* 450 E. E. 3 qr. 2 na.

6. Reduce 12 ells French, 5 qr. 3 na. to nails. *Ans.* 311 nails.

7. Reduce 622 nails to ells French. *Ans.* 25 ells, 5 qr. 2 na.

#### APPLICATION OF TABLE VI.—WINE MEASURE.

Ex. 1. Reduce 3 hhd. 42 gal. 2 qt. 1 pt. to pints.

## REDUCTION.

91

## PERFORMED.

$$\begin{array}{r}
 3. \ 42. \ 2. \ 1. \\
 63 \\
 \hline
 11 \\
 22 \\
 \hline
 231 \\
 4 \\
 \hline
 926 \\
 2 \\
 \hline
 \end{array}$$

1853 Ans.

The same reversed:  $2 \overline{) 1853}$  $4 \overline{) 926} - 1 \text{ remains.}$  $63 \overline{) 231} - 2 \quad "$  $3 - 42 \quad "$ 

Ans. 3 hhd. 42 gal. 2 qt. 1 pt.

2. Reduce 6 pipes, 1 hogshead, and 1 gill, to gills. Ans. 26209 gills.

3. Reduce 30000 gills to pipes. Ans. 7 pipes, 55 gal. 2 qt.

4. Reduce 20 tuns to gills. Ans. 161280 gills.

5. Reduce 5 hhd. 56 gal. 2 qt. to pints. Ans. 2972 pints.

6. Reduce 16 barrels, 21 gal. to quarts. Ans. 2100.

## APPLICATION OF TABLE VII.—ALE OR BEER MEASURE.

1. Reduce 4 hhd. 45 gal. 3 qt. to pints.

$$\begin{array}{r}
 4. \ 45. \ 3. \\
 54 \\
 \hline
 21 \\
 24 \\
 \hline
 261 \\
 4 \\
 \hline
 1047 \\
 2 \\
 \hline
 2094 \text{ Ans.}
 \end{array}$$

The same reversed:  $2 \overline{) 2094}$

$4 \overline{) 1047}$

$54 \overline{) 261}$ —3 qt. remains.

4—45 gal. "

2. Reduce 47 barrels of beer to pints. *Ans.* 13536 pints.
3. Reduce 13672 pt. to barrels, &c. *Ans.* 47 bar. 17 gal.
4. Reduce 451 bar. 7 gal. to quarts. *Ans.* 64972 quarts.
5. Reduce 21 hhd. to quarts. *Ans.* 4536 quarts.
6. Reduce 6562 pints to hogsheds. *Ans.* 15 hogsheds, 10 gal. 1 qt.

#### APPLICATION OF TABLE VIII.—DRY MEASURE.

- Ex. 1. Reduce 6 ch. 9 bu. 3 pk. to gills. *Ans.* 57792 gills.
2. Reduce 87762 gills to chaldrons. *Ans.* 9 ch. 18 bu. 3 pk. 2 qt. 2 gi.
3. In 136 bushels, how many pecks, quarts, and pints? *Ans.* 544 pk. 4352 qt. 8704 pt.
4. Reduce 10640 pints to bushels. *Ans.* 166 bu. 1 pk.
5. Reduce 3 pecks to gills. *Ans.* 192 gills.
6. Reduce 720 quarts to bushels. *Ans.* 22 bu. 2 pk.

#### APPLICATION OF TABLE IX.—LONG MEASURE.

1. Reduce 8 leagues, 2 miles, 6 furlongs, 16 rods, 3 yards, 2 feet, 9 inches, and 2 barley corus, to barley corns. *Ans.* 5094569 b. c.
2. How many barley corns will reach round the earth, it being 360 degrees? *Ans.* 4755801600.
3. Reduce 48765000 barley corns to miles, &c. *Ans.* 256 m. 4 fur. 15 rods, 15 ft. 10 inches.
4. Reduce 26431 rods to miles, &c. *Ans.* 82 m. 4 fur. 31 rods.
5. Reduce 1710720 inches to miles. *Ans.* 27 miles.
6. Reduce 7 fur. 36 rods, and 9 ft. to inches. *Ans.* 62676.

#### APPLICATION OF TABLE X.—LAND OR SQUARE MEASURE.

1. Reduce 500 acres to square rods. *Ans.* 80000 rods.
2. Reduce 32000 rods or poles to acres. *Ans.* 200 acres.
3. Reduce 3 square miles to square rods. *Ans.* 307200.
4. Reduce 458000 square rods to square miles, &c. *Ans.* 4 sq. m. 302 acres, 2 roods.

5. Reduce 6272640 square-inches to acres. *Ans.* 1 acre.

*Note.*—For figures to illustrate square and solid measure, the scholar is referred to square and cube root.

#### APPLICATION OF TABLE XI.—SOLID MEASURE.

1. In a pile of wood containing 2 cords and 64 feet, how many solid inches? *Ans.* 552960.

2. Reduce 884736 solid inches of wood to cords. *Ans.* 4 cords.

3. Reduce 6117120 cubic inches to tons of round timber. *Ans.* 88 tons, 20 ft.

4. Reduce 72 tons of hewn timber to cubic inches. *Ans.* 6220800.

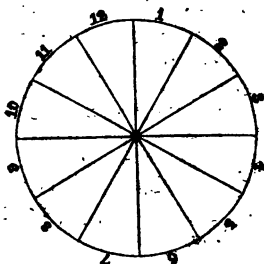
#### APPLICATION OF TABLE XII.—CIRCULAR MOTION.

1. Reduce 6 signs, 21 degrees, 40 minutes, to seconds. *Ans.* 726000 seconds.

2. Reduce 432000 seconds to signs. *Ans.* 4 signs.

3. Reduce 1, circle, 6 signs, 25 degrees, to minutes. *Ans.* 33900 m.

4. Reduce 45200 minutes to circles, &c. *Ans.* 2 cir. 1 S. 3°. 20'.



It is desirable that the scholar should obtain correct views of this measure. The adjoining figure will illustrate its application. The circle is regarded as an integral object. Its first division is into 12 signs. These are represented by the figures, 1, 2, 3, 4, &c. Each sign is divided into 30 equal parts: these constitute degrees; and as the circle contains 12 signs, it is evident the whole circle must contain 360 degrees. These again are divided into minutes, and the minutes into seconds. The degrees, minutes, and seconds are not represented in the figure. This measure is obviously applied to bodies moving in a circle; such as all wheels in machinery, the revolutions of the heavenly bodies, &c.

## APPLICATION OF TABLE XIII.—TIME.

1. Reduce 360 years, 300 days, 20 hours, 50 minutes, and 37 seconds, to seconds. *Ans.* 11386731037.

In the above sum, 365 days and 6 hours are allowed to the year.

2. Reduce 662709600 seconds to years, &c. allowing the year to be as above. *Ans.* 21 years.

3. Reduce 49 weeks to seconds. *Ans.* 29635200 sec.

4. Reduce 59270400 seconds to years, &c. *Ans.* 1 year, 46 weeks.

5. Suppose my age to be 21 years, how many seconds have I lived? *Ans.* 662709600.

6. Reduce 1325419200 seconds to hours. *Ans.* 368172 hours.

The following particulars require to be introduced here. The year, as given above, contains 365 days and 6 hours. In four years, this six hours amounts to 24 hours, or one day. Hence every fourth year has 366 days. This is called Bis-sextile or Leap year. As it requires four years to gain this one day, it is obvious that the Leap year may be found by dividing the given year by 4. If it be Leap year it will divide without remainder. If in dividing there be 1 remaining, the year under consideration is the first after Leap year. If there be 2 remaining, it is the second; and if 3 remain, it is the third after Leap year. Thus 1824, is Leap year, for  $1824 \div 4 = 456$ , and *nothing remains*. The same operation shows 1826 to be the second after Leap year. In the table the year is divided into 13 months. These are lunar months. It is much more usually divided into 12 calendar months, containing each the following number of days, viz. April, June, September, and November, 30; January, March, May, July, August, October, December, 31; and February, 28. To this last month, (February,) the additional day of the Leap year is added, so that every fourth year, this month has 29 days.

## PROMISCUOUS EXAMPLES.

1. In 64126 gills, how many bushels? *Ans.* 250 bu. 1 pk. 7 qt. 1 pt. 2 gi.

2. In 26709912 barley corns, how many leagues? *Ans.* 46 l. 2 m. 4 fur. 6 rods, 1 yd.

3. In 161280 gills, how many tuns of wine? *Ans.* 20.

4. In 10 cords of wood, how many solid inches? *Ans.* 2211840.

5. In 20 hhd. of sugar, each 12 cwt. how many pounds? *Ans.* 26880.

6. How many gills in 250 bu. 1 pk. 7 qt. 1 pt. 2 gills? *Ans.* 64126.

7. How many pence are there in 16 bags, containing each 24 guineas, 16 shillings, and 8 pence, the guinea being 28 s.? *Ans.* 132224.

8. In 15840 yards, how many leagues? *Ans.* 3.

9. In 1876742 solid inches, how many cords, &c.? *Ans.* 8 cords, 62 ft. 134 in.

Thus far, examples have been avoided, which in their solution required both multiplication and division. They will here be introduced. Let us take the following. How many ells French in 15 pieces of cloth, containing each 20 yards? Now it is evident that yards cannot be changed to ells French, at a single step. When the question is, how many times one quantity is contained in another, a simple operation of division is often all that is required, to obtain the answer. But that will not suffice here, and the reason is, the two quantities are not in the same denomination. The first step then, is, to bring the two quantities given to the same kind. The scholar will therefore turn to Table 5th, Cloth Measure. He will there find that the ell French and the yard, may both be brought into quarters; the former, by being multiplied by 6, and the latter, by 4. Therefore,

$$\begin{array}{r}
 1.5 \text{ pieces.} \\
 20 = \text{yd. in a piece.} \\
 \hline
 300 = \text{yd. in 15 pieces.} \\
 4 = \text{qr. in a yard.} \\
 \hline
 6) 1200 = \text{qr. in 300 yards, or 15 pieces.} \\
 \hline
 200 = \text{ells French, the quantity required.}
 \end{array}$$

There are evidently as many ells French as there are 6's in 1200 qr. as 6 qr. make one ell French.

The scholar will perceive that the given quantity must be brought first into a denomination, from which it may be changed to the required denomination.

The above work may be abbreviated by applying to the solution the principle explained in the rule for canceling. The numbers 15, 20, and 4, are multiplied together and produce the dividend, 1200. This is then divided by 6, and the result is the answer. The scholar may therefore turn back to the rule for canceling; and he will see that the following statement is in accordance with it; viz.  $\frac{15. 20. 4}{6}$ . This statement, by Sec.

5th, of the same rule, may be reduced to  $\frac{5. 20. 4}{2}$ ; and again, by Sec. 4th, to  $\frac{5. 20. 2}{0}$ ; and by 6th and 7th Sec. to 200, answer as before.

We will now give the solution at a single statement, thus :

$$\frac{5}{15} \cdot \frac{2}{20} \cdot \frac{4}{4} = 200 \text{ ells French, the answer.}$$

2. How many barrels, each holding 2 bushels and 3 pecks, are required to contain 880 bushels of corn?

The 2 bushels and 3 pecks equal 11 pecks. The question then is, how many times are 11 pecks contained in 880 bushels. By the preceding solution, it will be seen that the bushels, as they are to be divided by 11 pecks, must also be brought into pecks. Therefore,

$$\begin{array}{r} 880 \\ 4 = \text{pecks in one bushel.} \\ \hline \text{One barrel} = 11 \text{ pecks, therefore, } 11 \overline{) 3520} = \text{pecks in 880 bushels.} \\ \hline 320 = \text{number barrels required.} \end{array}$$

The same canceled. The scholar will read over the explanation of the preceding sum, if he does not yet understand the work.

Statement,  $\frac{880. 4}{11}$ . Canceled, (see Sec. 4, rule for canceling,)  $\frac{880. 4}{11}$ , and  $80 \times 4 = 320$  barrels, the same as before.

3. In 33 guineas, at 28 shillings each, how many pistoles, each 22 shillings?

The simple question is, how many pistoles are there in 33



guineas. The guineas cannot be divided by the pistoles, for they are of different value. They must be brought upon some common ground, and the nearest is that of shillings; therefore,

$$\begin{array}{r} 33 \\ 28 \\ \hline 264 \\ 66 \\ \hline \end{array}$$

22 s. = 1 pistole, therefore, 22 ) 924 = shillings in 33 guineas.

42 = number of pistoles in the guineas.

The same canceled :  $\frac{33.28}{22}$ . (See rule for canceling, Sec. 4th

and 5th.) Performed,  $\frac{314}{22}$ , and  $14 \times 3 = 42$ , the number of pistoles, as before.

4. Purchased 24 hogsheads of wine, at 1 s. 8 d per quart, and paid for the same with cloth, for which I was allowed 3 s. 4 d. per yard. How much cloth was required?

The price of the quart being given, it is obvious that the 24 hhd. must first be brought to quarts. This is done by multiplying the 24 by 63 and by 4. Each quart is worth 1 s. 8 d. = 20 d. Therefore, multiplying the quarts by 20 d. gives the cost of the wine in pence. Again, each yard of cloth is worth 3 s. 4 d. = 40 d. If, then, the pence the wine cost be divided by the price of one yard of cloth, the quotient obtained must be the number of yards required; therefore, statement for canceling,  $\frac{24.63.4.20}{40}$ . Let this statement be compared with the

above analysis of the sum, and with the rule for stating sums for canceling. Statement repeated and canceled,  $\frac{24.63.4.20}{40}$ ;

therefore,  $24 \times 63 \times 2 =$  the number of yards required; viz. 3024 yds.

We will now give the usual solution of this sum, and observe the increased facility of canceling.

$$\begin{array}{r} 24 \\ 63 \\ \hline \end{array}$$

$$72$$

$$144$$

$$1512 = \text{the gallons in 24 hhd.}$$

$$4$$

$$6048 = \text{qt. in the same.}$$

$$20$$

$$4,0) 12096,0 = \text{cost of the same in pence.}$$

3024 = yds. of cloth required, the same as before.  
40 d. = price of 1 yd. of cloth; therefore I divide by 40.

The scholar will readily perceive the object to be attained in sums of this character; viz. to exchange dissimilar quantities, or quantities of different denominations, but of equal value.

**RULE FOR THE COMMON MODE OF OPERATION.**—Reduce the quantity to be exchanged to the denomination in which the price, or the equivalent of exchange of the other kind, is given; then divide by this price, or equivalent of exchange, and perform such operations of reduction as the nature of the case may require.

**RULE FOR CANCELING.**—Consider what is the quantity to be exchanged, and place it over a horizontal line towards the left. Then on the right of this, also above the line, place such numbers as are required to reduce this quantity to the denomination in which the price, or equivalent of exchange of the other kind is given. Write also under the line those numbers which are necessary to reduce this price, or equivalent of exchange, to the required denomination. Proceed to cancel, multiply, and divide, as directed in the rule for canceling, and the number obtained will be the one sought.

**Note 1.**—In stating for canceling, care should be taken to introduce into the statement every number required for the complete solution of the same, including all operations of reduction, &c., since each number introduced increases the opportunity for canceling, and thus abbreviates the operation.

*Note 2.*—If any one term consists of more than one denomination, it should be reduced to the lowest denomination given before stating.

5. How many times will a wheel 18 feet in circumference, turn round in traveling 84 miles?

The thing to be done is to change 84 miles into revolutions of the wheel. To do this, 84 miles must be reduced to feet, because the distance required for one revolution is given in feet, viz. 18. Therefore,

$$\begin{array}{r}
 84 \\
 \times 8 \\
 \hline
 672 = \text{the furlongs in 84 miles.} \\
 40 \\
 \hline
 26880 = \text{the rods in 84 miles.} \\
 16\frac{1}{2} \\
 \hline
 161280 \\
 26880 \\
 13440 \\
 \hline
 443520 = \text{the feet in 84 miles.}
 \end{array}$$

Then,  $18 \overline{) 443520} (24640 \text{ Ans.}$

$$\begin{array}{r}
 36 \\
 \hline
 83 \\
 72 \\
 \hline
 115 \\
 108 \\
 \hline
 72 \\
 72 \\
 \hline
 000
 \end{array}$$

The preceding solution is by the first rule. We will now solve the sum by canceling. Statement,  $\frac{84. 8. 40. 16\frac{1}{2}}{18}$ . Observe that the numbers above the line are those multiplied together for a dividend in the preceding operation, and that the one below the line was the divisor.

To avoid the fraction in the statement,  $16\frac{1}{2}$  may be written  $\frac{33}{2}$ ; for since 1 unit = 2 halves or  $\frac{2}{2}$ , 16 units = 32 halves or  $\frac{32}{2}$ , and  $16\frac{1}{2} = 33$  halves or  $\frac{33}{2}$ . The preceding statement may

therefore be written,  $\frac{84. 8. 40. 33}{18. 2}$ . Canceled,  $\frac{14. 4. 11}{84. 8. 40. 33}$   
 $\frac{18. 2}{8}$

Then,  $\begin{array}{r} 40 \\ 11 \end{array}$

$\begin{array}{r} 440 \\ 4 \end{array}$

$\begin{array}{r} 1760 \\ 14 \end{array}$

$\begin{array}{r} 7040 \\ 1760 \end{array}$

$\begin{array}{r} 24640 \end{array}$

24640 The same Ans. as before.

The scholar will observe that the 4th and 5th Sec. of the rule for canceling, have been applied in the preceding solution.

6. In 30 purses containing 20 guineas each, how many pounds? *Ans.* 840.

Statement for canceling,  $\frac{30. 20. 28}{20}$ . The 28 above the line expresses the shillings in a guinea, and the 20 below it, the shillings in a pound. The scholar may perform the solution.

7. How many pounds in money will 9 tuns of wine cost at 3 s. 4 d. per gallon? 3 s. 4 d. = 40 d. Statement for canceling,  $\frac{9. 2. 2. 63. 40}{12. 20}$ . *Ans.* 378 £.

For the terms in this statement the scholar is referred to Tables 1st and 6th, of the Compound Numbers.

8. How many times will a wheel 12 feet 6 inches in circumference, revolve in traveling 124-miles? *Ans.* 53377 $\frac{1}{2}$ .

Statement—12 ft. 6 in. = 150 in.  $\frac{124. 8. 40. 16\frac{1}{2}. 12}{150}$ . (See Table 9th, Compound Numbers.)

The last three sums have been stated by the rule for cancel-

ing only, because that is regarded as *superior* to the common mode of solution. The scholar will feel at liberty to adopt either mode of operation. The following sums are not stated, that the scholar may exercise his own judgment.

9. How long will it take to count 6000000, at the rate of 75 per minute. *Ans.* 55 $\frac{1}{2}$  days.

10. In 107520 pounds of sugar, how many hogsheads, each containing 6 cwt. *Ans.* 160.

11. If one quart of melasses cost 10 pence, how much will 12 hogsheads cost? *Ans.* 126 £.

12. How many dollars, each 8 s., will it cost to ride 45 leagues, at 6 pence a mile? *Ans.* \$8.437+.

13. How much will 540 yards of cloth cost at 3 s. 4 d. per yard, in dollars, at 6 shillings each? *Ans.* \$300.

14. How many dozen of gallon, quart, and pint bottles, of each an equal number, are contained in a cistern holding 144 gallons. *Ans.* 8 $\frac{1}{3}$  dozen.

15. How many casks, each containing 1 bushel 1 peck, are required to hold 145 bushels? *Ans.* 116.

16. I have five hogsheads of wine, 63 gallons each, which I wish to put into gallon, quart, and pint bottles, of each an equal number; how many will be required? *Ans.* 229, and 1 pint of wine would be left.

17. In 16 cwt. 3 qr. 20 lb., how many parcels, each containing 36 lb? *Ans.* 52 $\frac{1}{3}$ .

18. In 56 ells Flemish, how many yards? *Ans.* 42.

19. In 144 yards, how many ells French? *Ans.* 96.

20. In 472 parcels of sugar, each 72 pounds, how many cwt.? *Ans.* 303 cwt. 1 qr. 20 lb.

21. If 15 casks of flour contain 4000 lb., how many cwt. are there in each? *Ans.* 2 cwt. 1 qr. 14 $\frac{1}{2}$  lb.

22. In 8 lb. of drugs, how many parcels, each 12 drams? *Ans.* 64.

23. In 80 parcels, each 15 drams, how many pounds? *Ans.* 12 $\frac{1}{2}$ .

24. How many revolutions will a wheel 18 feet 4 inches in circumference, make in traveling 300 miles? *Ans.* 86400.

25. How many cups, each weighing 22 oz. may be made of 25 lb. 6 oz. of silver. *Ans.* 13 cups, and 20 oz. silver remain.

26. How much would 1008 nails of cloth cost, at 10 pence per yard, in dollars, at 6 shillings each? *Ans.* \$8.75.

27. In 4 bales of cloth, each 15 pieces, and each piece 16 ells English, how many ells French? *Ans.* 800.

28. In 6 bales, each 12 pieces, and each piece 18 yards, how many ells Flemish? *Ans.* 1728.

29. In 4 ingots of silver, each weighing 2 lb. 6 oz. 11 pwt. how many grains? *Ans.* 59656.

30. How many hours, minutes, and seconds in one year? *Ans.* 8766 hr. 525960 min. and 31557600 sec.

31. In 1697 quarts, how many bushels, &c.? *Ans.* 49 bu. 3 pk. and 5 qt.

**QUESTIONS.**—What are compound numbers? How do they increase? What are included under this head? Let the 14 tables of Compound Numbers be made familiar, before the scholar proceeds with Reduction.

What is Reduction? How are high denominations brought into low denominations? And how are low denominations brought into high? What is the rule when high denominations are brought into low? And what is it, when low denominations are brought into high? What should the scholar notice before commencing to reduce any quantity? In circular measure, how is the circle regarded? What are the divisions of the circle? To what is this measure applied? How many days and hours does the year contain? To what do the 6 hours amount in 4 years? How many days does every fourth year contain? What is this fourth year called? How may it be found? In dividing the given year by four, what does the figure that remains (if any) show? What is the more usual division of the year? How many days are contained in each of the 12 months? When quantities are to be exchanged, what is the rule for the common mode of operation? What is the rule for canceling? What is Note 1st? What is Note 2d?

## COMPOUND RULES.

The scholar will recollect, that in simple numbers, the denominations increase in value in the constant ratio of 10.

The peculiarity of the numbers in the preceding rule, and in the four next following, is, that they have no uniform ratio of increase, common to all denominations; but each denomination has its own peculiar ratio. These ratios are all represented in the tables of Compound Numbers.

In simple numbers, 10 units make 1 ten; 10 tens make 1 hundred; 10 hundred make 1 thousand, &c. In operations with these numbers, we therefore carry for 10.

In the table of English money, 4 farthings make 1 penny; 12 pence 1 shilling, and 20 shillings 1 pound. For the same

reason, therefore, that we carry for 10 in simple numbers, we carry for 4, 12, and 20, in operations with pounds; shillings, pence, and farthings; that is, from farthings to pence, we carry for 4, because 4 farthings make 1 penny; from pence to shillings, by 12, for a similar reason; and from shillings to pounds, for 20. The same general principle and reasoning may be applied to the other compound tables.

There is one peculiarity noticeable in *writing* compound numbers. In simple numbers, we always know that any figure sustains a *ten-fold* relation to the figures next it; that is, the one on the left of it is of 10 times more value; and the one on the right, of 10 times less value than they would be in its own place. Hence, all that is here necessary, is that the figures preserve their proper order. In *compound numbers*, each denomination is known *only* by its own appropriate mark. There is, therefore, an obvious necessity for each denomination to be separately written.

## COMPOUND ADDITION.

Compound Addition is an operation by which several numbers of different denominations, as pounds, shillings, pence, &c. are united together. The rule to be observed in writing down these numbers is, to place those of the same name under each other.

Let it be required to add together 3 £. 15 s. 9 d. 3 qr.; 5 £. 6 s. 8 d. 2 qr.; 8 £. 13 s. 11 d. 3 qr.; and 10 £. 12 s. 8 d. 2 qr.. The following is a convenient mode of writing them:

| £.    | s. | d. | qr. |
|-------|----|----|-----|
| 3     | 15 | 9  | 3   |
| 5     | 6  | 8  | 2   |
| 8     | 13 | 11 | 3   |
| 10    | 12 | 8  | 2   |
| <hr/> |    |    |     |
| 28    | 9  | 2  | 2   |

The amount of the right hand column is 10 farthings = 2 d. and 2 qr. Like simple numbers, the 2 qr. are set down; and the 2 d. added to the column of pence, the amount of which = 38 d. = 3 s. and 2 d. Setting down the 2 d. and carrying the 3 s. to the column of shillings, we make this column = 49 s. = 2 £. and 9 s. Lastly, setting down

and carrying as before, we find the amount of the column of pounds to be 28, which we write at the foot of the column. We therefore find the amount of the four given numbers to be 28 £. 9 s. 2 d. 2 qr.

- From the preceding example, the scholar will see the appropriateness of the following rule :

**RULE.**—Write the numbers so that each denomination shall occupy a separate column. Then, commencing with the lowest denomination, add each column by itself.

Notice at the addition of each column, to how many of the denomination next above, the amount obtained is equal, and how many remain. Write down those that remain, and carry the other number to the next column. Proceed thus through all the denominations.

**Note.**—The whole amount of the left hand column must be written down, if it be in the highest denomination. If it be not in the highest denomination it should be reduced as far as practicable.

2.

| £.    | s. | d. | qr. |
|-------|----|----|-----|
| 27    | 15 | 6  | 3   |
| 13    | 7  | 8  | 1   |
| 24    | 16 | 9  | 0   |
| 3     | 4  | 11 | 2   |
| <hr/> |    |    |     |
| 69    | 4  | 11 | 2   |

The column of farthings amounts to 6 qr. = 1 d. and 2 qr. The column of pence is 35 d. = 2 s. 11 d. The column of shillings is 44 s. = 2 £. and 4 s. and the column of pounds = 69 £. Carrying and setting down agreeably to rule, we obtain the annexed amount.

3.

| £.    | s. | d. | qr. |
|-------|----|----|-----|
| 0     | 15 | 7  | 0   |
| 0     | 14 | 6  | 3   |
| 0     | 8  | 3  | 1   |
| 0     | 18 | 11 | 2   |
| <hr/> |    |    |     |
| 2     | 17 | 4  | 2   |

The farthings are 6 = 1 d. 2 qr. The pence are 28 = 2 s. 4 d. The shillings are 57 = 2 £. 17 s. which is written agreeably to the above note.

4.

| £.    | s. | d. | qr. |
|-------|----|----|-----|
| 8     | 12 | 9  | 2   |
| 31    | 6  | 11 | 0   |
| 42    | 18 | 3  | 1   |
| 2     | 3  | 8  | 3   |
| <hr/> |    |    |     |
| 85    | 1  | 8  | 2   |

5.

| £.    | s. | d. | qr. |
|-------|----|----|-----|
| 57    | 11 | 11 | 1   |
| 27    | 13 | 2  | 3   |
| 48    | 9  | 6  | 1   |
| 73    | 10 | 9  | 2   |
| <hr/> |    |    |     |

6.

| £.    | s. | d. |
|-------|----|----|
| 67    | 18 | 10 |
| 150   | 19 | 6  |
| 175   | 16 | 8  |
| 37    | 14 | 7  |
| <hr/> |    |    |



| 7.  |    |    | 8.  |    |    | 9.   |    |    |     |
|-----|----|----|-----|----|----|------|----|----|-----|
| £.  | s. | d. | £.  | s. | d. | £.   | s. | d. | qr. |
| 1   | 1  | 1  | 54  | 7  | 9  | 444  | 4  | 11 | 3   |
| 10  | 10 | 10 | 0   | 19 | 11 | 26   | 16 | 4  | 1   |
| 100 | 0  | 7  | 144 | 3  | 10 | 372  | 0  | 10 | 2   |
| 73  | 4  | 9  | 132 | 18 | 0  | 1780 | 14 | 6  | 2   |
| 43  | 8  | 11 | 43  | 6  | 8  | 200  | 10 | 10 | 3   |

| 10. |    |    | 11. |    |    | 12. |    |    |     |
|-----|----|----|-----|----|----|-----|----|----|-----|
| £.  | s. | d. | £.  | s. | d. | £.  | s. | d. | qr. |
| 76  | 0  | 0  | 56  | 18 | 8  | 875 | 16 | 10 | 3   |
| 0   | 19 | 1  | 73  | 11 | 11 | 783 | 19 | 7  | 2   |
| 86  | 11 | 7  | 22  | 12 | 2  | 59  | 17 | 7  | 3   |
| 43  | 4  | 8  | 77  | 17 | 7  | 85  | 13 | 11 | 1   |
| 750 | 18 | 6  | 88  | 18 | 8  | 387 | 14 | 9  | 3   |

13. Bought a horse for 26 £. 12 s.; a yoke of oxen for 31 £. 17 s. 8 d.; a cow for 7 £. 16 s. 9 d.; and paid 15 s. 8 d. for a bridle. How much did they all cost me? *Ans.* 67 £. 2 s. 1 d.

14. Bought cloth for 29 £. 6 s. 10 d.; ribbon for 3 s. 4 d. 3 qr.; a pair of boots for £ 1. 6 s. 3 d.; and paid 2 s. 8 d. 2 qs. for mending a pair of shoes. What was my bill for the whole? *Ans.* 30 £. 19 s. 2 d. 1 qr.

15. Bought at one time goods to the amount of 175 £. 16 s. 11 d.; at another, to the amount of 35 £. 19 s. 8 d.; paid for carting 7 £. 8 s. 9 d. 3 qr.; and for insurance 3 £. 9 s. 7 d. What was the amount of my bills? *Ans.* 222 £. 14 s. 11 d. 3 qr.

16. Sold at one time goods to the amount of 35 £. 11 s. 6 d. 3 qr.; at another, to the amount of 56 £. 19 s. 7 d. 1 qr.; at a third time, to the amount of 75 £. 1 s. 3 d.; and at a fourth, to the amount of 63 £. 13 s. 4 d. 2 qr. What was the whole amount of my sales? *Ans.* 231 £. 5 s. 9 d. 2 qr.

17. Bought a quantity of corn for 113 £. 11 s. 11 d.; of rye, for 32 £. 19 s. 3 d.; of wheat, for 136 £. 16 s. 8 d.; and of oats, for 22 £. 14 s. 9 d. What was the whole amount? *Ans.* 306 £. 2 s. 7 d.

18. A man sold his farm for 850 £.; his sheep, for 67 £. 17 s.; his swine, for 19 £. 19 s. 11 d.; and his grain, for 36 £. 13 s. 2 d. How much money did he receive? *Ans.* 980 £. 10 s. 1 d.

## TROY WEIGHT.

1.

| lb.   | oz. | pwt. | gr. |
|-------|-----|------|-----|
| 15    | 6   | 13   | 14  |
| 11    | 3   | 11   | 19  |
| 17    | 4   | 3    | 21  |
| 42    | 11  | 17   | 15  |
| <hr/> |     |      |     |
| 87    | 2   | 6    | 21  |

The column of grains amounts to 69 gr. = 2 pwt. 21 gr. The pwt. amount to 46 = 2 oz. and 6 pwt. The ounces amount to 26 = 2 lb. and 2 oz. The pounds amount to 87; the whole of which is to be written down.

The scholar will observe, that as we have left the table of English money, we have no longer to carry by 20, 12, and 4. Our carrying numbers now are 12, 20, and 24.

2.

| lb. | oz. | pwt. | gr. |
|-----|-----|------|-----|
| 10  | 9   | 11   | 16  |
| 17  | 9   | 6    | 8   |
| 28  | 11  | 16   | 21  |
| 36  | 7   | 17   | 22  |

---

94 2 12 19

3.

| lb. | oz. | pwt. | gr. |
|-----|-----|------|-----|
| 29  | 7   | 13   | 19  |
| 17  | 6   | 11   | 11  |
| 31  | 6   | 16   | 23  |
| 71  | 1   | 18   | 7   |

---

149 11 00 12

4.

| lb. | oz. | pwt. | gr. |
|-----|-----|------|-----|
| 1   | 8   | 18   | 12  |
| 9   | 6   | 19   | 9   |
| 11  | 5   | 3    | 21  |
| 22  | 6   | 10   | 9   |
| 15  | 5   | 12   | 21  |

5.

| lb. | oz. | pwt. | gr. |
|-----|-----|------|-----|
| 0   | 9   | 11   | 16  |
| 12  | 7   | 16   | 11  |
| 3   | 0   | 0    | 0   |
| 71  | 0   | 16   | 9   |
| 16  | 9   | 17   | 23  |

6.

| lb. | oz. | pwt. | gr. |
|-----|-----|------|-----|
| 76  | 11  | 0    | 21  |
| 3   | 3   | 3    | 3   |
| 11  | 7   | 19   | 0   |
| 13  | 9   | 11   | 19  |
| 14  | 11  | 17   | 12  |

7. Purchased at one time, 4 lb. 3 oz. 16 pwt. 15 gr. of silver, and, at another, 7 lb. 8 oz. 18 pwt. 23 gr.; besides a quantity of jewelry, weighing 5 lbs. 11 oz. and 13 pwt. What was the whole weight? *Ans.* 18 lb. 8 pwt. 14 gr.

8. Add 3 lb. 9 oz. 13 pwt. 19 gr.; 2 lb. 10 oz. 9 pwt. 17 gr.; 6 lb. 11 oz. 18 pwt. 22 gr.; 9 oz. 11 pwt. 12 gr.; and 8 lb. 3 oz. 6 pwt. 20 grs. *Ans.* 22 lbs. 9 oz. 18 gr.

9. Again, add 6 lb. 2 oz. 16 pwt. 14 gr.; 3 lb. 8 pwt. 2 gr.; 12 lb. 4 oz. 15 pwt. 22 gr.; 8 oz. 16 gr.; 5 lb. 13 gr.; and 23 gr. *Ans.* 27 lb. 4 oz. 2 pwt. 18 gr.

## A VOIRDUPOIS WEIGHT.

1.

| T.    | cwt. | qr. | lb. | oz. |
|-------|------|-----|-----|-----|
| 7     | 16   | 3   | 20  | 13  |
| 4     | 12   | 1   | 25  | 11  |
| 3     | 9    | 2   | 16  | 9   |
| 12    | 18   | 0   | 14  | 12  |
| <hr/> |      |     |     |     |
| 28    | 17   | 0   | 21  | 13  |

The amount of the ounces is 45  
 $= 2$  lb. 13 oz. The pounds amount  
to 77  $= 2$  qr. 21 lb. The qr. are  
 $8 = 2$  cwt. The cwt. are 57  $= 2$  tons,  
17 cwt. and the tons are 28.

2.

| T. | cwt. | qr. | lb. | oz. |
|----|------|-----|-----|-----|
| 3  | 19   | 0   | 16  | 15  |
| 9  | 0    | 3   | 24  | 12  |
| 1  | 18   | 1   | 26  | 14  |
| 14 | 5    | 2   | 12  | 9   |

3.

| cwt. | qr. | lb. | oz. | dr. |
|------|-----|-----|-----|-----|
| 16   | 3   | 24  | 15  | 12  |
| 32   | 0   | 18  | 9   | 14  |
| 7    | 2   | 21  | 7   | 9   |
| 12   | 3   | 11  | 11  | 12  |

4.

| cwt. | qr. | lb. | oz. | dr. |
|------|-----|-----|-----|-----|
| 6    | 2   | 27  | 12  | 9   |
| 12   | 3   | 16  | 9   | 12  |
| 14   | 1   | 24  | 14  | 6   |
| 17   | 3   | 8   | 15  | 8   |

5.

| cwt. | qr. | lb. | oz. | dr. |
|------|-----|-----|-----|-----|
| 17   | 0   | 9   | 4   | 8   |
| 13   | 3   | 27  | 15  | 12  |
| 18   | 2   | 17  | 13  | 6   |
| 29   | 1   | 23  | 12  | 13  |

6. Purchased at one time, 16 tons, 3 qr. 21 lb. of hay; at another, 9 tons, 16 cwt. 2 qr. 17 lb.; and at another, 27 tons, 13 cwt. 1 qr. 17 lb. How much did I purchase? *Ans.* 53 tons, 10 cwt. 3 qr. 27 lb.

7. Bought 36 cwt. 3 qr. 24 lb. of wool; but finding the demand large, I made three successive purchases, at each of which I bought 45 cwt. 2 qr. 16 lb. What was the amount of my purchases? *Ans.* 173 cwt. 3 qr. 16 lb.

## APOTHECARIES' WEIGHT.

1.

| lb.   | ℥. | ʒ. | ʒ. | ʒ. |
|-------|----|----|----|----|
| 2     | 8  | 3  | 1  |    |
| 5     | 6  | 2  | 2  |    |
| 6     | 4  | 7  | 2  |    |
| 8     | 6  | 5  | 2  |    |
| <hr/> |    |    |    |    |
| 23    | 2  | 3  | 1  |    |

2.

| ℥. | ʒ. | ʒ. | ʒ. | gr. |
|----|----|----|----|-----|
| 6  | 5  | 2  | 12 |     |
| 9  | 6  | 1  | 15 |     |
| 3  | 4  | 0  | 9  |     |
| 2  | 7  | 2  | 13 |     |

3.

| lb. | ℥. | ʒ. | ʒ. | ʒ. | gr. |
|-----|----|----|----|----|-----|
| 8   | 9  | 4  | 1  | 14 |     |
| 14  | 6  | 7  | 0  | 12 |     |
| 1   | 9  | 5  | 1  | 6  |     |
| 8   | 6  | 7  | 2  | 5  |     |

4. A physician purchased the following quantities of medicine, at three different times; viz. 1 pound, 4 ounces, 5 drams; 3 pounds, 11 ounces, 6 drams, 2 scruples, 15 gr.; and 7 drams, 1 scruple, and 12 grains; what was their whole weight? *Ans.* 5 lb. 5  $\frac{3}{4}$  33. 1  $\frac{1}{2}$  7 gr.

## CLOTH MEASURE.

| 1.  |     |     | 2.  |     |     | 3.   |     |     | 4.   |     |     |
|-----|-----|-----|-----|-----|-----|------|-----|-----|------|-----|-----|
| yd. | qr. | na. | yd. | qr. | na. | E.E. | qr. | na. | E.F. | qr. | na. |
| 7   | 3   | 2   | 1   | 1   | 1   | 16   | 4   | 3   | 21   | 3   | 0   |
| 9   | 2   | 3   | 6   | 3   | 2   | 12   | 3   | 3   | 13   | 5   | 2   |
| 6   | 1   | 0   | 12  | 2   | 3   | 18   | 2   | 2   | 16   | 4   | 3   |
| 8   | 3   | 3   | 15  | 1   | 2   | 20   | 3   | 1   | 19   | 2   | 1   |

5. Bought at one purchase, 32 yd. 3 qr.; at another, 16 yd. 2 qr. 2 na.; and 24 yd. 1 qr. at another. How many yards did I purchase? *Ans.* 73 yd. 2 qr. 2 na.

6. Bought of one man, 12 ells English, 4 qr. 3 na.; and of each of two others, 27 ells English, 3 qr. and 3 na. How many Ells did I purchase? *Ans.* 68 E.E. 2 qr. 1 na.

7. Received from France 12 ells, 4 qr. of broadcloth; 17 ells, 5 qr. 3 na. of cassimere; and 19 ells, 2 qr. 3 na. of silks. How many ells were there in the three articles purchased? *Ans.* 50 ells. 0 qr. 2 na.

## DRY MEASURE.

| 1.  |     |     |     | 2.  |     |     |     | 3.  |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| bu. | pk. | qt. | pt. | bu. | pk. | qt. | pt. | bu. | pk. | qt. | pt. |
| 3   | 2   | 5   | 1   | 8   | 3   | 7   | 0   | 15  | 1   | 6   | 1   |
| 7   | 3   | 7   | 2   | 9   | 2   | 4   | 1   | 127 | 5   | 7   | 1   |
| 11  | 2   | 4   | 1   | 6   | 3   | 6   | 1   | 12  | 2   | 5   | 0   |
| 6   | 1   | 3   | 0   | 4   | 2   | 2   | 0   | 16  | 2   | 7   | 1   |

## WINE MEASURE.

| 1. |    |      |      |     | 2.   |      |     |     |     | 3.   |      |     |     |     |
|----|----|------|------|-----|------|------|-----|-----|-----|------|------|-----|-----|-----|
| T. | P. | hhd. | gal. | qt. | hhd. | gal. | qt. | pt. | gi. | hhd. | gal. | qt. | pt. | gi. |
| 4  | 1  | 1    | 42   | 3   | 15   | 27   | 3   | 1   | 2   | 140  | 46   | 2   | 1   | 1   |
| 6  | 0  | 1    | 24   | 2   | 20   | 13   | 2   | 0   | 3   | 127  | 15   | 3   | 0   | 3   |
| 8  | 1  | 0    | 18   | 2   | 12   | 16   | 1   | 1   | 1   | 263  | 29   | 1   | 1   | 2   |
| 12 | 0  | 1    | 23   | 0   | 132  | 54   | 3   | 0   | 3   | 42   | 27   | 3   | 0   | 3   |

## LONG MEASURE.

| 1.        |             |             |            | 2.         |             |             |             |
|-----------|-------------|-------------|------------|------------|-------------|-------------|-------------|
| <i>L.</i> | <i>m.</i>   | <i>fur.</i> | <i>rd.</i> | <i>m.</i>  | <i>fur.</i> | <i>rd.</i>  | <i>yd.</i>  |
| 12        | 2           | 5           | 36         | 9          | 6           | 12          | 3           |
| 9         | 1           | 7           | 24         | 4          | 7           | 26          | 2           |
| 15        | 0           | 6           | 17         | 12         | 4           | 32          | 5           |
| 30        | 2           | 4           | 26         | 7          | 1           | 38          | 4           |
| <hr/>     |             |             |            | <hr/>      |             |             |             |
| 3.        |             |             |            | 4.         |             |             |             |
| <i>m.</i> | <i>fur.</i> | <i>rd.</i>  | <i>yd.</i> | <i>ft.</i> | <i>in.</i>  | <i>b.c.</i> |             |
| 4         | 4           | 23          | 5          | 2          | 9           | 2           |             |
| 9         | 3           | 30          | 6          | 1          | 10          | 1           |             |
| 6         | 2           | 36          | 4          | 2          | 8           | 2           |             |
| 5         | 2           | 27          | 4          | 2          | 8           | 2           |             |
| <hr/>     |             |             |            | <hr/>      |             |             |             |
| <i>L.</i> | <i>m.</i>   | <i>fur.</i> | <i>rd.</i> | <i>yd.</i> | <i>ft.</i>  | <i>in.</i>  | <i>b.c.</i> |
| 18        | 2           | 5           | 18         | 3          | 2           | 7           | 1           |
| 21        | 0           | 4           | 30         | 4          | 1           | 9           | 2           |
| 32        | 1           | 7           | 31         | 5          | 2           | 4           | 1           |
| 76        | 2           | 3           | 39         | 4          | 2           | 10          | 2           |

## LAND OR SQUARE MEASURE.

| 1.        |              |            |            |            | 2.         |           |              |            |
|-----------|--------------|------------|------------|------------|------------|-----------|--------------|------------|
| <i>A.</i> | <i>rood.</i> | <i>rd.</i> | <i>yd.</i> | <i>ft.</i> | <i>in.</i> | <i>A.</i> | <i>rood.</i> | <i>rd.</i> |
| 7         | 2            | 36         | 24         | 6          | 72         | 9         | 3            | 21         |
| 8         | 3            | 23         | 20         | 4          | 91         | 12        | 2            | 37         |
| 5         | 1            | 15         | 17         | 8          | 108        | 1         | 1            | 39         |
| 12        | 3            | 12         | 13         | 6          | 22         | 15        | 3            | 12         |
| 15        | 0            | 17         | 9          | 7          | 136        | 8         | 1            | 9          |
| <hr/>     |              |            |            |            | <hr/>      |           |              |            |
| <i>A.</i> | <i>rood.</i> | <i>rd.</i> | <i>yd.</i> | <i>ft.</i> |            | <i>A.</i> | <i>rood.</i> | <i>rd.</i> |
| 46        | 0            | 29         | 11         | 7          |            | 34        | 2            | 33         |
| 27        | 3            | 26         | 6          | 4          |            | 44        | 0            | 30         |
| 18        | 2            | 32         | 6          | 4          |            | 15        | 3            | 29         |
| 25        | 3            | 30         | 7          | 5          |            | 33        | 3            | 36         |
| 6         | 1            | 16         | 6          | 8          |            | 44        | 3            | 37         |
| <hr/>     |              |            |            |            | <hr/>      |           |              |            |
| <i>A.</i> | <i>rood.</i> | <i>rd.</i> | <i>yd.</i> | <i>ft.</i> |            | <i>A.</i> | <i>rood.</i> | <i>rd.</i> |
| 34        | 2            | 33         | 7          | 6          |            | 34        | 2            | 33         |
| 44        | 0            | 30         | 7          | 5          |            | 44        | 0            | 30         |
| 15        | 3            | 29         | 10         | 5          |            | 15        | 3            | 29         |
| 33        | 3            | 36         | 8          | 7          |            | 33        | 3            | 36         |
| 44        | 3            | 37         | 8          | 7          |            | 44        | 3            | 37         |

## SOLID MEASURE.

| 1.         |            | 2.         |            |            | 3.         |            |            |
|------------|------------|------------|------------|------------|------------|------------|------------|
| <i>ft.</i> | <i>in.</i> | <i>cd.</i> | <i>ft.</i> | <i>in.</i> | <i>cd.</i> | <i>ft.</i> | <i>in.</i> |
| 99         | 420        | 11         | 72         | 726        | 3          | 99         | 777        |
| 78         | 864        | 12         | 16         | 317        | 66         | 77         | 333        |
| 320        | 740        | 113        | 17         | 36         | 122        | 116        | 1240       |
| 950        | 222        | 4          | 117        | 1372       | 372        | 108        | 1617       |
| 48         | 12         | 116        | 8          | 96         | 12         | 96         | 456        |
| <hr/>      |            | <hr/>      |            |            | <hr/>      |            |            |

## TIME.

| 1.  |     |    |    |    |    |      | 2. |    |    |      |
|-----|-----|----|----|----|----|------|----|----|----|------|
| yr. | mo. | w. | d. | h. | m. | sec. | d. | h. | m. | sec. |
| 2   | 9   | 3  | 6  | 13 | 22 | 56   | 15 | 21 | 43 | 51   |
| 8   | 11  | 0  | 3  | 21 | 43 | 21   | 23 | 17 | 55 | 56   |
| 1   | 12  | 2  | 4  | 23 | 69 | 52   | 6  | 19 | 59 | 49   |
| 2   | 8   | 8  | 5  | 17 | 36 | 8    | 16 | 15 | 43 | 36   |

| 3. |    |    |    |      | 4. |    |    |    |      |
|----|----|----|----|------|----|----|----|----|------|
| w. | d. | h. | m. | sec. | w. | d. | h. | m. | sec. |
| 5  | 6  | 9  | 7  | 59   | 13 | 10 | 17 | 8  | 47   |
| 6  | 5  | 21 | 39 | 27   | 14 | 9  | 20 | 40 | 43   |
| 22 | 4  | 23 | 36 | 42   | 15 | 8  | 22 | 45 | 23   |
| 11 | 3  | 16 | 17 | 18   | 16 | 11 | 23 | 18 | 22   |

## CIRCULAR MOTION.

| 1. |    |    |    | 2. |    |    |    | 3. |    |    |    |
|----|----|----|----|----|----|----|----|----|----|----|----|
| S. | °  | '  | "  | S. | °  | '  | "  | S. | °  | '  | "  |
| 6  | 23 | 42 | 39 | 4  | 9  | 29 | 41 | 3  | 22 | 40 | 37 |
| 8  | 26 | 54 | 36 | 6  | 7  | 43 | 2  | 11 | 29 | 59 | 57 |
| 3  | 19 | 11 | 9  | 11 | 8  | 51 | 59 | 4  | 23 | 18 | 38 |
| 2  | 21 | 13 | 23 | 12 | 13 | 27 | 17 | 8  | 6  | 13 | 15 |

## APPLICATION.

Ex. 1. What is the amount of 23 £. 11 d.; 13 £. 17 s. 3 qr.; 16 s. 8 d.; and 11 d. 3 qr.? *Ans.* 37 £. 15 s. 7 d. 2 qr.

2. Bought the following quantities of oil; viz. 12 gal. 3 qt.; 2 hhd. 42 gal. 2 qt. 1 pt.; and 13 hhd. 56 gal. What was the whole amount? *Ans.* 16 hhd. 48 gal. 1 qt. 1 pt.

3. Add together 250 £. 18 s. 9 d. 3 qr.; 16 £. 7 s. 2 qr.; 21 £. 19 s. 3 d.; 18 s. 6 d.; and 36 £. *Ans.* 326 £. 3 s. 7 d. 1 qr.

4. What is the amount of 5 cwt. 3 qr. 27 lb.; 2 qr. 29 lb.; 12 cwt. 1 qr. 17 lb.; and 36 cwt. 16 lb. *Ans.* 55 cwt. 1 qr. 5 lb.

5. Bought at one time, 7 bu. 3 pk. of wheat; at another, 9 bu. 1 pk. and had previously in each of two bins, 6 bu. and 3 pk. What was the whole amount? *Ans.* 30 bu. 2 pk.

6. Sold one cow for 10 £. 15 s. 6 d.; another for 6 £. 19 s.

11 d.; and a colt for 12 £. 6 s. 4 d. How much did they all amount to? *Ans.* 30 £. 1 s. 9 d.

7. Bought four casks of wine, of which the first contained 42 gal. 2 qt. 1 pt.; the second, 65 gal. 1 pt.; the third, 50 gal. 3 qt.; and the fourth, 55 gal. 1 qt. 1 pt. How many gallons did I purchase? *Ans.* 213 gal. 3 qt. 1 pt.

8. Purchased three pieces of land. The first contained 17 acres, 1 rood, and 35 rods; the second, 36 acres, 2 roods, 21 rods; and the third, 46 acres and 37 rods. How much land did I purchase? *Ans.* 100 acres, 1 rood, 13 rods.

QUESTIONS.—In what ratio do simple numbers increase? What is the peculiarity of Compound Numbers? Where are the ratios of increase and decrease of compound numbers given? Why do you carry for 10 in simple numbers? Why do you carry for 4, 12, and 20, in the table of English money? What peculiarity noticeable in *writing* compound numbers? What only is necessary in writing simple numbers? How are compound numbers known? How must each denomination therefore be written? What is Compound Addition? How are numbers to be written? What is the rule? What is the note following the rule?

## COMPOUND SUBTRACTION.

The scholar has now become acquainted with Compound Addition; and he was previously acquainted with the Simple rules. He needs, therefore, to be informed only, that Compound Subtraction sustains the same relation to Compound Addition, that Simple Subtraction does to Simple Addition. It is the subtracting of numbers of different denominations.

In this rule, instead of constantly borrowing 10, when the lower figure is the larger, he must borrow as many units as are required of the denomination he is subtracting, to make a unit of the next higher denomination; that is, when it becomes necessary to borrow a number in subtracting *farthings*, 4 is the number always required; in subtracting *pence*, 12 is the number; and in *shillings*, 20; and in like manner in other denominations.

RULE.—Place the less quantity under the greater, so that each denomination shall stand under one of its own name or kind. Begin at the right, and proceed in all respects as in

*Simple Subtraction, except in borrowing when the lower figure is the larger; in doing which, instead of 10, (the number borrowed in Simple Subtraction,) borrow as many units as make one of the next higher denomination. Whenever a number is borrowed, one is to be carried to the next lower figure.*

|             |    |    |    |     |
|-------------|----|----|----|-----|
| Ex. 1. From | £. | s. | d. | qr. |
|             | 16 | 18 | 8  | 1   |
| Take        | 8  | 16 | 11 | 3   |

First, I cannot take 3 qr. from 1 qr. I therefore add 4 qr. to the upper figure, and take the 3

from the amount, 5, and obtain a remainder of 2. I carry 1 to the next lower figure, viz. 11, which makes it 12, and proceed to take it from the figure above, but find it impracticable. I therefore add 12 to the upper figure, 8, making it 20; and from this amount, subtract 12, and obtain the 8 in the answer. Again, I carry 1 to the next figure, 16, which makes it 17, and take this from the figure above, and obtain a remainder of 1. I here borrowed nothing and have nothing to carry; therefore, 8 from 16 leaves 8.

2.

|    |    |    |     |
|----|----|----|-----|
| £. | s. | d. | qr. |
| 35 | 11 | 9  | 3   |
| 17 | 9  | 11 | 2   |

3.

|    |    |    |     |
|----|----|----|-----|
| £. | s. | d. | qr. |
| 46 | 13 | 7  | 1   |
| 21 | 17 | 9  | 3   |

4.

|    |    |    |     |
|----|----|----|-----|
| £. | s. | d. | qr. |
| 74 | 0  | 9  | 2   |
| 72 | 19 | 11 | 3   |

5.

|    |    |    |     |
|----|----|----|-----|
| £. | s. | d. | qr. |
| 99 | 16 | 8  | 3   |
| 77 | 17 | 7  | 1   |

6.

|    |    |    |     |
|----|----|----|-----|
| £. | s. | d. | qr. |
| 33 | 12 | 3  | 1   |
| 13 | 8  | 9  | 3   |

7.

|    |    |    |
|----|----|----|
| £. | s. | d. |
| 94 | 11 | 8  |
| 72 | 9  | 11 |

8. A certain man owed 75 £. 13 s. 9 d., and paid of this sum 39 £. 19 s. 11 d. How much remained due? *Ans.* 35 £. 13 s. 10 d.

9. Received of three individuals the following sums of money; viz. of A., 16 £. 12 s. 8 d. 3 qr.; of B., 21 £. 17 s. 9 d.; and of C., 46 £. 19 s. I afterwards paid D. 58 £. 13 s. 9 d. 2 qr. How much had I left? *Ans.* 26 £. 15 s. 8 d. 1 qr.

10. The following sums are due to A.; viz. 136 £. 15 s. 11 d.; 450 £. 8 s. 6 d.; 356 £. 17 s. 10 d. 2 qr.; and 12 £. 9 s. 4 d. He is indebted to B., 67 £. 14 s. 9 d. 2 qr.; to C.,



24 £. 11 s. 3 d.; and to D., 571 £. 11 s. 11 d. How much is due to him, more than he owes? *Ans.* 292 £. 13 s. 8 d.

TROY WEIGHT.

| 1.    |     |      |     | 2.    |     |      |     | 3.    |      |      |     |
|-------|-----|------|-----|-------|-----|------|-----|-------|------|------|-----|
| lb.   | oz. | pwt. | gr. | lb.   | oz. | pwt. |     | lb.   | oz.  | pwt. | gr. |
| 14    | 9   | 19   | 16  | 46    | 11  | 13   |     | 9     | 11   | 11   | 21  |
| 10    | 11  | 16   | 23  | 13    | 9   | 17   |     | 4     | 3    | 19   | 23  |
| <hr/> |     |      |     | <hr/> |     |      |     | <hr/> |      |      |     |
| 4.    |     |      |     | 5.    |     |      |     | 6.    |      |      |     |
| lb.   | oz. | pwt. | gr. | lb.   | oz. | pwt. | gr. | oz.   | pwt. | gr.  |     |
| 36    | 7   | 14   | 17  | 8     | 9   | 16   | 11  | 11    | 9    | 18   |     |
| 17    | 9   | 17   | 22  | 2     | 11  | 19   | 23  | 10    | 16   | 23   |     |
| <hr/> |     |      |     | <hr/> |     |      |     | <hr/> |      |      |     |

AVOIRDUPOIS WEIGHT.

| 1.      |     |     |     |       | 2.      |     |     |     |       |
|---------|-----|-----|-----|-------|---------|-----|-----|-----|-------|
| cwt.    | qr. | lb. | oz. | dr.   | cwt.    | qr. | lb. | oz. | dr.   |
| 12      | 3   | 19  | 13  | 14    | 31      | 1   | 23  | 14  | 15    |
| 9       | 2   | 21  | 11  | 6     | 26      | 3   | 25  | 15  | 8     |
| <hr/>   |     |     |     |       | <hr/>   |     |     |     |       |
| 3.      |     |     |     |       | 4.      |     |     |     |       |
| T. cwt. | qr. | lb. | oz. | dr.   | T. cwt. | qr. | lb. | oz. | dr.   |
| 6       | 13  | 0   | 11  | 12 13 | 9       | 17  | 3   | 20  | 15 8  |
| 4       | 17  | 3   | 5   | 13 14 | 2       | 15  | 2   | 26  | 12 15 |
| <hr/>   |     |     |     |       | <hr/>   |     |     |     |       |

7. Having in my possession 45 cwt. 3 qr. 17 lb. of cheese, I sold 32 cwt. 27 lb. How much remained? *Ans.* 13 cwt. 2 qr. 18 lb.

APOTHECARIES' WEIGHT.

| 1.    |    |    |    |     | 2.    |    |    |    |     | 3.    |    |    |    |     |
|-------|----|----|----|-----|-------|----|----|----|-----|-------|----|----|----|-----|
| lb.   | ℥. | ℥. | ℥. | gr. | lb.   | ℥. | ℥. | ℥. | gr. | lb.   | ℥. | ℥. | ℥. | gr. |
| 5     | 9  | 5  | 2  | 16  | 12    | 6  | 5  | 2  | 15  | 31    | 17 | 1  | 1  | 12  |
| 3     | 11 | 6  | 2  | 15  | 8     | 9  | 4  | 1  | 17  | 20    | 10 | 5  | 2  | 15  |
| <hr/> |    |    |    |     | <hr/> |    |    |    |     | <hr/> |    |    |    |     |

CLOTH MEASURE.

| 1.    |     |     |  | 2.    |     |     |  | 3.    |     |     |  | 4.    |     |     |  |
|-------|-----|-----|--|-------|-----|-----|--|-------|-----|-----|--|-------|-----|-----|--|
| yd.   | qr. | na. |  | E.F.  | qr. | na. |  | E.E.  | qr. | na. |  | E.F.  | qr. | na. |  |
| 9     | 3   | 3   |  | 10    | 5   | 1   |  | 21    | 1   | 2   |  | 16    | 1   | 3   |  |
| 4     | 2   | 3   |  | 6     | 3   | 3   |  | 16    | 4   | 3   |  | 8     | 2   | 2   |  |
| <hr/> |     |     |  | <hr/> |     |     |  | <hr/> |     |     |  | <hr/> |     |     |  |

8

## DRY MEASURE.

| 1.  |     |     |         | 2.  |     |     |         | 3.  |     |     |         |
|-----|-----|-----|---------|-----|-----|-----|---------|-----|-----|-----|---------|
| bu. | pk. | qt. | pt. gi. | bu. | pk. | qt. | pt. gi. | bu. | pk. | qt. | pt. gi. |
| 15  | 3   | 3   | 0 2     | 26  | 0   | 5   | 0 1     | 30  | 2   | 7   | 1 3     |
| 12  | 2   | 5   | 1 3     | 23  | 3   | 7   | 1 2     | 16  | 3   | 5   | 1 3     |

## WINE MEASURE.

| 1. |    |      |      |     |         | 2.   |      |     |         |  |  |
|----|----|------|------|-----|---------|------|------|-----|---------|--|--|
| T. | P. | hhd. | gal. | qt. | pt. gi. | hhd. | gal. | qt. | pt. gi. |  |  |
| 3  | 1  | 1    | 27   | 3   | 1 1     | 27   | 19   | 3   | 0 1     |  |  |
| 2  | 0  | 2    | 47   | 1   | 1 3     | 16   | 43   | 1   | 1 3     |  |  |

| 3.   |      |     |     |     |  | 4.   |      |     |     |     |  |
|------|------|-----|-----|-----|--|------|------|-----|-----|-----|--|
| hhd. | gal. | qt. | pt. | gi. |  | hhd. | gal. | qt. | pt. | gi. |  |
| 137  | 42   | 1   | 0   | 3   |  | 175  | 59   | 0   | 1   | 3   |  |
| 128  | 56   | 3   | 1   | 1   |  | 21   | 37   | 3   | 1   | 2   |  |

## LONG MEASURE.

| 1. |      |     |     |     |     |      | 2. |      |     |     |     |     |  |
|----|------|-----|-----|-----|-----|------|----|------|-----|-----|-----|-----|--|
| m. | fur. | rd. | yd. | ft. | in. | b.c. | m. | fur. | rd. | yd. | ft. | in. |  |
| 15 | 4    | 27  | 4   | 2   | 11  | 1    | 32 | 5    | 39  | 1   | 2   | 3   |  |
| 12 | 3    | 36  | 3   | 1   | 9   | 2    | 27 | 2    | 4   | 3   | 1   | 11  |  |

| 3. |    |      |     |     |     |     | 4. |    |      |     |     |     |          |
|----|----|------|-----|-----|-----|-----|----|----|------|-----|-----|-----|----------|
| L. | m. | fur. | rd. | yd. | ft. | in. | L. | m. | fur. | rd. | yd. | ft. | in. b.c. |
| 17 | 2  | 3    | 19  | 3   | 1   | 7   | 31 | 0  | 3    | 15  | 4   | 2   | 7 2      |
| 12 | 1  | 7    | 35  | 4   | 2   | 8   | 27 | 2  | 5    | 17  | 1   | 2   | 9 2      |

## LAND OR SQUARE MEASURE.

| 1. |      |     |         | 2. |       |     |     | 3. |       |     |     |
|----|------|-----|---------|----|-------|-----|-----|----|-------|-----|-----|
| A. | rod. | rd. | yd. ft. | A. | rood. | rd. | yd. | A. | rood. | rd. | yd. |
| 9  | 36   | 14  | 8       | 74 | 3     | 27  | 16  | 12 | 1     | 16  | 15  |
| 4  | 39   | 6   | 4       | 64 | 2     | 31  | 12  | 9  | 2     | 17  | 16  |

## SOLID MEASURE.

| 1. |     |     | 2. |     |      | 3. |     |      |
|----|-----|-----|----|-----|------|----|-----|------|
| C. | ft. | in. | C. | ft. | in.  | C. | ft. | in.  |
| 21 | 62  | 856 | 56 | 110 | 1462 | 8  | 100 | 8    |
| 16 | 115 | 972 | 19 | 36  | 472  | 1  | 101 | 1560 |

## TIME.

| 1.  |    |    |    |    |
|-----|----|----|----|----|
| yr. | m. | w. | d. | h. |
| 16  | 8  | 3  | 5  | 13 |
| 7   | 9  | 2  | 6  | 21 |

---

| 2. |    |    |    |    |
|----|----|----|----|----|
| m. | w. | d. | h. | m. |
| 12 | 2  | 1  | 15 | 21 |
| 9  | 3  | 2  | 16 | 22 |

---

| 3.  |     |    |    |      |
|-----|-----|----|----|------|
| yr. | d.  | h. | m. | sec. |
| 19  | 152 | 13 | 42 | 21   |
| 16  | 256 | 19 | 36 | 56   |

---

| 4.  |    |    |    |      |
|-----|----|----|----|------|
| yr. | d. | h. | m. | sec. |
| 45  | 67 | 17 | 50 | 15   |
| 36  | 36 | 22 | 46 | 45   |

---

## CIRCULAR MOTION.

| 1. |    |    |    | 2. |    |    |    | 3. |    |    |    |
|----|----|----|----|----|----|----|----|----|----|----|----|
| S. | °  | '  | "  | S. | °  | '  | "  | S. | °  | '  | "  |
| 8  | 18 | 42 | 36 | 9  | 27 | 36 | 51 | 11 | 15 | 16 | 31 |
| 6  | 26 | 11 | 52 | 1  | 29 | 42 | 52 | 8  | 19 | 17 | 42 |

---

| 4. |    |    |    |
|----|----|----|----|
| S. | °  | '  | "  |
| 11 | 21 | 49 | 59 |
| 6  | 27 | 13 | 21 |

---

| 5. |    |    |    |
|----|----|----|----|
| S. | °  | '  | "  |
| 8  | 19 | 38 | 46 |
| 6  | 21 | 42 | 50 |

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## PROMISCUOUS EXAMPLES.

Ex. 1. I have in my possession 46 £. 19 s. 11 d. How much shall I have left, after paying a debt of 27 £. 13 s. 9 d. ?  
*Ans.* 19 £. 6 s. 2 d.

2. Received 156 £. 3 s. 8 d. after which I paid out 137 £. 15 s. 10 d. How much remained in my possession ? *Ans.* 18 £. 7 s. 10 d.

3. Lent a friend 16 £. 17 s. 6 d. On the following day he paid me 5 £. 13 s. 11 d. ; one week after he made another payment of 7 £. 5 s. 10 d. How much then remained due ?  
*Ans.* 3 £. 17 s. 9 d.

4. Bought a wagon for 9 £. 11 s. and sold the same for 13 £. 5 s. How much did I gain ? *Ans.* 3 £. 14 s.

5. Bought a horse for 56 £. 15 s. and exchanged the same

for a colt, and received 46 £. 11 s. in money. How much did the colt cost me? *Ans.* 10 £. 4 s.

6. A man having 15 tons, 13 cwt. 3 qr. of hay, sold 5 tons, 10 cwt. and gave 3 cwt. 3 qr. to a friend? How much had he left? *Ans.* 10 tons.

7. Bought 7 cwt. 3 qr. 16 lb. of rice, at one purchase, and 9 cwt. 1 qr. 27 lb. at another; of this, 2 cwt. 16 lb. was stolen, and of the remainder, I sold 11 cwt. 2 qr. 21 lb. How much had I left? *Ans.* 3 cwt. 2 qr. 6 lb.

8. Three men bought a piece of land for 450 £. 16 s. 10 d. of which two of them paid each 69 £. 17 s. 6 d.; what did the third man pay? *Ans.* 311 £. 1 s. 10 d.

9. I owned a tract of land containing 356 acres, 3 roods, and 30 rods; from this I sold to A. 127 acres, 2 roods; and to B. 27 acres, 1 rood, and 36 rods. How much remained? *Ans.* 201 acres, 3 roods, 34 rods.

10. A father, 46 years, 9 months, and 27 days old, has two sons; the elder of whom is 19 years, 3 months, and 13 days old; and the younger, 7 years, 10 months, and 21 days. How much does the father's age exceed the sum of his sons'? *Ans.* 19 y. 7 m. 23 d.

11. Bought a quantity of cotton, which, at the price agreed upon, came to 20 £. 4 s. In pay for this I gave a quantity of rice worth 15 £. 18 s. and the balance in cash. How much money did I pay? *Ans.* 4 £. 6 s.

12. A merchant bought a piece of cloth containing 40 yards, from which he sold 36 yd. 1 qr. 2 na. How much had he left? *Ans.* 3 yd. 2 qr. 2 na.

13. Sold from a pile of wood containing 40 cords, 64 feet, 39 cords, 32 feet. How much remained? *Ans.* 1 cord 32 ft.

14. Bought 560 acres of land for 940 £. From this I sold to A. 120 acres, 2 roods, and 16 rods, for 300 £.; and to B. 150 acres, 1 rood, and 24 rods, for 297 £. 10 s. and 6 d. How much land remains in my possession, and how much has it cost me? *Ans.* 269 acres; cost, 342 £. 9 s. 6 d.

**QUESTIONS.**—What is Compound Subtraction? Instead of 10; how many are to be borrowed here? What is the number borrowed in subtracting farthings? Why? What in subtracting pence? Shillings? And why? What is the Rule?

## COMPOUND MULTIPLICATION.

The peculiarity of Compound Numbers having been fully explained, and multiplication of Simple Numbers being also understood, no other definition of this rule is needed than is conveyed by the *name*.

A simple inquiry presents itself, viz. are both the given numbers compound? To answer this, the scholar needs only to be informed, that in multiplication, the *multiplier* is always a simple number, showing how many times the multiplicand is to be repeated. The multiplicand, therefore, *only* is compound. The product, as it is formed by repeating the multiplicand, must necessarily be of the same denomination with it.

Take the following illustration. Multiply 8 s. 9 d. 2 qr. by 4.

$$\begin{array}{r} \text{PERFORMED.} \\ 8 \text{ s. } 9 \text{ d. } 2 \text{ qr.} \\ \quad 4 \\ \hline \end{array}$$

$$1 \text{ £. } 15 \text{ s. } 2 \text{ d. } 0 \text{ qr.}$$

In this example, we say, four times 2 farthings are 8 farthings, and 8 qr. = 2 d. and 0 qr. remain. We therefore write down a cypher, and carry 2. Again, four times 9 d. are 36 d. and 2 d. to carry make 38 d. = 3 s. and 2 d. The 2 d. is written down, and the 3 s. carried. Lastly, four times 8 s. are 32 s. and 3 s. to carry make 35 s. = 1 £. 15 s. which is written down as seen in the answer. If now, in the above example, 8 s. 9 d. 2 qr. had been given as the price of one yard of cloth, and the scholar had been required to find the price of 4 yards, the operation would have been the same. The number of yards only decides how many *times* the price of one yard is to be repeated.

**CASE 1st.**—WHEN THE MULTIPLIER OR SIMPLE NUMBER IS NOT GREATER THAN 12.

**RULE.**—Multiply the Compound number by the Simple one, commencing with the lowest denomination and carrying from one denomination to another, as in the preceding Compound Rules.

Ex. 1. Multiply 9 £. 16 s. 8 d. 2 qr. by 9.

PERFORMED.

$$\begin{array}{r} 9 \text{ £. } 16 \text{ s. } 8 \text{ d. } 2 \text{ qr.} \\ \times 9 \\ \hline \end{array}$$

88 £. 10 s. 4 d. 2 qr. Ans.

*Explanation.*—9 times 2 qr. = 18 qr. = 4 d. 2 qr. 9 times 8 d. = 72 d. and 4 d. from the farthings added, make 76 d. = 6 s. 4 d. Again, 9 times 16 s. = 144 s. and the 6 s. obtained from the pence make 150 s. = 7 £. 10 s.; and lastly, 9 times 9 pounds = 81 £. and 81 £. + 7 £. = 88. The product therefore is, as given above, viz. 88 £. 10 s. 4 d. 2 qr.

2. Multiply 16 £. 11 s. 9 d. 3 qr. by 3.

PERFORMED.

$$\begin{array}{r} 16 \text{ £. } 11 \text{ s. } 9 \text{ d. } 3 \text{ qr.} \\ \times 3 \\ \hline \end{array}$$

49 £. 15 s. 5 d. 1 qr. Ans.

3. Multiply 1 £. 11 s. 6 d. 2 qr. by 5. Ans. 7 £. 17 s. 8 d. 2 qr.

4. Multiply 11 s. 9 d. by 3. Ans. 1 £. 15 s. 3 d.

5. Multiply 15 £. 10 s. 8 d. by 2. Ans. 31 £. 1 s. 4 d.

6. Multiply 5 s. 6 d. by 9. Ans. 2 £. 9 s. 6 d.

7. What cost 4 gallons of wine at 8 s. 7 d. per gallon? Ans. 1 £. 14 s. 4 d.

8. What cost 5 cwt. of raisins at 1 £. 7 s. 9 d. 2 qa per cwt.? Ans. 6 £. 18 s. 11 d. 2 qr.

9. What cost 8 yards of broadcloth at 1 £. 2 s. 3 d. per yard? Ans. 8 £. 18 s.

10. What cost 11 tons of hay at 2 £. 1 s. 10 d. per ton? Ans. 23 £. 0 s. 2 d.

11. What cost 12 bushels of wheat at 9 s. 10 d. per bushel? Ans. 5 £. 18 s.

CASE 2d.—WHEN THE MULTIPLIER OR SIMPLE NUMBER IS A COMPOSITE NUMBER GREATER THAN 12.

*RULE.*—Separate the simple number or multiplier into its component parts, and multiply first by one of these parts, and the product of this multiplication by the others in succession. The last product will be the answer required.

*Note.*—It will generally be found more expeditious to divide the multiplier into two parts only. Should it, however, be large, as 125 or 1728, it may be divided into more than two parts, viz. 125 into three 5's, and 1728 into three 12's.

**Ex. I.** Multiply 6 s. 10 d. by 28.  $28 = 4 \times 7$ . Therefore,

$$\begin{array}{r} 6 \text{ s. } 10 \text{ d.} \\ \quad \quad \quad 7 \\ \hline 2 \text{ £. } 7 \text{ s. } 10 \text{ d.} = \text{product of } 7. \\ \quad \quad \quad 4 \end{array}$$

$$9 \text{ £. } 11 \text{ s. } 4 \text{ d.} = \text{prod. of } 4 \text{ times } 7 = 28.$$

2. What cost 27 yards of cloth at 7 s. 6 d. per yard?  $27 = 9 \times 3$ . Therefore,

$$\begin{array}{r} 7 \text{ s. } 6 \text{ d.} \\ \quad \quad \quad 9 \\ \hline 3 \text{ £. } 7 \text{ s. } 6 \text{ d.} = \text{price of } 9 \text{ yards.} \\ \quad \quad \quad 3 \end{array}$$

$$10 \text{ £. } 2 \text{ s. } 6 \text{ d.} = \text{price of } 3 \text{ times } 9 \text{ yd.} = 27 \text{ yd.}$$

3. What cost 32 yards at 9 s. 9 d. per yard? *Ans.* 15 £. 12 s.

4. What cost 20 yards at 3 s. 6 d. per yard? *Ans.* 3 £. 10 s.

5. What cost 36 gallons at 5 s. 8 d. per gallon? *Ans.* 10 £. 4 s.

6. What cost 63 yards at 7 s. 6 d. 2 qr. per yard? *Ans.* 23 £. 15 s. 1 d. 2 qr.

7. What cost 72 yards at 3 s. 11 d. per yard? *Ans.* 14 £. 2 s.

8. What cost 144 yards at 1 £. 4 s. 2 d. per yard? *Ans.* 174 £.

9. Sold 21 men each 3 qr. 16 lb. of sugar. How many cwt. did I sell? *Ans.* 18 cwt. 3 qr.

10. Suppose 27 young lads to have lived each 6 years, 9 months, 8 days, and 11 hours; how many days, &c., have they all lived, allowing 30 days to a month? *Ans.* 182 yr. 10 mo. 18 d. 9 h.

**CASE 3d.**—WHEN THE MULTIPLIER IS MORE THAN 12, AND IS NOT A COMPOSITE NUMBER.

**RULE.**—Take any two or more numbers whose product will come as near as possible to the given number or multiplier, without exceeding it, and having multiplied the given price of one yard, pound, &c. by them, retain their product. Then multiply the same given price by the number wanting to make up the entire multiplier, and add the product to the preceding product. Their sum will be the product required.

**Note.**—If preferred, the given quantity or multiplier may be multiplied by each denomination of the compound quantity separately, and the several products, reduced to the highest denomination, may be added together.

**Ex. 1.** What will 51 yards of cloth cost at 3 s. 6 d. per yard? The two numbers whose product comes nearest to 51, are 7 and 7, and their product is 49. Consequently, if we multiply 3 s. 6 d. by 7, and their product by 7 again, we shall obtain the cost of 49 yards. There will then be the cost of two yards wanting. This will be obtained by multiplying 3 s. 6 d. the price of 1 yard, by 2. The operation is thus performed:

$$\begin{array}{r}
 3 \text{ s. } 6 \text{ d.} \\
 7 \\
 \hline
 1 \text{ £. } 4 \text{ s. } 6 \text{ d.} = \text{the price of 7 yards.} \\
 7 \\
 \hline
 8 \text{ £. } 11 \text{ s. } 6 \text{ d.} = \text{price of 49 yards.} \\
 7 \text{ s. } 0 \text{ d.} = \text{price of 2 yards.} \\
 \hline
 8 \text{ £. } 18 \text{ s. } 6 \text{ d.} = \text{price of 51 yards.}
 \end{array}$$

Or, the sum may be solved by the note, thus: 51 yards, at 6 d. per yard, would = 306 d. = 1 £. 5 s. 6 d.; and 51 yards at 3 s. per yard = 153 s. = 7 £. 13 s.; and 1 £. 5 s. 6 d. added to 7 £. 13 s. as before, gives 8 £. 18 s. 6 d.

2. What cost 47 yards of cloth, at 17 s. 9 d. per yard? The two numbers required by the rule are 6 and 7, and there is a remainder of 5. Therefore,



17 s. 9 d.

6

5 £. 6 s. 6 d. = price of 6 yards.

7

37 £. 5 s. 6 d. = price of 42 yards.

4 £. 8 s. 9 d. = price of 5 yards added.

41 £. 14 s. 3 d. = price of 47 yards.

Or, 47 yds.  $\times$  9 d. = 1 £. 15 s. 3 d.

47 yds  $\times$  17 s. = 39 £. 19 s. 0 d.

41 £. 14 s. 3 d. as before.

3. What cost 23 gallons of melasses at 3 s. 6 d. per gallon? *Ans.* 4 £. 6 d.

4. What cost 94 yards of cloth at 1 £. 9 s. 4 d. per yard? *Ans.* 137 £. 17 s. 4 d.

5. What cost 59 yards of baize at 3 s. 4 d. per yard? *Ans.* 9 £. 16 s. 8 d.

6. What cost 29 cwt. of sugar at 17 s. 8 d. per cwt.? *Ans.* 25 £. 12 s. 4 d.

7. What cost 78 yards of cloth at 9 s. 3 d. per yard? *Ans.* 36 £. 1 s. 6 d.

8. What cost 65 cwt. of sugar at 19 s. 3 d. per cwt.? *Ans.* 62 £. 11 s. 3 d.

9. Seventeen men brought each a load of hay to market, weighing 17 cwt. 3 qr. and 21 lb. and received each for his load, 5 £. 8 s. 3 d. What quantity of hay did they all bring; and how much money did they all receive? *Ans.* They brought 15 T. 4 cwt. 3 qr. 21 lb. and received 92 £. 0 s. 3 d.

#### EXAMPLES OF WEIGHTS AND MEASURES.

1. What is the weight of 5 hogshheads of sugar, each weighing 7 cwt. 3 qr. 16 lb.? *Ans.* 39 cwt. 1 qr. 24 lb.

2. What is the weight of 9 chests of tea, each weighing 3 cwt. 2 qr. 9 lb.? *Ans.* 32 cwt. 25 lb.

3. In 8 piles of wood, each containing 4 cords and 56 feet, how many cords and feet? *Ans.* 35 cords, 64 feet.

4. Multiply 15 yards, 3 qr. and 2 nails, by 9. *Ans.* 142 yd. 3 qr. 2 nails.

5. Multiply 20 years, 5 months, 3 weeks, and 6 days, by 14. *Ans.* 286 yr. 11 mo. 2 w.

6. In 10 fields, containing each 12 acres, 2 roods, and 16 rods, how many acres? *Ans.* 126.

7. In 7 casks, containing each 42 gallons, 3 quarts, and 1 pint, how many gallons, &c? *Ans.* 300 gal. 1 pint.

**QUESTIONS.**—What is always the nature of the multiplier? Which is the compound number, the multiplier or multiplicand? What is the nature of the product? In case the quantity by which you multiply is yards, what does the number of yards decide? What is Case 1st? What is the rule? What is Case 2d? What is the rule? What is the note under Case 2d? What is Case 3d? What is the rule? What note follows the rule? No direct definition has been given of Compound Multiplication; will the scholar give one?

## COMPOUND DIVISION.

This is the last of the Compound Rules, and is the reverse of the preceding. In this rule a compound number is given as a dividend, and a simple number as a divisor; and by the operation the dividend is resolved into as many equal parts as there are units in the divisor. The quotient is always one of these equal parts, and is therefore a compound number; each figure of which is of the same denomination as the figure or figures in the dividend from which it was obtained.

**CASE 1st.**—WHEN THE DIVISOR OR SIMPLE NUMBER IS 12, OR LESS THAN 12.

**RULE.**—*Divide the highest denomination first. If after dividing this, there be a remainder, reduce it to the next lower denomination, adding the figures of the dividend in that denomination to it, and divide again. Proceed in the same manner through all denominations; the number obtained will be the one required.*

**Ex. 1.** Divide 17 £. 11 s. 5 d. by 8.

**PERFORMED.**  
8 ) 17 £. 11 s. 5 d.

2 £. 3 s. 11½ d.

17 £.  $\div 8 = 2$  and 1 remains. What remains of any number or quantity, must obviously be of the same kind as the quantity itself. Therefore, the 1 is one pound = 20 s. and 20 s. + 11 s. = 31 s. and  $31 \div 8 = 3$  s. and 7 s. remain. Again, 7 s. = 84 d. and  $84 \text{ d.} + 5 \text{ d.} = 89 \text{ d.}$ ; and  $89 \div 8 = 11$  and 1 remainder =  $11\frac{1}{8} \text{ d.}$  Therefore the answer is as given above, viz. 2 £. 3 s.  $11\frac{1}{8} \text{ d.}$

2. Divide 25 £. 18 s. 9 d. by 6.

PERFORMED.

$$6 \overline{) 25 \text{ £. } 18 \text{ s. } 9 \text{ d.}}$$

$$4 \text{ £. } 6 \text{ s. } 5 \text{ d. } 2 \text{ qr.}$$

3. Divide 13 £. 19 s. 4 d. by 4. Ans. 3 £. 9 s. 10 d.

4. Divide 140 £. 12 s. 9 d. by 12. Ans. 11 £. 14 s. 4 d. 3 qr.

5. Divide 73 £. 16 s. 11 d. 3 qr. by 9. Ans. 8 £. 4 s. 1 d.  $1\frac{1}{3} \text{ qr.}$

6. Divide 12 cwt. 3 qr. 12 lb. by 10. Ans. 1 cwt. 1 qr. 4 lb.

7. Eleven men own equal shares of 36 hhd. 42 gal. and 2 qt. of wine; what is each man's share? Ans. 3 hhd. 21 gal. 0 qt. 0 pt.  $1\frac{5}{11} \text{ gills.}$

8. Seven men bought 16 hhd. 24 gal. 3 qt. of wine, for which they paid 45 £. 18 s. 6 d.; each man paying the same money and consequently entitled to an equal share of wine. What was each man's share, and how much money did he pay? Ans. His share was 2 hhd. 21 gal.  $2\frac{1}{4} \text{ qt.}$  and paid 6 £. 1 s. 2 d.  $2\frac{1}{4} \text{ qr.}$

CASE 2d.—WHEN THE DIVISOR IS A COMPOSITE NUMBER GREATER THAN 12.

*RULE.*—Resolve the divisor into its component parts, and divide the compound number by each of these parts in succession. The quotient arising from the first division will form a dividend for the second; and so on.

EX. 1. Divide 26 £. 16 s. 8 d. by 21. The factors of 21 are 7 and 3, because  $7 \times 3 = 21$ . Therefore,

$$7 \overline{) 26 \text{ £. } 16 \text{ s. } 8 \text{ d.}}$$

$$3 \overline{) 3 \text{ £. } 16 \text{ s. } 8 \text{ d.}}$$

$$1 \text{ £. } 5 \text{ s. } 6\frac{1}{3} \text{ d.} = \text{the quotient of } 26 \text{ £. } 16 \text{ s. } 8 \text{ d.} \div 21, \text{ and is the answer.}$$

2. Divide 47 £. 15 s. 8 d. by 24. *Ans.* 1 £. 19 s. 9 d.  $3\frac{1}{4}$  qr.
3. Divide 85 £. 11 s. 11 d. by 16. *Ans.* 5 £. 6 s. 11 d.  $3\frac{1}{4}$  qr.
4. Divide 128 £. 9 s. by 42. *Ans.* 3 £. 1 s. 2 d.
5. Divide 15 £. 18 s. 9 d. by 72. *Ans.* 4 s.  $5\frac{1}{2}$  d.
6. Divide 5 £. 10 s. 3 d. by 81. *Ans.* 1 s. 4 d.  $1\frac{1}{2}$  qr.
7. Divide 7 £. 19 s. 9 d. by 96. *Ans.* 1 s. 7 d.  $3\frac{1}{4}$  qr.
8. Divide 27 £. 16 s. by 32. *Ans.* 17 s. 4 d. 2 qr.

CASE 3d.—WHEN THE DIVISOR IS LARGE AND NOT A COMPOSITE NUMBER.

*RULE.*—Divide the whole compound quantity by the whole divisor; reducing the remainders after the division of each denomination to the next lower denomination, as directed in Case 1st.

Ex. 1. Divide 8 £. 18 s. 6 d. by 51.

$$\begin{array}{r}
 \text{PERFORMED.} \\
 51 \overline{) 8 \text{ £. } 18 \text{ s. } 6 \text{ d. (0 £. } 3 \text{ s. } 6 \text{ d.}} \\
 \underline{20} \\
 178 = \text{the shillings in 8 £. 18 s.} \\
 \underline{153} \\
 25 = \text{shillings remaining.} \\
 \underline{12} \\
 306 = \text{pence in 25 s. 6 d.} \\
 \underline{306} \\
 000
 \end{array}$$

The pounds are first reduced to shillings, and the given shillings are added. The 178 shillings are thus produced. This divided by 51 gives 3 as a quotient figure and 25 as a remainder. After a second reduction, 306 pence are obtained, which contains the divisor six times. Thus the answer obtained is 3 s. 6 d.

2. Divide 41 £. 14 s. 3 d. by 47. *Ans.* 17 s. 9 d.
3. Divide 4 £. 1 s. 5 d. 2 qr. by 23. *Ans.* 3 s. 6 d. 2 qr.
4. Divide 137 £. 17 s. 4 d. by 94. *Ans.* 1 £. 9 s. 4 d.
5. Divide 36 £. 1 s. 6 d. by 78. *Ans.* 9 s. 3 d.
6. Divide 10 £. 5 s. 8 d. by 59. *Ans.* 3 s. 5 d. 3 qr. +
7. Divide 25 £. 12 s. 4 d. by 29. *Ans.* 17 s. 8 d.
8. Divide 61 £. 12 s. by 65. *Ans.* 18 £. 11 s. 1 qr. +

## EXAMPLES IN WEIGHTS AND MEASURES.

1. Divide 5 hhd. 42 gal. 3 qt. equally among 4 men. *Ans.* 1 hhd. 26 gal. 1 qt. 1 pt. 2 gi.
2. Divide 14 cwt. 1 qr. 12 lb. by 5. *Ans.* 2 cwt. 3 qr. 13 lb. 9 oz.  $9\frac{3}{5}$  dr.
3. Divide 27 yd. 1 qr. 2 na. by 7. *Ans.* 3 yd. 3 qr.  $2\frac{4}{7}$  na.
4. Divide 156 bushels, 3 pk. 6 qt. by 18. *Ans.* 8 bu. 2 pk. 7 qt.
5. Divide 9 hhd. 28 gal. 2 qt. by 12. *Ans.* 49 gal. 2 qt. 1 pt.
6. Divide 16 cwt. 3 qr. 18 lb. by 32. *Ans.* 2 qr. 3 lb. 3 oz.
7. If 27 loads of hay weigh 30 tons, 8 cwt. 2 qr. 23 lb. what is the weight of one load? *Ans.* 1 ton, 2 cwt. 2 qr. 5 lb.
8. A man traveled 17 leagues, 1 mile, 4 furlongs, and 21 poles, in 21 hours. At what rate did he travel per hour? *Ans.* 2 m. 4 fur. and 1 pole.
9. Nine men own 56 lb. 6 oz. and 17 pwt. of silver. What will each man receive if the whole quantity be equally divided among them? *Ans.* 6 lb. 3 oz. 8 pwt.  $18\frac{1}{3}$  gr.
10. Bought 15 loads of hay, the whole weight of which was 12 tons, 15 cwt. 3 qr. 16 lb. Supposing them all to have been equal, what was the weight of each? *Ans.* 17 cwt.  $6\frac{1}{3}$  lb.
11. If a man's income be 86 £. 18 s. 10 d. per year, what is it per calendar month? *Ans.* 7 £. 4 s.  $10\frac{1}{3}$  d.
12. If I pay 15 £. 3 s. 8 d. for 56 pairs of gloves, what is one pair worth? *Ans.* 5 s. 5 d.  $0\frac{2}{3}$  qr.
13. If a hogshead of wine cost 33 £. 12 s., what is the price of a gallon? *Ans.* 10 s. 8 d.
14. If 42 yards of cloth cost 21 £. 18 s. 8 d., what was the cost per yard? *Ans.* 10 s. 5 d.  $1\frac{1}{3}$  qr.
15. If 16 men cut 53 cords, 69-feet of wood in 2 days, what did each man cut per day? *Ans.* 1 cord,  $86\frac{1}{2}$  feet.

## APPLICATION OF THE FOUR PRECEDING RULES.

1. A silversmith sold to his customer 3 dozen silver spoons, each weighing 3 oz. 3 pwt. 16 gr.;  $1\frac{1}{2}$  dozen tea spoons, each weighing 14 pwt. 20 gr.; 3 silver cups, each weighing 20 oz. 18 pwt. In return, he received old silver to the amount of 8 lb. 11 oz. 19 pwt.; for how much ought he to receive pay? *Ans.* 6 lb. 10 oz. 14 pwt.
2. Bought the following articles at the prices mentioned; viz.

4 cwt. of sugar at 2 £. 4 s. 8 d. per cwt.  
 3 hhd. of molasses at 2 s. 4 d. per gallon,  
 5 lb. of green tea at 7 s. 6 d. per pound,  
 12 lb. of raisins at 2 s. per pound,  
 42 yards of cotton cloth at 1 s. 6 d. per yard,  
 27 pounds of ham at 1 s. 3 d. per pound ;

What was the amount of my bill? *Ans.* 38 £. 17 s. 11 d.

3. Bought of James Rankin, £. s. d.  
 27 yards of broadcloth at 1 £. 9 s. per yard,  
 42 yards of Irish linen at 4 s. 6 d. per yard,  
 36 hats valued each at 18 s.  
 30 pairs of shoes at 7 s. 8 d. per pair ;

What was the amount of my bill? *Ans.* 92 £. 10 s. 0 d.

4. A. owning 100 acres of land, divided it into 8 equal parts, and sold each part for \$22.50 per acre. How many acres were there in each part, and what was the value of the same?  
*Ans.* 12 A. 2 R. ; value, \$281.25.

5. Out of a pipe of wine a merchant sold 36 gallons, 3 quarts, and 1 pint at each of three different times ; he then filled 15 bottles, holding 1 pint and 2 gills each ; how much remained?  
*Ans.* 12 gal. 2 qt. and 2 gills.

6. Bought 144 pairs of shoes for 96 £. ; what was the price of one pair? *Ans.* 13 s. 4 d.

7. A person dying, left real estate to the amount of 2356 £. 19 s. 9 d. and personal property to the amount of 3184 £. 12 s. 8 d. In his will, he directed that his wife should receive one sixth of the whole, and that the remainder should be equally divided among his four daughters. What was the share of each? *Ans.* The widow, 923 £. 12 s.  $\frac{1}{2}$  d. and the daughters each 1154 £. 10 s.  $1\frac{1}{4}$  d.

**QUESTIONS.**—How does Compound Division compare with Compound Multiplication? What is given as a dividend? What as a divisor? Into what is the dividend resolved by the operation? What is always one of these equal parts? Is the quotient a Simple or Compound Number? How may you know the denomination of each figure in the quotient? What is Case 1st? What is the rule? What is Case 2d? What is the rule? What is Case 3d? What rule?

## VULGAR FRACTIONS.

1. When a unit or single object is divided into a number of equal parts, each of these parts is a fraction.

If it be divided into two equal parts, each part is called a *half*, and is thus written,  $\frac{1}{2}$ .

If it be divided into three equal parts, each is called a *third*, and is thus written,  $\frac{1}{3}$ .

If the whole be separated into six equal parts, each part is called a *sixth*, and if into eight equal parts, an *eighth*, of the whole, and are thus written,  $\frac{1}{6}$ ,  $\frac{1}{8}$ .

When more parts than one are to be expressed, the figure above the line designates their number, thus,  $\frac{4}{6}$ ; by which expression, we are to understand that the unit is divided into six equal parts, and that five of these parts are included in the fraction.

The fraction therefore is used to express *parts of units*, and is represented by two numbers, one standing below and the other above a short horizontal line. The number below the line is called the *denominator*, and shows the number of equal parts into which the unit is divided. The number above the line is called the *numerator*, and shows how many of these equal parts are included in the fraction, or make up its value. Thus of the fraction  $\frac{5}{9}$ , the lower number shows a unit to be divided into nine equal parts; and the upper number, that five of these parts are included in the fraction.

These two numbers, when spoken of collectively, are called the *terms* of the fraction.

2. Fractions are divided into six kinds; viz. Proper, Improper, Simple, Compound, Mixed, and Complex.

A Proper Fraction is one whose numerator is less than its denominator, as,  $\frac{3}{4}$ .

An Improper Fraction is one whose numerator equals or exceeds its denominator, as,  $\frac{5}{4}$ .

A Simple Fraction consists of one expression, and is either proper or improper, as,  $\frac{3}{4}$  or  $\frac{5}{4}$ .

A Compound Fraction is the fraction of a fraction, as,  $\frac{1}{2}$  of  $\frac{3}{4}$ . It may consist of any number of simple fractions.

A Mixed Number consists of a whole number and fraction written together, as,  $6\frac{3}{4}$ ,  $25\frac{1}{2}$ , &c.

A Complex Fraction is one that has a fraction in its numerator or denominator, or both, as,  $\frac{2\frac{1}{2}}{3}$ ,  $\frac{4}{7\frac{1}{4}}$ ,  $\frac{9\frac{1}{2}}{8\frac{1}{4}}$ , &c.

3. The denominator shows the number of equal parts into which the unit is divided; and the numerator, how many of these parts are expressed in the fraction. Consequently, the greater the numerator, the denominator being given, the greater the value of the fraction; and the less the numerator, the less the value of the fraction. If the denominator be 8, and the numerator 1, the value expressed is  $\frac{1}{8}$ , or one eighth part of a unit; if the numerator be 2, the value expressed is  $\frac{2}{8}$ , or two eighth parts of a unit; if it be 4, the value is  $\frac{4}{8}$ , or four eighth parts of a unit; and if it be 6, the value is  $\frac{6}{8}$ , or six eighth parts of a unit.

*The value of a fraction is therefore the quotient arising from dividing the numerator by the denominator, and always increases in the same ratio as the numerator, so long as the denominator remains unaltered.*

We may therefore express any value, not only less than a unit, but equal to and even greater than a unit, by a fraction. Thus, if we take 9 as the denominator of a fraction, and any number less than 9 as a numerator of the same, the value expressed is always less than a unit, as,  $\frac{8}{9}$ ; or if 9 be taken as the numerator, we obtain the fraction  $\frac{9}{9}$ , which, as the unit was divided into 9 parts only, is obviously equal to 1. Again, we may suppose more than a single unit of the same kind to be divided in the same manner, and their parts united in one fraction, and thus obtain fractions of any value more than a unit. If two units be thus divided into seven equal parts, and three parts of the one be united to all the parts of the other, the fraction would be  $\frac{10}{7}$ ; or if all the parts of each be united, it would be  $\frac{14}{7}$ , which is equal to 2; or if three units were thus divided, all their parts would produce the fraction  $\frac{21}{7} = 3$ .

The only consideration which limits the value of a fraction, is the number of equal parts united in the same expression.

From the preceding it is obvious that the value of a fraction is increased in the same ratio as the numerator; hence,

4. *A fraction is multiplied by a whole number, by multiplying the numerator only.*



In accordance with the above principle, the scholar may multiply the following examples :

1. Multiply  $\frac{1}{2}$  by 3. *Ans.*  $\frac{3}{2}$ .
2. Multiply  $\frac{2}{3}$  by 5. *Ans.*  $\frac{10}{3}$ .
3. Multiply  $\frac{3}{4}$  by 3. *Ans.*  $\frac{9}{4}$ .
4. Multiply  $\frac{1}{6}$  by 9. *Ans.*  $\frac{9}{6}$ .
5. Multiply  $\frac{2}{12}$  by 6. *Ans.*  $\frac{12}{12}$ .
6. Multiply  $\frac{1}{12}$  by 9. *Ans.*  $\frac{9}{12}$ .
7. Multiply  $\frac{1}{12}$  by 7. *Ans.*  $\frac{7}{12}$ .
8. Multiply  $\frac{2}{8}$  by 12. *Ans.*  $\frac{24}{8}$ .
9. Multiply  $\frac{1}{12}$  by 12. *Ans.*  $\frac{12}{12}$ .
10. Multiply  $\frac{1}{8}$  by 8. *Ans.*  $\frac{8}{8}$ .
11. Multiply  $\frac{1}{3}$  by 3. *Ans.*  $\frac{3}{3}=1$ .
12. Multiply  $\frac{1}{7}$  by 7. *Ans.*  $\frac{7}{7}=1$ .

From the last two examples, it is obvious that a fraction is multiplied by a number equal to its own denominator, by rejecting that denominator and retaining only the numerator.

It should always be an object with the scholar to preserve the terms of a fraction *as small as is possible and express the true value*. This was not regarded in the above examples. A little experience will show that to increase or diminish the value of a fraction, it is only necessary to make the numerator larger or smaller *compared with the denominator*. Suppose it be required to multiply the fraction  $\frac{1}{6}$  by 2. By the above rule the product would be  $\frac{2}{6}$ , which is equal in value to  $\frac{1}{3}$ , and this is at once obtained by dividing the denominator by 2 instead of multiplying the numerator as above, thus,  $\frac{1}{6 \div 2} = \frac{1}{3}$ ; therefore,

*A fraction is multiplied by a whole number, by dividing the denominator by that number.*

The following examples will illustrate this principle :

1. Multiply  $\frac{2}{3}$  by 2. *Ans.*  $\frac{2}{3}$ .
2. Multiply  $\frac{2}{6}$  by 3. *Ans.*  $\frac{2}{2}$  or 1.
3. Multiply  $\frac{2}{8}$  by 4. *Ans.*  $\frac{2}{2}$ .
4. Multiply  $\frac{2}{10}$  by 5. *Ans.*  $\frac{2}{2}$ .
5. Multiply  $\frac{2}{16}$  by 8. *Ans.*  $\frac{2}{2}$ .
6. Multiply  $\frac{2}{12}$  by 7. *Ans.*  $\frac{7}{6}$ .
7. Multiply  $\frac{2}{12}$  by 4. *Ans.*  $\frac{2}{3}$ .
8. Multiply  $\frac{1}{12}$  by 9. *Ans.*  $\frac{3}{4}$ .
9. Multiply  $\frac{1}{20}$  by 5. *Ans.*  $\frac{1}{4}$ .
10. Multiply  $\frac{1}{128}$  by 8. *Ans.*  $\frac{1}{16}$ .

*The value of a fraction may therefore be multiplied by a whole number, either by multiplying the numerator or dividing the denominator by that number.*

*Note.*—The denominator should always be divided, whenever it can be done without a remainder.

5. *A fraction is divided by a whole number, by dividing the numerator by that number.*

This needs no explanation. If we divide a number by 2, we take a half, and if by 3, a third of that number; that is, the divisor always shows what part of the dividend is taken; therefore,  $\frac{12}{13} \div 2 = \frac{6}{13}$ , and  $\frac{12}{13} \div 3 = \frac{4}{13}$ .

The following examples will illustrate the operation of the above principle :

Ex. 1. Divide  $\frac{3}{8}$  by 3. *Ans.*  $\frac{1}{8}$ .

2. Divide  $\frac{4}{7}$  by 2. *Ans.*  $\frac{2}{7}$ .

3. Divide  $\frac{9}{15}$  by 8. *Ans.*  $\frac{3}{40}$ .

4. Divide  $\frac{14}{15}$  by 6. *Ans.*  $\frac{7}{45}$ .

5. Divide  $\frac{1}{8}$  by 4.

6. Divide  $\frac{20}{3}$  by 5.

7. Divide  $\frac{3}{4}$  by 7.

In this last example, the scholar will find a difficulty. He cannot divide the numerator in any way, except to place it over the 7 in the form of a fraction, as will hereafter be explained; and this would make one fraction the numerator of another fraction. When, therefore, the divisor will not divide the numerator without a remainder, a more convenient mode of operating is desirable. It will be remembered, that division is the reverse of multiplication; and since we can multiply fractions by dividing the denominator, we will try the effect of dividing fractions by multiplying the denominator. Let it be required to divide  $\frac{3}{12}$  by 3. By dividing as above, we obtain  $\frac{1}{12}$  as the quotient, viz.  $\frac{3}{12} \div 3 = \frac{1}{12}$ . By the mode we propose to try, we obtain  $\frac{3}{36}$ . It therefore remains to show that  $\frac{1}{12} = \frac{3}{36}$ . If any object be first divided into 12 equal parts, and then each of these 12 parts be divided into 3 equal parts, it is plain that the whole would be divided into 36 equal parts, and that each twelfth part would make 3 thirty-sixth parts; therefore,  $\frac{1}{12} = \frac{3}{36}$ ; hence, *a fraction is divided by a whole number by multiplying its denominator by that number.*

This principle may be applied to the following sums :

Ex. 1. Divide  $\frac{3}{4}$  by 6. Ans.  $\frac{1}{8}$ .

2. Divide  $\frac{1}{2}$  by 2. Ans.  $\frac{1}{4}$ .

3. Divide  $\frac{1}{3}$  by 3. Ans.  $\frac{1}{9}$ .

4. Divide  $\frac{1}{4}$  by 5. Ans.  $\frac{1}{20}$ .

5. Divide  $\frac{1}{5}$  by 8.

6. Divide  $\frac{1}{6}$  by 7.

7. Divide  $\frac{1}{7}$  by 11.

8. Divide  $\frac{1}{8}$  by 6.

By uniting the two preceding principles, we have the following more comprehensive principle, viz. :

*The value of a fraction is divided by a whole number, by dividing the numerator, or by multiplying the denominator by that number.*

*Note.*—The numerator should always be divided, when it can be done *without a remainder*. In all other cases the denominator should be multiplied.

From the preceding remarks and illustrations we learn, *that whatever operation is performed on the numerator of a fraction, the SAME OPERATION IS PERFORMED on the VALUE of the fraction ; but that the effect produced on the VALUE of any fraction, is the REVERSE of the OPERATION PERFORMED ON ITS DENOMINATOR.*

6. A fraction is multiplied by a fraction, by multiplying the numerators together for a new numerator, and the denominators for a new denominator.

For example, let it be required to multiply  $\frac{1}{2}$  by  $\frac{3}{4}$ . Agreeably to the principles already explained, if I multiply the denominator of the fraction  $\frac{1}{2}$ , by 4, the other denominator, I shall obtain  $\frac{1}{4}$  of that quantity, viz.  $\frac{1}{8}$ ; and if I multiply this quantity, viz.  $\frac{1}{8}$ , by 3, the other numerator, I shall make this value three times as large, that is, it will become  $\frac{3}{8}$ ; therefore,  $\frac{3}{8}$  is  $\frac{3}{4}$  of  $\frac{1}{2}$ , or  $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$ .

In accordance with the above, the scholar may multiply the following fractions :

1. Multiply  $\frac{1}{2}$  by  $\frac{5}{8}$ . Ans.  $\frac{5}{16}$ .

2. Multiply  $\frac{3}{4}$  by  $\frac{2}{3}$ . Ans.  $\frac{1}{2}$ .

3. Multiply  $\frac{2}{3}$  by  $\frac{1}{4}$ . Ans.  $\frac{1}{6}$ .

4. Multiply  $\frac{9}{11}$  by  $\frac{1}{2}$ . Ans.  $\frac{9}{22}$ .

5. Multiply  $\frac{2}{3}$  by  $\frac{1}{12}$ . Ans.  $\frac{1}{18}$ .
6. Multiply  $\frac{1}{3}$  by  $\frac{1}{3}$ . Ans.  $\frac{1}{9}$ .
7. Multiply  $\frac{1}{3}$  by  $\frac{2}{3}$ .
8. Multiply  $\frac{1}{12}$  by  $\frac{1}{3}$ .
9. Multiply  $\frac{1}{6}$  by  $\frac{1}{10}$ .
10. Multiply  $\frac{1}{7}$  by  $\frac{1}{10}$ .

7. A fraction is divided by another fraction, by inverting the divisor, and multiplying them together as before.

A unit is contained in the fraction  $\frac{3}{4}$ , three fourths of once;  $\frac{1}{4}$  is consequently contained in the same fraction twice as often, viz.  $\frac{2}{4}$  of a time; and  $\frac{1}{8}$ , three times as often, viz.  $\frac{3}{4}$  of a time, which fractions are obviously obtained by multiplying  $\frac{3}{4}$  by  $\frac{1}{4}$  and  $\frac{1}{8}$  inverted. Again, suppose it be required to find how many times  $\frac{3}{4}$  is contained in  $\frac{7}{8}$ . As before, a unit or 1 is contained in  $\frac{7}{8}$ , seven eighths of a time;  $\frac{1}{4}$  would be contained in it four times as often, viz.  $\frac{28}{8}$  of a time, and  $\frac{3}{4}$  would be contained in the same only one third as often as  $\frac{1}{4}$ , viz.  $\frac{28}{3}$  of a time =  $1\frac{4}{3}$ , or  $1\frac{1}{3}$ . This result is obtained by inverting the divisor  $\frac{3}{4}$ , and multiplying it into the dividend  $\frac{7}{8}$ ; thus,  $\frac{7}{8} \times \frac{4}{3} = \frac{28}{24}$ .

The following examples may now be performed :

- Ex. 1. Divide  $\frac{1}{2}$  by  $\frac{2}{3}$ . Ans.  $\frac{3}{4}$ .
2. Divide  $\frac{5}{6}$  by  $\frac{1}{8}$ . Ans.  $\frac{40}{3}$ .
3. Divide  $\frac{4}{8}$  by  $\frac{9}{10}$ . Ans.  $\frac{40}{9}$ .
4. Divide  $\frac{3}{4}$  by  $\frac{1}{12}$ . Ans.  $\frac{9}{1}$ .
5. Divide  $\frac{1}{12}$  by  $\frac{1}{6}$ . Ans.  $\frac{6}{12}$ .
6. Divide  $\frac{1}{10}$  by  $\frac{1}{20}$ . Ans.  $\frac{20}{10}$ .
7. Divide  $\frac{3}{8}$  by  $\frac{1}{4}$ .
8. Divide  $\frac{9}{7}$  by  $\frac{9}{11}$ .
9. Divide  $\frac{6}{7}$  by  $\frac{1}{2}$ .
10. Divide  $\frac{1}{3}$  by  $\frac{1}{6}$ .

8. If the numerator and denominator of any fraction be both multiplied or both divided by the same number, the value of the fraction will not be altered.

Of this principle no explanation is necessary. The value of the fraction being the quotient arising from dividing the numerator by the denominator, it is obvious that if both the terms be doubled, or repeated any number of times, the value of the quotient will not be affected.

## REDUCTION OF FRACTIONS.

**CASE 1st.**—TO REDUCE FRACTIONS TO THEIR LOWEST TERMS;  
OR, TO FIND THE LOWEST TERMS BY WHICH THE VALUE OF  
A GIVEN FRACTION CAN BE EXPRESSED.

**RULE.**—Divide both numerator and denominator by any number that will divide them both WITHOUT REMAINDER; then divide the quotients obtained, in the same manner, and so continue to do till there is no number greater than 1, that will divide them. The last quotient will be the numerator and denominator required.

**Ex. 1.** Reduce  $\frac{48}{64}$  to its lowest terms. Operation,  $\frac{48}{64} \div 8 = \frac{6}{8}$ , and  $\frac{6}{8} \div 2 = \frac{3}{4}$ , its lowest term.

2. Reduce  $\frac{124}{198}$  to its lowest terms. *Ans.*  $\frac{31}{49}$ .
3. Reduce  $\frac{144}{184}$  to its lowest terms. *Ans.*  $\frac{9}{11}$ .
4. Reduce  $\frac{96}{108}$  to its lowest terms. *Ans.*  $\frac{8}{9}$ .
5. Reduce  $\frac{12}{16}$  to its lowest terms. *Ans.*  $\frac{3}{4}$ .
6. Reduce  $\frac{240}{312}$  to its lowest terms. *Ans.*  $\frac{20}{26}$ .
7. Reduce  $\frac{48}{60}$  to its lowest terms. *Ans.*  $\frac{4}{5}$ .
8. Reduce  $\frac{465}{718}$  to its lowest terms. *Ans.*  $\frac{1}{2}$ .
9. Reduce  $\frac{894}{1177}$  to its lowest terms. *Ans.*  $\frac{1}{2}$ .
10. Reduce  $\frac{678}{880}$  to its lowest terms. *Ans.*  $\frac{1}{2}$ .
11. Reduce  $\frac{68}{108}$  to its lowest terms. *Ans.*  $\frac{17}{27}$ .
12. Reduce  $\frac{104}{56}$  to its lowest terms. *Ans.*  $\frac{13}{7}$ .

**CASE 2d.**—TO REDUCE A WHOLE NUMBER OR A MIXED QUANTITY TO AN IMPROPER FRACTION.

**RULE.**—If the given quantity be a whole number, multiply it by the proposed denominator; the product will be the numerator: but if it be a mixed quantity, multiply the whole number by the denominator of the fraction, and to the product add the given numerator; then under the number thus produced, write the denominator.

**Ex. 1.** Reduce 21 to a fraction whose denominator is 9. Operation,  $21 \times 9 = 189$ , the numerator; the fraction therefore is  $\frac{189}{9}$ .

2. Reduce  $8\frac{1}{3}$  to an improper fraction. Operation,  $8 \times 3 = 24$ , and  $24 + 1 = 25$ , the numerator; therefore,  $\frac{25}{3}$  is the answer.
3. Reduce  $16\frac{1}{2}$  to an improper fraction. Ans.  $\frac{33}{2}$ .
4. Reduce  $17\frac{1}{3}$  to an improper fraction. Ans.  $\frac{52}{3}$ .
5. Reduce  $47\frac{1}{5}$  to an improper fraction. Ans.  $\frac{236}{5}$ .
6. Reduce  $135\frac{1}{4}$  to an improper fraction. Ans.  $\frac{541}{4}$ .
7. Reduce  $1\frac{1}{2}$  to an improper fraction. Ans.  $\frac{3}{2}$ .
8. Reduce  $1728\frac{1}{2}$  to an improper fraction. Ans.  $\frac{4667}{2}$ .
9. Reduce  $9\frac{1}{2}$  to an improper fraction. Ans.  $\frac{19}{2}$ .
10. Reduce  $12\frac{1}{2}$  to an improper fraction. Ans.  $\frac{25}{2}$ .
11. Reduce 8 to a fraction whose denominator shall be 9. Ans.  $\frac{72}{9}$ .
12. Reduce 16 to a fraction whose denominator shall be 12. Ans.  $\frac{192}{12}$ .

CASE 3d.—TO REDUCE AN IMPROPER FRACTION TO A WHOLE OR MIXED NUMBER.

RULE.—Divide the numerator by the denominator; the quotient will be the whole number. If there be any remainder, place it over the denominator at the right of the whole number.

Note.—The true quotient includes both the whole number and fraction. In all cases of division, therefore, the remainder (if any) constitutes the numerator of a fraction of which the divisor is the denominator.

Ex. 1. Reduce  $\frac{141}{17}$  to a mixed number.

OPERATION.

$$\begin{array}{r} 17 \overline{) 141} \quad (8 \\ \underline{136} \phantom{0} \\ 5 \phantom{0} \end{array}$$

5 rem.; therefore,  $8\frac{5}{17}$  is the answer.

2. Reduce  $\frac{346}{23}$  to a mixed quantity. Ans.  $15\frac{1}{23}$ .
3. Reduce  $\frac{19}{8}$  to a mixed quantity. Ans.  $2\frac{3}{8}$ .
4. Reduce  $\frac{49}{5}$  to a mixed quantity or whole number. Ans. 7.
5. Reduce  $\frac{45}{8}$  to a mixed quantity. Ans.  $5\frac{5}{8}$ .
6. Reduce  $\frac{346}{43}$  to a mixed number. Ans.  $10\frac{194}{43}$ .
7. Reduce  $\frac{346}{5}$  to a mixed number. Ans.  $69\frac{16}{5}$ .
8. Reduce  $\frac{1246}{5}$  to a mixed number. Ans.  $249\frac{16}{5}$ .
9. Reduce  $\frac{106}{8}$  to a whole number. Ans. 12.
10. Reduce  $\frac{12}{13}$  to a mixed number. Ans.  $6\frac{1}{13}$ .
11. Reduce  $\frac{2789}{11}$  to a whole or mixed number. Ans. 253.
12. Reduce  $\frac{56789}{11}$  to its proper number. Ans.  $5162\frac{7}{11}$ .

CASE 4th.—TO REDUCE COMPOUND FRACTIONS TO SIMPLE ONES.

RULE 1st.—Multiply all the numerators together for a new numerator, and all the denominators for a new denominator, and reduce the new fraction to its lowest terms, by Case 1st.

Ex. 1. Reduce  $\frac{2}{3}$  of  $\frac{3}{4}$  of  $\frac{5}{6}$  to a simple fraction. Performed,  $2 \times 3 \times 5 = 30$ , the new numerator; and  $3 \times 4 \times 6 = 72$ , the new denominator; therefore,  $\frac{30}{72}$  is the fraction required, but susceptible of being expressed in lower terms; therefore,  $\frac{30}{72} \div 6 = \frac{5}{12}$ .  
Ans.

Compound fractions may be reduced to simple ones, however, much more expeditiously, by canceling. The labor of reducing to lower terms is thereby avoided.

RULE.—Draw a horizontal line and place all the numerators above the line and all the denominators below it. Cancel the numbers as far as practicable, as taught in the Rule for Canceling; then make the product of the numbers remaining above the line the new numerator, and the product of those remaining below, the new denominator.

Note 1st.—If there be nothing remaining above the line after canceling, 1 will always be the numerator of the new fraction. The same is true of the denominators.

Ex. 2. Reduce  $\frac{1}{2}$  of  $\frac{3}{4}$  of  $\frac{4}{5}$  to a simple fraction.

Statement,  $\frac{1. 3. 4}{2. 4. 5}$ . Canceled,  $\frac{1. 3. 4}{2. 4. 5}$ . Ans.  $\frac{3}{10}$ .

Example 1st stated and solved by canceling:

$\frac{2. 3. 5}{3. 4. 6}$ . Canceled,  $\frac{2. 3. 5}{2. 4. 6}$ . Ans.  $\frac{5}{12}$ .

3. Reduce  $\frac{6}{7}$  of  $\frac{1}{2}$  of  $\frac{14}{24}$  to a simple fraction.

Statement,  $\frac{6. 1. 14}{7. 2. 24}$ . Canceled,  $\frac{6. 1. 14}{7. 2. 24}$ . Ans.  $\frac{1}{4}$ .

Note 2d.—Whenever the product of any two numbers on one side of the line will cancel any number on the opposite side, they may be so canceled; as in the last example, 7 and 2 below the line cancel 14 above it.

4. Reduce  $\frac{5}{9}$  of  $\frac{12}{13}$  of  $\frac{1}{4}$  of  $\frac{7}{8}$  to a simple fraction.

$$\text{Statement, } \frac{5. 12. 4. 7}{9. 13. 5. 8} \quad \text{Canceled, } \frac{\overset{5}{\cancel{5}}. \overset{2}{\cancel{12}}. \overset{4}{\cancel{4}}. \overset{7}{\cancel{7}}}{\underset{3}{\cancel{9}}. \underset{2}{\cancel{13}}. \underset{2}{\cancel{5}}. \underset{2}{\cancel{8}}}$$

and  $7 \times 2 = 14$ , numerator; and  $13 \times 3 = 39$ , denominator; therefore, the new fraction is  $\frac{14}{39}$ .

5. Reduce  $\frac{5}{7}$  of  $\frac{1}{8}$  of  $\frac{1}{12}$  of  $\frac{2}{15}$  of  $\frac{7}{13}$  to a simple fraction. *Ans.*  $\frac{1}{360}$ .

6. Reduce  $\frac{1}{2}$  of  $\frac{3}{4}$  of  $\frac{2}{3}$  of  $\frac{4}{5}$  of  $\frac{5}{6}$  of  $\frac{6}{7}$  of  $\frac{7}{8}$  of  $\frac{8}{9}$  of  $\frac{9}{10}$  to a simple fraction. *Ans.*  $\frac{1}{10}$ .

7. Reduce  $\frac{1}{8}$  of  $\frac{5}{7}$  of  $\frac{9}{10}$  of  $\frac{2}{3}$  of  $\frac{6}{5}$  of  $\frac{4}{3}$  to a simple fraction. *Ans.*  $\frac{256}{315}$ .

*Note 3d.*—If any term of a compound fraction be a mixed number, it must be reduced to an improper fraction before stating.

8. Reduce  $\frac{1}{2}$  of  $\frac{5}{7}$  of  $4\frac{1}{2}$  of  $\frac{4}{5}$  of  $\frac{3}{4}$  to a simple fraction.  $4\frac{1}{2} = \frac{9}{2}$ , therefore, statement,  $\frac{1. 6. 14. 4. 3}{2. 7. 3. 5. 4}$ ; which canceled will give the answer,  $\frac{1}{5}$ .

9. Reduce  $\frac{2}{3}$  of  $7\frac{1}{2}$  of  $\frac{2}{3}$  of  $2\frac{1}{2}$  to a simple fraction. *Ans.*  $\frac{116}{315} = 6\frac{8}{315}$ .

10. Reduce  $\frac{1}{2}$  of  $\frac{3}{8}$  of  $\frac{2}{3}$  of  $\frac{5}{6}$  of  $\frac{7}{8}$  to a simple fraction. *Ans.*  $\frac{13}{144}$ .

11. Reduce  $\frac{9}{10}$  of  $\frac{5}{6}$  of  $\frac{1}{2}$  to a simple fraction. *Ans.*  $\frac{15}{80}$ .

12. Reduce  $\frac{9}{10}$  of  $\frac{5}{6}$  of  $\frac{7}{8}$  of  $\frac{1}{2}$  to a simple fraction. *Ans.*  $\frac{7}{160}$ .

13. Reduce  $\frac{1}{2}$  of  $\frac{1}{3}$  of  $\frac{5}{6}$  of  $\frac{1}{4}$  to a simple fraction. *Ans.*  $\frac{1}{48}$ .

14. Reduce  $\frac{5}{6}$  of  $\frac{4}{5}$  of  $\frac{9}{10}$  of  $\frac{1}{8}$  to a simple fraction. *Ans.*  $\frac{3}{40}$ .

15. Reduce  $\frac{3}{2}$  of  $8\frac{1}{2}$  of  $5\frac{1}{2}$  to a simple fraction. *Ans.*  $\frac{531}{8}$ , or  $70\frac{1}{8}$ .

**CASE 5th.**—To CHANGE FRACTIONS FROM ONE DENOMINATION TO ANOTHER, WITHOUT ALTERING THE VALUE.

1st. To reduce fractions of low denominations to those of higher value.

**RULE.**—Divide the fraction, or what is the same thing, multiply the denominator by such numbers as are required to reduce the given quantity from the GIVEN to the REQUIRED DENOMINATION.

**Ex. 1.** Reduce  $\frac{5}{8}$  of a penny to the fraction of a pound. The numbers required to reduce pence to pounds, are 12 and 20; therefore,  $\frac{5}{8}$  of a penny is to be divided by these numbers; and since this can be effected in the present case only by multiplying



the denominator, the operation will be;  $\frac{5}{6} \times 12 \times 20 = \frac{5}{1440}$ , and this, by Case 1st, is reduced to  $\frac{1}{288}$ , which is the fraction required. Hence  $\frac{2}{3}$  of a penny equals  $\frac{1}{288}$  of a pound.

The canceling principle may however be successfully applied in the solution of sums of this character.

**RULE FOR CANCELING.**—Place the numerator of the given fraction above a horizontal line, and its denominator below it; then place also BELOW the line, such numbers as are necessary to reduce the denomination given to that required. Cancel, &c. as before.

We will solve the above example by this rule also. Statement,  $\frac{5}{6. 12. 20}$ . The scholar should compare the statement with the rule, to see that he understands its application. The above statement canceled,  $\frac{5}{6. 12. 20}$ ; and  $6 \times 12 \times 4 = 288$ , the denomi-

nator as before, and nothing remains as a numerator; therefore, as before,  $\frac{1}{288}$  of a pound is the answer. (See Note 1st, Case 4th.)

2. Reduce  $\frac{3}{4}$  of a farthing to the fraction of a shilling. By the common rule,  $\frac{3}{4 \times 4 \times 12} = \frac{3}{192}$ , which, by Case 1st, equals  $\frac{1}{64}$ , Ans. By the rule for canceling,  $\frac{3}{4. 4. 12}$ . The same canceled,  $\frac{3}{4. 4. 12} = \frac{1}{64}$ , Ans.

3. Reduce  $\frac{4}{5}$  of a penny to the fraction of a pound. Statement,  $\frac{4}{5. 12. 20}$ . Canceled,  $\frac{4}{5. 12. 20}$ ; and  $5 \times 3 \times 20 = 300$ , therefore, Ans.  $\frac{1}{300}$ .

4. Reduce  $\frac{6}{7}$  of a gallon to the fraction of a hogshead. Ans.  $\frac{2}{147}$ .

Statement,  $\frac{6}{7. 63}$ . The 63 below the line reduces the gallons to hogsheads.

5. Reduce  $\frac{3}{8}$  of an ounce Troy to the fraction of a pound. Ans.  $\frac{1}{16}$ .

6. Reduce  $\frac{16}{17}$  of a minute to the fraction of a day. Ans.  $\frac{1}{1530}$ .

7. Reduce  $\frac{8}{11}$  of a pound Avoirdupois to the fraction of a cwt. Ans.  $\frac{1}{154}$ .

8. Reduce  $\frac{1}{2}$  of a nail to the fraction of an ell English. *Ans.*  $\frac{1}{160}$ .
9. Reduce  $\frac{5}{12}$  of a penny to the fraction of a pound. *Ans.*  $\frac{1}{72}$ .
10. Reduce  $\frac{1}{3}$  of an hour to the fraction of a year. *Ans.*  $\frac{1}{9408}$ .

Secondly. To reduce fractions of high denominations to equivalent fractions of lower denominations.

**RULE.**—*Multiply the numerator by such numbers as are required to reduce the given quantity from the given to the required denomination, and then by Case 1st reduce the result to its lowest terms.*

**Ex. 1.** Reduce  $\frac{1}{96}$  of a shilling to the fraction of a farthing.  
To reduce shillings to farthings, we must multiply by 12 and 4; therefore,  $\frac{1}{96} \times 12 \times 4 = \frac{48}{96}$ ; and by Case 1st,  $\frac{48}{96} = \frac{1}{2}$ , *Ans.*

**RULE FOR CANCELING.**—*Place the numerator of the given fraction above a horizontal line, and the denominator below, as before; then place above the line such numbers as are necessary to reduce the denomination given to that required. Cancel, &c. as before.*

The above sum solved by this rule. Statement,  $\frac{1. \quad 12. \quad 4.}{96}$ .

The same canceled,  $\frac{1. \quad 12. \quad 4.}{\cancel{96} \quad \quad \quad}$ ; therefore,  $\frac{1}{2}$  of a farthing is the answer.

The scholar will carefully observe the difference between the statement here, and the one given for reducing low denominations to high.

2. Reduce  $\frac{1}{360}$  of a pound to the fraction of a penny. *Ans.*  $\frac{1}{3}$ .

Statement,  $\frac{1. \quad 20. \quad 12.}{360}$ .

3. Reduce  $\frac{1}{108}$  of a pound Troy to the fraction of a pwt.  
*Ans.*  $\frac{20}{9}$ , or  $2\frac{2}{9}$  pwt. Statement,  $\frac{1. \quad 12. \quad 20.}{108}$ .

4. Reduce  $\frac{1}{18}$  of a pound Troy to the fraction of an ounce.  
*Ans.*  $\frac{1}{3}$ .

5. Reduce  $\frac{1}{12}$  of a penny to the fraction of a farthing.  
*Ans.*  $\frac{1}{48}$ .
6. Reduce  $\frac{1}{560000}$  of a mile to the fraction of a barley corn.  
*Ans.*  $\frac{1}{280000}$ .
7. Reduce  $\frac{1}{112}$  of a cwt. to the fraction of a pound Avoirdupois.  
*Ans.*  $\frac{1}{112}$ .
8. Reduce  $\frac{1}{168}$  of an ell English to the fraction of a nail.  
*Ans.*  $\frac{1}{12}$ .
9. Reduce  $\frac{1}{31536}$  of a year to the fraction of an hour.  
*Ans.*  $\frac{1}{13}$ .

CASE 6th.—TO REDUCE FRACTIONS OF A HIGHER DENOMINATION TO THEIR VALUE IN WHOLE NUMBERS OF A LOWER DENOMINATION.

**RULE.**—Reduce the fraction to its next lower denomination by multiplying the numerator by the requisite number, and divide the product by the denominator; the quotient thus obtained will be a whole number of the lower denomination, and the remainder, if any, may be reduced and divided as before. This process may be continued till nothing remains, or till it is reduced to the lowest denomination.

Ex. 1. Reduce  $\frac{1}{3}$  of a pound sterling to its value in shillings, and pence.

OPERATION.

2 = number.

2 0

Div. by denom.  $3 \overline{) 4.0} = \text{shillings.}$

1 3 s. and 1 s. remains, which equals 12 d., and  $12 \text{ d.} \div 3 = 4 \text{ d.}$  Therefore, 13 s. 4 d. is the required number.

It is evident that  $\frac{1}{3}$  of a pound sterling is 20 times as many thirds of a shilling; viz.  $\frac{20}{3} = 13\frac{1}{3}$  shillings; and  $\frac{1}{3}$  of a shilling is 12 thirds of a penny, that is,  $\frac{12}{3} = 4 \text{ d.}$  Hence,  $\frac{1}{3}$  of a pound is 13 s. 4 d.

2. Reduce  $\frac{1}{60}$  of a pound sterling to its value in lower denominations.

Solution:  $\frac{1}{60}$  of a pound =  $\frac{20}{60}$  of a shilling, and  $\frac{20}{60}$  of a shilling =  $\frac{40}{60}$  of a penny = 4 d. *Ans.*

3. Reduce  $\frac{1}{4}$  of a pound Troy to its integral value. *Ans.* 9 oz.

4. Reduce  $\frac{1}{16}$  of a day to its integral value. *Ans.* 1 h. 20 m.
5. Reduce  $\frac{1}{16}$  of an hour to its value in whole numbers. *Ans.* 54 m.
6. Reduce  $\frac{1}{8}$  of a hogshead to its value in whole numbers. *Ans.* 2 qt. 1 pt. 1 gi.
7. Reduce  $\frac{1}{4}$  of a quart to its integral value. *Ans.* 1 pt. 1 gi.
8. Reduce  $\frac{1}{2}$  of an ell English to a whole number. *Ans.* 3 qr.
9. Reduce  $\frac{1}{4}$  of a yard to a whole number. *Ans.*  $3\frac{1}{2}$  qr.
10. Reduce  $\frac{1}{2}$  of an ell French to a whole number. *Ans.* 4 qr.

CASE 7th.—TO REDUCE THE LOWER DENOMINATIONS OF A COMPOUND NUMBER TO FRACTIONS OF A HIGHER DENOMINATION.

**RULE.**—Reduce the given quantity to the lowest denomination in that quantity, for a numerator of the fraction; and then reduce a unit of the higher denomination to the same denomination with the numerator for the denominator of the fraction. This fraction reduced to its lowest terms by Case 1st, will be the one required.

Ex. 1. Reduce 2 qr. 2 na. to the fraction of a yard.

Operation, 2 qr. 2 na. = 10 nails, the numerator; and 1 yd. = 4 qr., and 4 qr. = 16 nails, the denominator; therefore,  $\frac{10}{16}$  is the fraction, which equals  $\frac{5}{8}$ , *Ans.*

The operation may be much abbreviated by canceling; for which the following will be found a convenient rule.

**RULE FOR CANCELING.**—Reduce the given quantity to the lowest denomination mentioned, (if it consist of different denominations,) and place it above a horizontal line; and beneath the same line place the numbers required to reduce this denomination to the required denomination. Cancel, multiply, &c., and the terms of the required fraction will be obtained.

Ex. 2. Reduce 3 s. 4 d. to the fraction of a pound.

Operation, 3 s. 4 d. = 40 d., and pence are reduced to pounds by dividing by 12 and 20. Therefore,

$$\text{Statement, } \frac{40}{12 \cdot 20} \quad \text{Canceled, } \frac{\overset{2}{40}}{\underset{6}{12 \cdot 20}}$$

Nothing remains above the line, the numerator of the required fraction therefore is 1; and 6 remains below the line, and consequently is the denominator; therefore,  $\frac{1}{6}$  is the fraction required, and 3 s. 4 d. is  $\frac{1}{6}$  of a pound.

3. Reduce 2 rods and 30 rods to the fraction of an acre.

2 rods and 3 rods = 110 rods; and rods are reduced to acres, by dividing by 40 and by 4; the statement therefore is,  $\frac{110}{40 \cdot 4}$ ; and the same canceled is,  $\frac{110}{48 \cdot 4} = \frac{11}{16}$ , *Ans.*

4. Reduce 12 ounces to the fraction of a pound Avoirdupois. *Ans.*  $\frac{3}{4}$ .

5. Reduce 26 gal. 2 qt. to the fraction of a hogshead. *Ans.*  $\frac{58}{128}$ .

6. Reduce 3 fur. 20 rods to the fraction of a mile. *Ans.*  $\frac{7}{16}$ .

7. Reduce 3 qr. 2 na. to the fraction of an ell English. *Ans.*  $\frac{7}{10}$ .

8. Reduce 2 dr. 2 sc. to the fraction of an ounce. *Ans.*  $\frac{1}{4}$ .

9. Reduce 2 qr. 24 lb. to the fraction of a cwt. *Ans.*  $\frac{5}{7}$ .

10. Reduce 6 oz. 10 pwt. to the fraction of a pound Troy. *Ans.*  $\frac{13}{24}$ .

11. Reduce 6 qt. to the fraction of a bushel. *Ans.*  $\frac{3}{16}$ .

12. Reduce 12 h. 30 m. to the fraction of a day. *Ans.*  $\frac{25}{48}$ .

13. Reduce 4 d. 12 h. to the fraction of a week. *Ans.*  $\frac{9}{14}$ .

14. Reduce 15 deg. 30 m. to the fraction of a Sign. *Ans.*  $\frac{31}{60}$ .

**CASE 8th.**—TO REDUCE FRACTIONS HAVING DIFFERENT DENOMINATORS TO EQUIVALENT FRACTIONS HAVING A COMMON DENOMINATOR.

**RULE.**—Multiply all the denominators together for a new denominator, and each numerator into all the denominators except its own, for a new numerator to each fraction. The several numerators placed over the common denominator will give the required fractions.

**Note.**—The fractions should be reduced to their lowest terms before multiplying. If the scholar looks carefully into the nature of this rule, he will see that it is only multiplying numerators and denominators by the same numbers; and he has already learned, that this does not affect the value of the fraction.

**Ex. 1.** Reduce  $\frac{2}{3}$ ,  $\frac{1}{4}$ , and  $\frac{6}{11}$ , to a common denominator.

## PERFORMED.

$7 \times 5 \times 11 = 385$ , the common denominator.

$6 \times 5 \times 11 = 330$ , the numerator for  $\frac{6}{7}$ , which therefore equals  $\frac{330}{385}$ .

$4 \times 7 \times 11 = 308$ , the numerator for  $\frac{4}{7}$ , therefore,  $\frac{4}{7} = \frac{308}{385}$ .

$8 \times 5 \times 7 = 280$ , the numerator for  $\frac{8}{11}$ , therefore,  $\frac{8}{11} = \frac{280}{385}$ .

The required fractions, therefore, are  $\frac{330}{385}$ ,  $\frac{308}{385}$ ,  $\frac{280}{385}$ .

2. Reduce  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ , to a common denominator. *Ans.*  
 $\frac{60}{120}$ ,  $\frac{40}{120}$ ,  $\frac{30}{120}$ , and  $\frac{24}{120}$ .

3. Reduce  $\frac{6}{8}$ ,  $\frac{2}{3}$ ,  $\frac{1}{4}$ , and  $\frac{2}{5}$ , to a common denominator. *Ans.*  
 $\frac{168}{240}$ ,  $\frac{72}{240}$ ,  $\frac{64}{240}$ ,  $\frac{63}{240}$ .

4. Reduce  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ , and  $\frac{2}{5}$ , to a common denominator. *Ans.*  
 $\frac{2304}{4800}$ ,  $\frac{10560}{4800}$ ,  $\frac{2400}{4800}$ ,  $\frac{1620}{4800}$ .

5. Reduce  $\frac{3}{8}$ ,  $\frac{1}{4}$ ,  $\frac{2}{5}$ , and  $\frac{3}{10}$ , to a common denominator. *Ans.*  
 $\frac{810}{1620}$ ,  $\frac{1080}{1620}$ ,  $\frac{1440}{1620}$ ,  $\frac{486}{1620}$ .

6. Reduce  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ , and  $\frac{2}{11}$  to a common denominator. *Ans.*  
 $\frac{891}{1485}$ ,  $\frac{495}{1485}$ ,  $\frac{1155}{1485}$ ,  $\frac{1080}{1485}$ .

At the commencement of this rule, the scholar was instructed relative to the peculiar form and nature of fractions, and made acquainted with certain principles of universal application. In the course of the preceding eight cases, he has been shown the various changes of which fractions are susceptible, while their *value* remains unaffected. His attention will now be directed to those operations by which their *value is affected*.

## ADDITION OF FRACTIONS.

If the scholar will turn back to Simple Addition, he will there find it stated, that numbers, or quantities of the same kind only, can be reduced to a single number or quantity by adding. The same is true of fractions. It is obvious that  $\frac{1}{4}$  of a shilling, and  $\frac{1}{4}$  of a penny, make neither  $\frac{1}{2}$  of a shilling nor  $\frac{1}{2}$  of a penny. But  $\frac{1}{4}$  of a shilling makes  $\frac{1}{2}$  of a penny; and to this, we can add  $\frac{1}{4}$  of a penny, and the amount will be  $\frac{3}{4}$  of a penny. *Therefore, before we can add fractions, they must be reduced to the same denomination.* (See Case 5th.)

It is equally impossible to add fractions whose denominators are unlike.  $\frac{1}{2}$  of a shilling added to  $\frac{1}{4}$  of a shilling, makes neither  $\frac{2}{4}$  of a shilling nor  $\frac{2}{2}$  of a shilling. But  $\frac{1}{2}$  of a shilling  $= \frac{2}{4}$  of a shilling; and  $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$  of a shilling. Fractions must therefore be reduced to a common denominator, before they can be united. (See Case 8th.) Hence we have the following rule.

**RULE.**—Reduce all the fractions to the same denomination, and also to a common denominator; then add their numerators and place their sum over the common denominator. If the fractions produced be improper, reduce them to a whole number or mixed quantity.

**Note.**—If any of the fractions are compound, they must be reduced to simple ones, before they can be reduced to a common denominator. (See Case 4th.)

**Ex. 1.** What is the sum of  $\frac{1}{2}$  and  $\frac{3}{4}$ ? These fractions, reduced to a common denominator by Case 8th, become  $\frac{2}{4}$  and  $\frac{3}{4}$ , and  $\frac{2}{4} + \frac{3}{4} = \frac{5}{4}$  or  $1\frac{1}{4}$ , *Ans.*

2. What is the sum of  $\frac{2}{3}$  of  $\frac{1}{2}$ , and  $\frac{1}{3}$  of  $\frac{2}{3}$ ? By Case 4th,  $\frac{2}{3}$  of  $\frac{1}{2} = \frac{1}{3}$ , and  $\frac{1}{3}$  of  $\frac{2}{3} = \frac{2}{9}$ , and  $\frac{1}{3} + \frac{2}{9} = \frac{3}{9} + \frac{2}{9} = \frac{5}{9}$ , (see Case 8th,) and  $\frac{5}{9} + \frac{4}{9} = \frac{9}{9}$  or  $1\frac{1}{9}$ .

3. What is the sum of  $\frac{1}{2}$  and  $\frac{1}{4}$ ? *Ans.*  $\frac{3}{4}$  or  $1\frac{1}{4}$ .

4. What is the amount of  $\frac{2}{3}$  of  $\frac{1}{2}$  and  $\frac{1}{3}$  of  $\frac{1}{2}$ ? *Ans.*  $\frac{2}{3}$ .

5. What is the amount of 18 and 16, and  $\frac{1}{2}$  of  $\frac{1}{2}$ ? *Ans.*  $34\frac{1}{4}$ .

**Note 2d.**—When whole numbers are combined in the same operation with fractions, add each separately and unite them, as in the above sum.

6. What is the amount of 21, 7,  $\frac{7}{8}$ , and  $\frac{1}{2}$  of  $\frac{1}{2}$ ? *Ans.*  $29\frac{1}{4}$ .

7. What is the amount of  $\frac{1}{2}$  of a penny added to  $\frac{1}{3}$  of a £.?

**Explanation.**— $\frac{1}{2}$  of a £.  $= \frac{2}{3}$  of a penny; and  $\frac{1}{3}$  of a penny  $+ \frac{2}{3}$  of a penny  $= \frac{4}{3}$  of a penny, and this equals 2 s. 3 d.  $1\frac{1}{2}$  qr. *Ans.*

8. What is the amount of  $\frac{3}{4}$  of a yard and  $\frac{1}{4}$  of a nail? *Ans.* 3 qr.  $0\frac{1}{4}$  na.

9. What is the sum of  $\frac{5}{8}$  of a pound added to  $\frac{1}{2}$  of a shilling? *Ans.*  $1\frac{1}{8}$  s.  $= 17$  s. 2 d.

10. What is the sum of  $\frac{1}{2}$  of  $\frac{1}{2}$  and  $\frac{1}{3}$  of  $\frac{1}{2}$ ? *Ans.*  $\frac{5}{6}$  or  $1\frac{1}{6}$ .

11. What is the sum of  $\frac{1}{2}$  of  $\frac{1}{2}$  of  $\frac{1}{2}$  of  $\frac{1}{2}$  of  $\frac{1}{2}$  of  $\frac{1}{2}$ , and  $\frac{1}{2}$ ? *Ans.*  $\frac{1}{2}$ .

12. What is the sum of  $\frac{5}{8}$  of a ton added to  $\frac{3}{4}$  of a cwt.?  
*Ans.*  $11\frac{3}{4}$  cwt.

13. What is the sum of  $\frac{4}{5}$  of a day added to  $\frac{1}{2}$  of an hour?  
*Ans.*  $19\frac{7}{10}$  hours.

14. What is the sum of  $\frac{1}{2}$  of a pound,  $\frac{2}{3}$  of a shilling, and  $\frac{1}{4}$  of a penny?  
*Ans.* 3 s. 2 d. 2 qr.

15. What is the sum of  $\frac{1}{2}$  of a week,  $\frac{2}{3}$  of a day, and  $\frac{1}{4}$  of an hour?  
*Ans.* 1 day, 22 hours, 15 m.

## SUBTRACTION OF FRACTIONS.

As we can subtract a quantity only from another of the same kind, it is obvious that the same preparations are necessary to perform operations in this rule as in the preceding; therefore,

*RULE.*—Prepare the fraction as in addition, then subtract the less numerator from the greater, and place the remainder over the common denominator.

It will be obvious that the difference of the numerators is the difference sought.

*Ex. 1.* From  $\frac{7}{8}$  take  $\frac{1}{4}$ . Operation,  $\frac{7}{8} - \frac{1}{4} = \frac{7}{8} - \frac{2}{8} = \frac{5}{8}$ ,  
*Ans.*

2. From  $\frac{8}{9}$  take  $\frac{2}{11}$ .  $\frac{8}{9} - \frac{2}{11} = \frac{188}{99} - \frac{72}{99} = \frac{116}{99}$ , *Ans.*

3. From  $\frac{6}{7}$  take  $\frac{4}{7}$ . *Ans.*  $\frac{2}{7}$ . In this last example it is evident, that, as the denominators are the same, the operation consists in subtracting the numerators only. The same is true of all similar examples, provided only that the fractions are of the same denomination.

4. From  $\frac{1}{2}$  take  $\frac{5}{12}$ . *Ans.*  $\frac{1}{6}$  or  $\frac{2}{12}$ .

5. From  $\frac{1}{3}$  take  $\frac{1}{6}$ . *Ans.*  $\frac{1}{6}$ .

6. From  $\frac{3}{4}$  take  $\frac{2}{6}$ . *Ans.*  $\frac{1}{4}$  or  $\frac{3}{12}$ .

7. From  $\frac{3}{4}$  take  $\frac{1}{2}$ . *Ans.*  $\frac{1}{4}$  or  $\frac{2}{8}$ .

8. From  $\frac{1}{2}$  of a pound take  $\frac{2}{3}$  of a shilling.  $\frac{1}{2}$  of a pound =  $20$  of a shilling; and  $20 = \frac{40}{2}$  of a shilling. Therefore,  $\frac{40}{2} - \frac{20}{3} = \frac{80}{3}$  of a shilling, and  $\frac{80}{3}$  of a shilling = 9 s. 2 d.

9. From  $\frac{3}{4}$  of a league take  $\frac{1}{2}$  of a mile. 1 league = 3



miles; therefore,  $\frac{3}{4}$  of a league =  $\frac{9}{4}$  of a mile; and  $\frac{1}{2}$  of a mile =  $\frac{2}{4}$  of the same; hence,  $\frac{9}{4} - \frac{2}{4} = \frac{7}{4}$  or  $1\frac{3}{4}$  of a mile, which is the distance required.

10. From  $\frac{1}{2}$  of a shilling take  $\frac{3}{4}$  of a penny. *Ans.*  $2\frac{1}{4}$  or  $5\frac{1}{2}$  pence.

11. From  $\frac{7}{8}$  of a day take  $\frac{3}{7}$  of an hour. *Ans.*  $14\frac{1}{4}$ , or  $20\frac{1}{2}$  hours.

12. From  $\frac{9}{10}$  of a pound Avoirdupois take  $\frac{3}{4}$  of an ounce. *Ans.*  $27\frac{3}{8}$ , or  $13\frac{1}{2}$  ounces.

13. From  $\frac{3}{4}$  take  $\frac{1}{2}$  of  $\frac{3}{4}$  of  $\frac{3}{4}$ . *Ans.*  $\frac{1}{2}$ .  $\frac{1}{2}$  of  $\frac{3}{4}$  of  $\frac{3}{4} = \frac{1}{4}$ , and  $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$  or  $\frac{1}{2}$ .

14. From  $1\frac{1}{2}$  of an ell English take  $\frac{1}{2}$  of a yard. *Ans.*  $2\frac{1}{2}$  or  $2\frac{1}{2}$  yards.

15. From  $9\frac{3}{4}$  take  $6\frac{1}{4}$ . *Ans.*  $3\frac{1}{4}$ .

16. From 19 yards take  $5\frac{1}{2}$  yards. *Ans.*  $13\frac{1}{2}$ .

17. From 7 Ells English take  $4\frac{1}{2}$  yards. *Ans.*  $1\frac{1}{2} = 4\frac{1}{2}$  yards, or  $3\frac{3}{4}$  Ells English.

18. From  $\frac{1}{2}$  of a pound sterling take  $\frac{3}{4}$  of a penny. *Ans.*  $2\frac{1}{2}$  of a penny, or  $2s. 1\frac{1}{2}d.$

19. From  $\frac{9}{10}$  of a rod take  $\frac{3}{4}$  of a foot. *Ans.*  $2\frac{1}{4}$  or  $10\frac{1}{4}$  feet.

20. From  $\frac{7}{8}$  of an ounce take  $\frac{3}{4}$  of a pwt. *Ans.*  $45\frac{3}{8}$  pwt. or  $16\frac{1}{4}$  pwt.

## MULTIPLICATION OF FRACTIONS.

A fraction may be multiplied by a whole number, either by multiplying the numerator or dividing the denominator by that number. This has been fully illustrated in Section 4th, of Fractions.

Ex. 1. Multiply  $\frac{1}{3}$  by 16. *Ans.*  $5\frac{1}{3}$ , or  $5\frac{1}{3}$ .

2. Multiply  $\frac{4}{5}$  by 8. *Ans.*  $6\frac{4}{5}$ , or  $6\frac{4}{5}$ .

3. Multiply  $\frac{5}{7}$  by 9. *Ans.*  $7\frac{4}{7}$ , or  $7\frac{4}{7}$ .

4. Multiply  $\frac{2}{3}$  by 3. *Ans.*  $2$ , or  $2$ .

5. Multiply  $2\frac{1}{3}$  by 7. *Ans.*  $16\frac{1}{3}$ , or  $16\frac{1}{3}$ .

6. Multiply  $3\frac{1}{4}$  by 8. *Ans.*  $28$ , or  $28$ .

7. Multiply  $3\frac{3}{8}$  by 12. *Ans.*  $42\frac{3}{4}$ , or  $42\frac{3}{4}$ .

8. Multiply  $2\frac{1}{4}$  by 3. *Ans.*  $7\frac{3}{4}$ , or  $7\frac{3}{4}$ .

A fraction is multiplied into a whole number equal to its denominator, by rejecting that denominator.

9. Multiply  $\frac{15}{21}$  by 21. By dividing  $\frac{15}{21}$  by 21, I take  $\frac{1}{21}$  part of 15; if, then, this  $\frac{1}{21}$  part be repeated 21 times, it is evident that the value of all the parts will equal 15.

10. Multiply  $\frac{72}{82}$  by 82. *Ans.* 72.

11. Multiply  $\frac{7}{9}$  by 9. *Ans.* 7.

12. Multiply  $\frac{41}{73}$  by 73. *Ans.* 41.

13. Multiply  $\frac{21}{43}$  by 43. *Ans.* 21.

#### TO MULTIPLY FRACTIONS BY FRACTIONS.

**RULE.**—Multiply the numerators of the given fractions together for the required numerator; and the denominators, for the required denominator; then by Case 1st reduce the terms as far as practicable.

**Notes.**—Mixed numbers are to be reduced to improper fractions before multiplying; or we may first multiply the integris and then the fractions, and add their products.

Ex. 1. Multiply  $\frac{3}{5}$  by  $\frac{7}{5}$ .  $\frac{3}{5} \times \frac{7}{5} = \frac{21}{25}$ , and  $\frac{21}{25} \div 3 = \frac{7}{25}$ , *Ans.*

2. Multiply  $\frac{3}{8}$  by  $\frac{7}{5}$ .  $\frac{3}{8} \times \frac{7}{5} = \frac{21}{40} = \frac{7}{10}$ , *Ans.*

3. Multiply  $7\frac{1}{2}$  by  $3\frac{1}{2}$ .  $7\frac{1}{2} = \frac{15}{2}$ , and  $3\frac{1}{2} = \frac{7}{2}$ , and  $\frac{15}{2} \times \frac{7}{2} = \frac{105}{4} = 26\frac{1}{4}$ , or 25, *Ans.*

**RULE FOR CANCELING.**—Place the numerators of the given fractions above a horizontal line, and their denominators below the same. Cancel, multiply, &c. as before.

One important advantage of the above rule will be found in the fact, that it always gives the answer in its lowest possible terms.

4. Multiply  $\frac{1}{6}$  by  $\frac{3}{5}$ .

$$\text{Statement, } \frac{1}{6} \cdot \frac{3}{5} \quad \text{Canceled, } \frac{1 \cdot 3}{6 \cdot 5} = \frac{1}{10}, \text{ Ans.}$$

5. Multiply  $\frac{3}{4}$  of  $\frac{1}{6}$  of  $\frac{2}{3}$  by  $\frac{1}{2}$  of  $\frac{3}{4}$  of  $\frac{11}{12}$ .

$$\text{Statement, } \frac{3 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 11}{4 \cdot 6 \cdot 3 \cdot 2 \cdot 9 \cdot 12} \quad \text{Canceled, } \frac{3 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 11}{4 \cdot 6 \cdot 3 \cdot 2 \cdot 9 \cdot 12};$$

therefore, 11 = numerator, and  $2 \times 6 \times 9 \times 12 = 1296$ , denominator, and  $\frac{11}{1296}$ , *Ans.*

6. Multiply  $\frac{9}{15}$  by  $\frac{3}{18}$ .

$$\text{Statement, } \frac{9}{15} \cdot \frac{3}{18} \quad \text{Ans. } \frac{1}{10}.$$

7. Multiply  $\frac{7}{8}$  by  $2\frac{1}{2}$ . *Ans.*  $\frac{7}{4}$ , or  $2\frac{3}{4}$ .  
 8. Multiply  $16\frac{1}{2}$  by  $12\frac{1}{4}$ . *Ans.*  $203\frac{1}{4}$ .  
 9. Multiply  $13\frac{1}{2}$  by  $9\frac{3}{4}$ . *Ans.*  $124\frac{7}{8}$ .  
 10. Multiply  $\frac{1}{2}$  of  $\frac{2}{3}$  of  $\frac{3}{4}$  of  $\frac{4}{5}$  of  $\frac{5}{6}$  of  $\frac{6}{7}$  of  $\frac{7}{8}$  by  $\frac{1}{2}$  of  $\frac{2}{3}$  of  $\frac{3}{4}$ . *Ans.*  $\frac{1}{8}$ .

In solving sums by canceling, like the above, the necessity of reducing compound fractions to simple ones is avoided.

11. Multiply  $\frac{1}{2}$  of 19 by  $\frac{3}{8}$ .

$$\text{Statement, } \frac{1}{2} \cdot \frac{19}{1} \cdot \frac{3}{8} \quad \text{Ans. } \frac{19}{4}, \text{ or } 4\frac{3}{4}.$$

If any whole numbers are given as parts of the dividend, it is only necessary to write them above the line as in the last example; or, if they are given as parts of the divisor, they only require to be placed below the line; the operation then proceeds as before.

12. Multiply  $\frac{3}{4}$  of 10 by  $\frac{2}{3}$ . *Ans.* 3.  
 13. Multiply 144 by  $\frac{1}{12}$ . *Ans.* 12.  
 14. Multiply 395 by  $\frac{1}{3}$  of  $\frac{2}{3}$ . *Ans.*  $14\frac{2}{3}$ , or  $52\frac{2}{3}$ .  
 15. Multiply  $\frac{2}{3}$  of 3 times  $\frac{9}{12}$  of 5 times  $\frac{1}{2}$  of  $\frac{1}{3}$  of  $\frac{1}{4}$ , by  $\frac{2}{3}$  of  $\frac{4}{5}$  of  $\frac{5}{6}$  of  $\frac{1}{12}$ . *Ans.*  $\frac{1}{8}$ .  
 16. What will  $2\frac{1}{2}$  tons of hay cost at  $16\frac{1}{2}$  dollars per ton? *Ans.*  $41\frac{1}{4}$  dollars.  
 17. What will  $2\frac{1}{2}$  barrels of sugar cost at  $18\frac{1}{2}$  dollars per barrel? *Ans.*  $42\frac{7}{2}$  dollars.  
 18. What will  $8\frac{3}{4}$  pounds of tea cost at  $1\frac{1}{4}$  dollars per pound? *Ans.*  $10\frac{1}{2}$  dollars.  
 19. What will  $4\frac{1}{2}$  cords of wood cost at  $3\frac{1}{2}$  dollars per cord? *Ans.*  $17\frac{3}{8}$  dollars.  
 20. What will 9 yards of cloth cost at  $\frac{7}{8}$  dollar per yard? *Ans.*  $7\frac{3}{8}$  dollars.  
 21. What will 34 gallons of wine cost at  $1\frac{3}{8}$  dollars per gallon? *Ans.*  $46\frac{3}{8}$  dollars.  
 22. What will  $12\frac{1}{2}$  barrels of sugar cost at  $15\frac{1}{4}$  dollars per barrel? *Ans.*  $190\frac{1}{8}$  dollars.  
 23. What will  $22\frac{1}{2}$  pounds of lard cost at  $\frac{1}{8}$  dollar per pound? *Ans.*  $2\frac{1}{4}$ .

## DIVISION OF FRACTIONS.

Division of Fractions is naturally divided into the three following kinds; viz. the division of a fraction by a whole number; the division of a whole number by a fraction; and the division of a fraction by a fraction.

Division of fractions by a whole number was fully illustrated in Sec. 5th of the remarks introductory to this rule. It is, therefore, necessary here merely to repeat, that a *fraction is divided by a whole number, either by dividing its numerator, or multiplying its denominator by that number.*

- Ex. 1. Divide  $\frac{3}{11}$  by 9. *Ans.*  $\frac{3}{99}$ .
2. Divide  $\frac{2}{5}$  by 7. *Ans.*  $\frac{2}{35}$ .
3. Divide  $\frac{4}{9}$  by 11. *Ans.*  $\frac{4}{99}$ .
4. Divide  $\frac{1}{12}$  by 3. *Ans.*  $\frac{1}{36}$ .
5. Divide  $\frac{8}{10}$  by 9. *Ans.*  $\frac{8}{90}$ , or  $\frac{4}{45}$ .
6. Divide  $\frac{1}{3}$  by 8. *Ans.*  $\frac{1}{24}$ .
7. Divide  $\frac{3}{12}$  by 6. *Ans.*  $\frac{3}{72}$ .
8. Divide  $\frac{1}{4}$  by 5. *Ans.*  $\frac{1}{20}$ .
9. Divide  $\frac{17}{20}$  by 12. *Ans.*  $\frac{17}{240}$ .
10. Divide  $\frac{12}{10}$  by 7. *Ans.*  $\frac{6}{35}$ .
11. Divide  $\frac{1}{2}$  by 40. *Ans.*  $\frac{1}{80}$ .
12. Divide  $\frac{1}{2}$  by 72. *Ans.*  $\frac{1}{144}$ .

It is obvious that the quotient arising from dividing a whole number by a fraction, must be as much larger than the number itself, as a unit or 1 is greater than the fraction; or, in other words, the given dividend must bear the same ratio to the required quotient, as the numerator of the fraction bears to its denominator. A unit or 1 is contained in 6, six times;  $\frac{1}{2}$  is contained in the same number, twelve times, and  $\frac{1}{3}$ , eighteen times; and  $\frac{1}{4}$ , half as many times as  $\frac{1}{2}$ , viz. nine times. The operation to obtain this last quotient is as follows,  $6 \times 3 = 18$ , the number of thirds in six; and  $18 \div 2 = 9$ .

For dividing a whole number by a fraction, we have then the following rule:

*Multiply the whole number by the denominator of the fraction, and divide the product by the numerator.*

Ex. 1. Divide 9 by  $\frac{3}{4}$ . Operation,  $9 \times 4 = 36$ , and  $36 \div 3 = 12$ , the quotient.

2. Divide 15 by  $\frac{5}{6}$ . Operation,  $15 \times 6 = 90$ , and  $90 \div 5 = 18$ , quotient.

Operations of a similar character may be performed by canceling.

**RULE FOR CANCELING.**—*Place the whole number above a horizontal line and invert the fractional divisor; that is, place the denominator above the line and the numerator below. Cancel, &c.*

3. Divide 21 by  $\frac{3}{7}$ . Statement,  $\frac{21. 7}{3}$ . Canceled,  $\frac{7}{21. 7}$ , and  $7 \times 7 = 49$ , the quotient required.

4. Divide 42 by  $\frac{4}{6}$ . Statement,  $\frac{42. 7}{6}$ . Ans. 49.

The following sums may be divided by either of the above rules.

5. Divide 18 by  $\frac{3}{4}$ . Ans. 42.
6. Divide 63 by  $\frac{7}{13}$ . Ans. 91.
7. Divide 63 by  $\frac{9}{11}$ . Ans. 77.
8. Divide 42 by  $\frac{8}{9}$ . Ans.  $47\frac{1}{2}$ .
9. Divide 66 by  $\frac{3}{4}$ . Ans. 99.
10. Divide 121 by  $\frac{7}{8}$ . Ans.  $138\frac{3}{4}$ .
11. Divide 101 by  $\frac{11}{12}$ . Ans.  $110\frac{11}{12}$ .
12. Divide 32 by  $\frac{4}{5}$ . Ans. 40.

That the scholar may be enabled to commence understandingly the division of fractions by fractions, he may turn back and review the 7th section of the introductory remarks of this rule. It is there said, that *a fraction is divided by another fraction by inverting the divisor and then multiplying them together as in multiplication.*

1. Divide  $\frac{3}{4}$  by  $\frac{3}{4}$ . Ans.  $\frac{3}{4}$  or  $1\frac{3}{4}$ .
2. Divide  $\frac{3}{4}$  by  $\frac{1}{2}$ . Ans.  $\frac{1}{2}$  or  $1\frac{1}{2}$ .

**RULE FOR CANCELING.**—*Proceed in all respects as in multiplication of fractions, in arranging the terms of the dividend,*

then invert the divisor; that is, place the numerators below and the denominators above the line. Proceed to cancel, &c.

Ex. 3. Divide  $\frac{15}{16}$  by  $\frac{5}{12}$ .

$$\text{Statement, } \frac{15}{16} \cdot \frac{12}{5} \quad \text{Canceled, } \frac{\overset{3}{15} \cdot \overset{3}{12}}{\underset{4}{16} \cdot \underset{1}{5}}$$

$3 \times 3 = 9$ ,  $\frac{9}{4}$  or  $2\frac{1}{4}$ , Ans.

*Note.*—When the divisor is a compound fraction, each fraction in the divisor must be inverted.

4. Divide  $\frac{3}{4}$  of  $\frac{4}{5}$  by  $\frac{1}{4}$  of  $\frac{5}{3}$ . Statement,  $\frac{3}{4} \cdot \frac{4}{5} \cdot \frac{4}{1} \cdot \frac{3}{5}$ . Canceled,  $\frac{3}{1} \cdot \frac{1}{5}$ ; therefore,  $\frac{3}{5}$  or  $\frac{3}{5}$  is the answer required.

5. Divide  $\frac{15}{17}$  of  $\frac{11}{12}$  by  $\frac{5}{3}$  of  $\frac{7}{6}$ . Statement,  $\frac{15}{17} \cdot \frac{11}{12} \cdot \frac{3}{5} \cdot \frac{6}{7}$ . Ans.  $\frac{1224}{1735}$ , or  $1\frac{704}{1735}$ .

6. Divide  $\frac{1}{4}$  of  $\frac{6}{14}$  of  $\frac{2}{3}$  by  $\frac{1}{4}$  of  $\frac{3}{7}$  of  $\frac{2}{10}$ . Ans.  $\frac{4}{5}$ , or  $1\frac{1}{5}$ .

7. Divide  $\frac{1}{3}$  by  $\frac{1}{6}$ . Ans. 2.

8. Divide  $\frac{8}{10}$  of  $\frac{3}{4}$  of  $\frac{14}{15}$  of  $\frac{18}{15}$  of  $\frac{28}{10}$  of  $\frac{1}{2}$  by  $\frac{1}{4}$  of  $\frac{7}{8}$  of  $\frac{2}{10}$  of  $\frac{1}{4}$ . Ans.  $\frac{504}{175} = 1\frac{29}{175}$ .

9. Divide  $4\frac{1}{2}$  by  $\frac{1}{4}$  of 6. Ans.  $1\frac{1}{8}$ .

10. Divide  $6\frac{1}{2}$  by  $\frac{1}{4}$  of 4. Ans.  $2\frac{1}{2}$ .

11. Divide  $\frac{2}{3}$  of  $\frac{3}{4}$  by  $\frac{1}{5}$  of  $6\frac{1}{2}$ . Ans.  $\frac{15}{252}$ .

12. Divide 6 by  $\frac{1}{3}$ . Ans. 18.

13. Divide  $3\frac{1}{2}$  by  $2\frac{1}{2}$ . Ans.  $1\frac{1}{5}$ .

14. Divide  $\frac{3}{4}$  by 12. Ans.  $\frac{1}{16}$ .

15. Divide  $\frac{1}{2}$  of  $\frac{6}{8}$  by  $\frac{3}{4}$  of  $\frac{1}{2}$ . Ans.  $1\frac{2}{5}$ .

16. Divide  $\frac{1}{4}$  of 72 by  $\frac{1}{8}$  of 56. Ans.  $7\frac{1}{2}$ .

17. Divide  $\frac{18}{10}$  by 6. Ans.  $\frac{3}{5}$ .

18. Divide  $\frac{3}{4}$  by 7. Ans.  $\frac{3}{28}$ .

19. Divide  $\frac{9}{10}$  by 3. Ans.  $\frac{3}{10}$ .

20. Divide  $\frac{1}{2}$  by 8. Ans.  $\frac{1}{16}$ .

21. Divide 8 by  $\frac{1}{2}$ . Ans. 16.

22. Divide 12 by  $\frac{1}{3}$ . Ans. 36.

23. Divide 41 by  $\frac{2}{3}$ . Ans.  $24\frac{3}{2}$ .

24. Divide 35 by  $\frac{1}{2}$ . Ans. 70.

25. Bought 8 lb. of coffee for  $2\frac{1}{2}$  of a dollar; what was the cost of one pound? Ans.  $\frac{3}{27}$  or  $\frac{1}{9}$  of a dollar.

26. Bought 9 pounds of sugar for  $1\frac{1}{2}$  of a dollar; what was the price of a pound? Ans.  $\frac{11}{108}$  of a dollar.

27. In  $8\frac{1}{2}$  weeks a family consumes  $84\frac{1}{2}$  pounds of butter; how much is that per week? *Ans.*  $10\frac{1}{6}$ .

**QUESTIONS.**—What is a fraction? If a unit be divided into two equal parts, what is each of these parts called? How is it written? If it be divided into three equal parts, what is each part called, and how is it written? Similar questions should be asked respecting other fractions. When more parts than one are to be expressed, how is it done? What do fractions therefore express? How are they represented? What is the number below the line called? What does the denominator show? What is the number above the line called? What does the numerator show? What are the two numbers called, when spoken of collectively? How many kinds of fractions are there? What are they? What is a proper fraction? Give an example. What is an improper fraction? Give an example. What is a simple fraction? Give an example. What is a compound fraction? Give an example. What is a mixed number? Give an example. What is a complex fraction? Give an example. What does the denominator show? If the denominator remain the same, how is the value of the fraction affected by increasing the numerator? How, by diminishing it? Give an illustration. What is therefore the value of a fraction? If the numerator of a fraction be less than the denominator, how is its value compared with a unit? If the numerator be equal to the denominator, how then does its value compare with a unit? And how, if the numerator be greater than the denominator? What is the only consideration which limits the value of a fraction? In what ratio is the value of a fraction increased? How then may fractions be multiplied? How is a fraction multiplied into a number equal to its denominator? In what form should the terms of the fraction always be preserved? What is necessary to increase or diminish the value of a fraction? How then may a fraction be multiplied by a whole number? In what two ways may fractions be multiplied by whole numbers? How may a fraction be divided by a whole number? How else may a fraction be divided by a whole number? In what two ways then may fractions be divided by whole numbers? In what case should the numerator always be divided? What operation is therefore performed on the value of a fraction, when the numerator is operated upon? And what, when the denominator is operated upon? How is a fraction multiplied by a fraction? How is a fraction divided by a fraction? If the numerator and denominator be both multiplied by the same number, how is the value of the fraction affected? What is the rule for reducing fractions to their lowest terms?

What is the rule for reducing a whole number or mixed quantity to an improper fraction? How is an improper fraction reduced to a whole or mixed number? In all cases of division, what disposition may be made of the remainder, when any occurs? How are compound fractions reduced to simple ones? What is the rule for canceling? What is note 1st? What is note 2d? What is note 3d? What is Case 5th? How are fractions of low denominations reduced to those of higher denominations? What is the rule for canceling?

How are fractions of high denominations reduced to those of a lower value? What is Case 6th? What is the rule for it? What is Case 7th? What is the rule for it? What is the rule for canceling?

What is Case 8th? What is the rule for it? What is the note? What must be done before fractions can be added? What else requires to be done? What is the rule for the addition of fractions? What note follows? What is note 2d? What preparations are necessary before fractions can be subtracted? What is the rule for subtracting fractions? How is a fraction multiplied by a whole number? How is a fraction multiplied into a quantity equal to its denominator? What is the rule for multiplying fractions by fractions? What is the rule for canceling? Into what three kinds is division of fractions naturally divided? How is a fraction divided by a whole number? What is the rule for dividing a whole number by a fraction? How are fractions divided by fractions? What is the rule for canceling? What note follows the rule?

## DECIMAL FRACTIONS.

In the preceding rule we have contemplated the unit as divided into *any number of equal parts*.

We are now to regard it as divided *first into ten equal parts, then each of these into ten other equal parts, or the whole unit into one hundred equal parts; and these parts again, each into ten other parts; or the whole into a thousand equal parts, &c.* The expressions obtained by these several divisions, therefore, *decrease in value in the constant ratio of ten, from the left to the right*, and are called decimals. Whole numbers, as was shown in Numeration, *increase in the same ratio from the right to the left*, and both commence their enumeration with the unit figure. The connection between them is therefore so intimate as to render them susceptible of being written together and subjected to the same operations. The only important consideration in writing them, in addition to what has already been explained, is to *distinguish the one from the other*. This is effected by the period, called in decimals, the point of separation, which is always placed between them. In the expression, 23.56, the 23 is the whole number, and the .56 the decimal.

It will be observed that decimals, although they express parts of units, do not, like vulgar fractions, require two terms to express them. The given decimal may however be regarded, as in truth it is, a numerator, with a denominator always under-



stood. What this denominator is, it shall be our next object to illustrate. The scholar may therefore, in the first place, carefully examine the following table of whole numbers and decimals. The decimals, it will be observed, are read or numerated from the left to the right.

|           |                        |                    |            |           |       |        |         |             |              |                      |                          |             |
|-----------|------------------------|--------------------|------------|-----------|-------|--------|---------|-------------|--------------|----------------------|--------------------------|-------------|
| Millions. | Hundreds of Thousands. | Tens of Thousands. | Thousands. | Hundreds. | Tens. | Units. | Tenths. | Hundredths. | Thousandths. | Tens of Thousandths. | Hundreds of Thousandths. | Millionths. |
| 8         | 7                      | 6                  | 5          | 4         | 3     | 2      | 3       | 4           | 5            | 6                    | 7                        | 8           |

By an examination of the preceding table, the scholar will see that the 3 on the right of the separatrix is so many tenths. In the preceding rule, this would be thus expressed,  $\frac{3}{10}$ , which, by section 8th of the introductory remarks of the same rule, equals  $\frac{30}{100}$ . He will also see, that the 4 on the same side of the separatrix, is so many hundredths, or  $\frac{4}{100}$ . Now it is obvious, that these two fractions united, would make  $\frac{34}{100}$ . The same process of reasoning will show, that the next figure, or 5, is so many thousandths; and since  $\frac{34}{100} = \frac{340}{1000}$ , if the 5 be added, the amount will be  $\frac{345}{1000}$ . From the preceding, we therefore learn, that if the decimal consist of one figure only, it is so many tenths; if it consist of two figures, it is so many hundredths, and if of three, it is so many thousandths; and from this we derive the following conclusion, viz. *that the denominator of a decimal fraction always consists of a figure 1, with as many cyphers annexed to it, as there are figures in the given decimal.*

The scholar may write a denominator to each of the following decimals, viz. .6; .356; .26; .7426; .98654; .71639.

From the preceding explanation of the nature of decimals, it is obvious, that cyphers added to the right of a decimal do not effect its value, while those placed on the left diminish its value in a ten-fold ratio. For .5 is the same in value as  $\frac{5}{10}$ , or  $\frac{1}{2}$ . Now if a cypher be added to the right of the .5, it becomes .50, which is of equal value with  $\frac{50}{100}$ , and this also equals  $\frac{1}{2}$ . (See Case 1st, Vulgar Fractions.) If, then, .5 and .50 are each equal to  $\frac{1}{2}$ , it is obvious, that the value of a decimal is not affected by cyphers placed on the

right of it. But if they be placed on the *left*, they *diminish the value of the decimal in a ten-fold ratio*. The decimals .5, .05, and .005, will serve as an illustration. From the explanation given above, the denominators of these several decimals are 10, 100, and 1000; or the decimals may be thus written:  $\frac{5}{10}$ ,  $\frac{5}{100}$ ,  $\frac{5}{1000}$ . But five hundredths equal only one tenth part of five tenths; and five thousandths, one tenth part of five hundredths. Hence, cyphers placed on the left of a decimal diminish its value as above specified.

The following numbers may now be expressed by figures, and then read.

1. Seventy-six and six tenths. *Ans.* 76.6.
2. One and three hundredths. *Ans.* 1.03.
3. Eighty and fifty-eight thousandths. *Ans.* 80.058.
4. One hundred and fifty-six, and thirty-nine thousandths. *Ans.* 156.039.
5. One hundred and one, and five thousandths. *Ans.* 101.005.

The scholar will observe, that if there is but one decimal figure, and that tenths, it requires the point only to be placed at the left of it, to express its true value; if it be hundredths, it requires a cypher to be placed at the left of it, and if it be thousandths, it requires two cyphers to be thus placed, with the decimal point on the left of the cyphers; and so on according to the denomination.

6. Write down nine, and three hundred thousandths. *Ans.* 9.00003.
7. Write down twelve and one millionth. *Ans.* 12.0000001.
8. Three hundred and seventy-five, and seven tens of thousandths. *Ans.* 375.0007.
9. Ninety-five hundredths.
10. Three hundred and sixteen thousandths.
11. Forty-five millionths.
12. Sixty-nine, and nine hundred and three thousandths.
13. Four hundred and fifty-six, and seventeen millionths.
14. Five hundred, and three tens of millionths.
15. One, and six hundred of millionths.
16. Eleven, and seven billionths.
17. Seven hundred and sixty-two billionths.
18. Four hundred and twenty-one, and nineteen thousandths.

19. Seven hundred and six, and one hundred and three millionths.

20. Twelve hundred and six trillionths.

The scholar is now requested to turn back to Federal Money, and compare the denominations there given, with those here brought to view. He will there find the dollar given as the unit money; the dime as the tenth part of the dollar; the cent as the tenth part of the dime, or the hundredth part of the dollar; and the mill as the tenth part of the cent, the hundredth part of the dime, and the thousandth part of the dollar. It is therefore obvious, that Federal Money and decimals are operated upon by the same general principles.

1. Reduce \$21, 8 dimes, and 6 mills, to mills. *Ans.* 21806.

2. Reduce 21806 mills to dollars, cents, and mills. *Ans.* \$21.806.

3. Reduce \$12, 3 dimes, 4 cents, and 9 mills, to mills. *Ans.* 12349.

4. Reduce 12349 mills to dollars, cents, and mills. *Ans.* \$12.349.

5. Reduce \$25 to cents. *Ans.* 2500.

6. Reduce \$9 to mills. *Ans.* 9000.

To reduce dollars to cents, we therefore add two cyphers, and to reduce them to mills, we add three.

7. Reduce 2567 cents to dollars. *Ans.* \$25.67, or \$25 and 67 cents.

8. Reduce 38679 mills to dollars, &c. *Ans.* \$38.679, or \$38, 67 cents, and 9 mills.

To reduce cents to dollars, we therefore cut off two figures, and to reduce mills to the same, we cut off three figures from the right of the given number.

9. Reduce \$2 to mills.

10. Reduce 99 cents to mills.

11. Reduce \$1.03 to mills.

12. Reduce 467 cents to dollars.

13. Reduce 12008 mills to dollars.

14. Reduce \$42 and 3 mills to mills.

15. Reduce 9000 mills to dollars.

## ADDITION OF DECIMAL FRACTIONS.

The scholar must here exercise a good degree of caution in writing the numbers to be added. He will recollect that, in adding vulgar fractions, it was necessary to reduce them all to the same name or denominator, before the numerators could be added; and that in Simple and Compound Addition, the same denominations only could be united. The same is true of Decimal Fractions. Hence, we have the following rule :

**RULE.**—Place the whole numbers as in Simple Addition, with units under units, and tens under tens, &c. Also place the decimals on the right of the whole numbers, with tenths under tenths, hundredths under hundredths, and thousandths under thousandths, &c.; then, beginning with the lowest denomination, add up and carry as in Simple Addition. Lastly, from the amount, point off as many decimals as are equal to the greatest number of decimals in any one of the given numbers.

**Ex. 1.** What is the amount of 3.56 ; 42.923 ; 125.6 ; 4.32 ; and 59.365.

$$\begin{array}{r}
 \text{ADDED.} \\
 3.56 \\
 42.923 \\
 125.6 \\
 4.32 \\
 59.365 \\
 \hline
 235.768
 \end{array}$$

The greatest number of decimals in either of the numbers given, is three ; therefore, three decimals are to be cut off from the sum. It will always be found correct to place the decimal point in the amount directly below those of the given numbers.

2. Add the following numbers, 325.63 ; 275.215 ; 1.02 ; 17.653 ; 136.1. *Amount*, 755.618.

3. What is the amount of 72.5 ; 32.071 ; 2.1574 ; 371.4 ; 2.75 ? *Ans.* 480.8794.

4. What is the amount of 225.75 ; 25.50 ; 8.255 ; 27.225 ?  
*Ans.* 286.73.

5. What is the amount of 35.175 ; 75.15 ; 13.31 ; 25.755 ?  
*Ans.* 149.39.

6. What is the amount of 304.39 ; 291.09 ; 136.99 ; 12.10 ?  
*Ans.* 744.57.

7. What is the amount of 365.541 ; 487.06 ; 94.67 ; 472.5 ; 439.089 ? *Ans.* 1858.860.

8. What is the amount of 2.151 ; 375.422 ; .675 ; .4567 ?  
*Ans.* 378.7047.

9. Add together the following decimals, viz. sixteen hundredths ; two hundred and thirty-five thousandths ; six tenths ; and one thousandth. *Ans.* .996.

10. What is the sum of one tenth ; two hundredths ; three thousandths ; four tens of thousandths ; five hundreds of thousandths, and six millionths ? *Ans.* .123456.

11. What is the amount of \$72.375 ; \$41.176 ; \$1.47 ; \$395.20 ; \$56.65 ; \$146.73, and \$0.16. *Ans.* \$713.761.

12. Bought a yoke of oxen for \$121.56 ; a horse for \$156.375 ; a cow for \$37.086 ; and a quantity of grain for \$95.739. What was the cost of the whole ? *Ans.* \$410.760.

## SUBTRACTION OF DECIMALS.

The scholar will need no explanation of the nature of this rule. His knowledge of Subtraction, and of the peculiarity of decimals, will enable him at once to make a correct application of the following rule:

**RULE.**—*Set down the less number under the greater, so that each figure in the lower number, or subtrahend, shall stand directly under one of its own name or denomination. Proceed to subtract as in simple numbers, and place the separatrix as in addition of decimals.*

**Ex. 1.** From 378.635 take 195.275.

OPERATION.

3 7 8 . 6 3 5

1 9 5 . 2 7 5

---

 1 8 3 . 3 6 0 remainder.

2. From 462.3 take 218.15. *Rem.* 244.15.
3. From 16.705 take 7.6845. *Rem.* 9.0205.
4. From 132.4 take 36.36. *Rem.* 96.04.
5. From 127.05 take 66.006. *Rem.* 61.044.
6. From 100.001 take 77.77. *Rem.* 22.231.
7. From five hundred and thirty-six, and fifteen hundredths, take two hundred and thirty-six, and eighteen hundredths. *Rem.* 299.97.
8. From six dollars take fifty-five cents. *Rem.* \$5.45.
9. From one dollar take one mill. *Rem.* .999 mills.
10. From 16 dollars take nineteen cents and one mill. *Rem.* \$15.809.

---

## MULTIPLICATION OF DECIMALS.

**RULE.**—Multiply as in whole numbers, and point off as many decimals in the product as there are decimals in both factors. Whenever the decimals in the product are not as many as those of the factors, the deficiency must be supplied by placing cyphers on the left of them.

Ex. 1. Multiply 25.16 by 3.45.

$$\begin{array}{r}
 25.16 \\
 3.45 \\
 \hline
 12580 \\
 10064 \\
 7548 \\
 \hline
 86.8020
 \end{array}$$

Four figures are cut off in the product as decimals, in ac-

cordance with the rule, there being four decimals in the two factors.

2. Multiply 175.2 by 45.72. *Ans.* 8010.144.
3. Multiply 15.75 by 1.05. *Ans.* 16.5375.
4. Multiply 37.99 by 25.77. *Ans.* 979.0023.
5. Multiply 100.00 by 0.01. *Ans.* 1.0000.
6. Multiply 3.45 by .16. *Ans.* .5520.
7. Multiply 25.238 by 12.17. *Ans.* 307.14646.
8. Multiply 27.56 by 12.22. *Ans.* 336.7832.
9. Multiply .01 by .01. *Ans.* .0001.
10. Multiply 7.02 by 5.27. *Ans.* 36.9954.
11. Multiply .001 by .001. *Ans.* .000001.
12. Multiply  $\frac{1}{2}$  25 cents by  $\frac{1}{2}$  25 cents. *Ans.* .0625, or 6 $\frac{1}{4}$  cts.

*Note.*—To multiply a decimal by 10, 100, 1000, &c., it is necessary only to remove the decimal point as many places to the right as there are cyphers in the multiplier.

13. Multiply 1.56 by 10. *Ans.* 15.6.
14. Multiply 36.541 by 100. *Ans.* 3654.1.
15. Multiply .42 by 100. *Ans.* 42.
16. Multiply 46.3789 by 1000. *Ans.* 46378.9.

## DIVISION OF DECIMALS.

We are now to reverse the preceding operation. In multiplying decimals, we were directed to point off as many decimal figures in the product as there were in both factors. In Division, the dividend corresponds to the product in Multiplication, and the divisor to one of the factors which produced that dividend, and we are required to obtain the other factor. Therefore the decimals of the quotient and divisor united must equal those of the dividend.

**RULE.**—Divide as in Simple Numbers, and point off from the right of the quotient as many decimals as are equal to

13\*

*the excess of decimals in the dividend, over those in the divisor.*

*Note 1st.*—If the decimal places in the divisor be more than those in the dividend, annex cyphers to make them equal.

*2d.* If, after dividing, there be a remainder, cyphers may be annexed to the remainder and the division continued. The cyphers thus added are decimals.

*3d.* If the decimals in the divisor and dividend are equal, and there is no remainder after dividing, the quotient will be a whole number.

*4th.* If the figures in the quotient do not equal the excess of decimal places in the dividend over those of the divisor, supply the defect by prefixing cyphers.

*5th.* To divide the decimal number by 10, 100, 1000, &c., it is necessary only to remove the point as many figures to the left, as there are cyphers in the divisor.

Ex. 1. Divide 34.317 by 21.75.

$$\begin{array}{r}
 \text{PERFORMED.} \\
 21.75 \overline{) 34.317(1.577 +} \\
 \underline{21.75} \phantom{00} \\
 12567 \\
 \underline{10875} \phantom{0} \\
 16920 \\
 \underline{15225} \phantom{0} \\
 16950 \\
 \underline{15225} \phantom{0} \\
 1725 \text{ remainder.}
 \end{array}$$

In the solution of this example, two cyphers have been added to the remainders of the dividend. By Note 2d, the whole number of decimals in the dividend is therefore five, and there are two in the divisor ; three should, therefore, be



cut off from the quotient. The plus sign in the quotient always implies a remainder.

2. Divide 30515.50 by 100. *Ans.* 305.1550.

For the solution of the preceding sum, see Note 5th.

3. Divide 483.125 by 386.5. *Ans.* 1.25.

4. Divide 198.15625 by 186.5. *Ans.* 1.0625.

5. Divide .56 by 1.12. *Ans.* .5.

6. Divide 99.99 by 33.3. *Ans.* 3.0027.+

7. Divide 1.00 by .12. *Ans.* 8.333.+

8. Divide 14325.16 by 1.33. *Ans.* 10770.721.+

9. Divide 36.5 by 10. *Ans.* 3.65.

10. Divide 36.5 by 100. *Ans.* .365.

11. Divide 981 by 1000. *Ans.* .981.

12. Divide 543.67 by 3.46. *Ans.* 157.13.+

## APPLICATION.

1. If 36.34 bushels of corn grow on an acre, how many acres will produce 674 bushels? *Ans.* 17.804 acres.

2. If 6 yards of cloth cost \$24.48, what was the price per yard? *Ans.* \$4.08.

3. Bought 56.87 yards of cloth at \$2.31 per yard; what was the whole cost? *Ans.* \$131.3697.

4. The first of three men possessed \$685.423; the second, \$746.03; and the third, \$10864.273. How much had they all? *Ans.* 12295.726.

5. What cost 9.6 yards of cloth at \$6.42 per yard? *Ans.* \$61.632.

6. If a man earn 2 dollars 2 mills per day, how much would he earn in 93.5 days? *Ans.* \$187.1870.

7. What cost .675 of a cord of wood at \$3 a cord? *Ans.* 2.025.

8. If a yard of cloth cost \$5.5625, how much will .25 of a yard cost? *Ans.* 1.3906.

## REDUCTION OF VULGAR AND DECIMAL FRACTIONS.

The value of a vulgar fraction is the quotient arising from dividing the numerator by the denominator. Therefore,

**CASE 1st.**—TO REDUCE A VULGAR FRACTION TO A DECIMAL.

**RULE.**—*Annex cyphers to the numerator, and divide it by the denominator.*

**Note.**—If the given fraction be proper, the quotient will always be a decimal, and will consist of figures equal in number to the cyphers annexed; or, if the number of figures be less, cyphers must be prefixed to complete the number.

**Ex. 1.** Reduce  $\frac{3}{4}$  to a decimal.

OPERATION.

$$\begin{array}{r} 4 \overline{) 3.00} \\ \underline{4} \phantom{00} \\ \phantom{4} 0 \phantom{0} \end{array}$$

.75 the decimal required.

**2.** Reduce  $\frac{1}{5}$  to a decimal.

OPERATION.

$$\begin{array}{r} 5 \overline{) 1.0} \\ \underline{5} \phantom{0} \\ \phantom{5} 0 \end{array}$$

.2 the decimal required.

**3.** Reduce  $\frac{1}{3}$  to a decimal. *Ans.* .333.+

**4.** Reduce  $\frac{1}{8}$  to a decimal. *Ans.* .125.

**5.** Reduce  $\frac{1}{16}$  to a decimal. *Ans.* .0625.

**6.** Reduce  $\frac{1}{10}$  to a decimal. *Ans.* .1.

**7.** Reduce  $\frac{1}{12}$  to a decimal. *Ans.* .083.+

**8.** Reduce  $\frac{9}{10}$  to a decimal. *Ans.* .9.

**9.** Reduce  $\frac{1}{3}$  to a decimal. *Ans.* .333.+

**CASE 2d.**—TO REDUCE A DECIMAL TO A VULGAR FRACTION.

**RULE.**—*Write down the given decimal as a numerator, and for a denominator, write 1 with as many cyphers annexed as there are figures in the numerator, and then reduce the fraction to its lowest terms. (See remarks introductory to Decimals.)*

1. Reduce the decimal .125 to a vulgar fraction. Performed,  $\frac{125}{1000}$ , (see Case 1st, Vulgar Fractions,)  $\frac{125}{1000} \div 5 = \frac{25}{200}$ , and again,  $\frac{25}{200} \div 25 = \frac{1}{8}$ , Ans.

2. Reduce .75 to a vulgar fraction. Performed,  $\frac{75}{100} \div 25 = \frac{3}{4}$ , Ans.

3. Reduce .9385 to a vulgar fraction. Ans.  $\frac{15}{16}$ .

4. Reduce .2 to a vulgar fraction. Ans.  $\frac{1}{5}$ .

5. Reduce .16 to a vulgar fraction. Ans.  $\frac{4}{25}$ .

6. Reduce .25 to a vulgar fraction. Ans.  $\frac{1}{4}$ .

7. Reduce .45 to a vulgar fraction. Ans.  $\frac{9}{20}$ .

8. Reduce .55 to a vulgar fraction. Ans.  $\frac{11}{20}$ .

9. Reduce .8 to a vulgar fraction. Ans.  $\frac{4}{5}$ .

10. Reduce .24 to a vulgar fraction. Ans.  $\frac{3}{12.5}$ .

11. Reduce .945 to a vulgar fraction. Ans.  $\frac{189}{200}$ .

12. Reduce .844 to a vulgar fraction. Ans.  $\frac{211}{250}$ .

### CASE 3d.—TO REDUCE LOWER DENOMINATIONS TO DECIMALS OF A HIGHER.

**RULE 1st.**—Write down the several denominations which are to be reduced to decimals of a higher denomination, one above another, with the lowest uppermost; then divide each denomination, commencing with the lowest, by that number which is required of each to make a unit of the next higher denomination, and at each division place the quotient as a decimal on the right of the next higher denomination. The number last obtained will be the required decimal.

**Note.**—It will be obvious that the division of the lowest denomination must be effected by adding cyphers to that denomination. Cyphers must also be added to each higher denomination to reduce them, unless the decimal figures previously obtained be sufficient.

The reason of the above rule is readily shown. Suppose it is required to reduce 7 pence to the fraction of a shilling. The fraction would be  $\frac{7}{12}$ , because the shilling is divided into 12 equal parts, and 7 of these parts are taken, and this vulgar fraction is reduced to a decimal by adding cyphers to its numerator and dividing by its denominator. (See Case 1st of Decimals.)

**Ex. 1.** Reduce 7 s. 6 d. to the decimal of a pound sterling. The statement would be,  $\frac{12}{20} \frac{6}{7}$ . That is, the 6 d. is to be divided by 12, and the 7 s. by 20. This must be effected by adding cyphers.

OPERATION.

$$\begin{array}{r|l} 12 & 6.0 \\ 20 & 7.5 \ 0 \ 0 \end{array}$$

. 3 7 5 = required decimal.

**2.** Reduce 9 d. 2 qr. to the decimal of a pound sterling.

OPERATION.

$$\begin{array}{r|l} 4 & 2.0 \\ 12 & 9.5 \ 0 \ 0 \ 0 \ 0 \ 0 \\ 20 & .7 \ 9 \ 1 \ 6 \ 6 \ 6 \end{array}$$

Ans. .0 3 9 5 8 3 3 +

**RULE 2d.**—Reduce the given quantity to its lowest denomination, and divide it by a unit of the denomination of the required fraction, reduced to the same denomination.

**Ex. 3.** Reduce 10 s. 9 d. 2 qr. to the decimal of a pound sterling. 10 s. 9 d. 2 qr. = 518 qr.; and by the rule, 518 qr. are to be divided by 1 £. reduced to qr. viz. by 960 qr. Therefore,  $\frac{518}{960}$  is the fractional answer, and  $518 \div 960 = .539583 +$ , Ans.

To understand the above operation the scholar should remember that 10 s. 9 d. 2 qr. or 518 qr. are to be divided into as many equal parts as there are farthings in 1 £. = 960 qr., and one of these parts =  $\frac{1}{960}$ , or the decimal .539583. +

**4.** Reduce 9 s. 8 d. to the decimal of a pound sterling. Ans. .4833. +

**5.** Reduce 3 qr. 16 lb. to the decimal of a cwt.? Ans. .8928571. +

**6.** Reduce 16 £. 12 s. 8 d. to a decimal expression. Ans. 16.633333. +

**7.** Reduce 3 qr. 2 na. to the decimal of a yard. Ans. .875.

**8.** Reduce 2 roods and 20 rods to the decimal of an acre. Ans. .625.

9. Reduce 3 furlongs 16 rods to the decimal of a mile.  
*Ans.* .425.

10. Reduce 12 hours, 15 minutes, and 30 seconds to the decimal of a day. *Ans.* .51076. +

11. Reduce 2 cwt. 3 qr. 24 lb. to the decimal of a ton. *Ans.* .14821428.

**CASE 4th.—TO FIND THE VALUE OF A DECIMAL IN INTEGERS OF LOWER DENOMINATIONS.**

**RULE.**—Multiply the decimal by the number required to reduce it to the next lower denomination, and from the right hand of the product cut off as many figures as there are in the given decimal. The figures on the left of the point will be integers of the denomination next below that given. Proceed in the same way through all the denominations, and the figures on the left of the several points will be the answer required.

This rule being directly the reverse of the preceding, needs no explanation.

**Ex. 1.** What is the value of the decimal .5638 of a pound sterling?

OPERATION.

.5 6 3 8

2 0

---

1 1.2 7 6 0 = the shillings and decimals of a shilling  
 .1 2 in .5638 of a pound sterling.

---

3.3 1 2 0 = the pence and decimals of a penny in  
 4 .2760 of a shilling.

---

1.2 4 8 0 = the farthings and decimals of a farthing in .3120 of a penny.

The value of the above decimal in shillings, pence, &c. is  
 11 s. 3 d. 1.2480 qr.

**Note.**—The integers on the left of the points are the numbers which compose the answer.

2. What is the value of .75 of a pound sterling? *Ans.* 15 s.

3. What is the value of .53854 of a pound sterling? *Ans.* 10 s. 9 d. 1 qr. nearly.

4. What is the value of .625 of an acre? *Ans.* 2 roods, 20 rods.
5. What is the value of .148712678 of a ton? *Ans.* 2 cwt. 3 qr. 25 lb. 1 oz. +
6. What is the value of .676 of a cwt.? *Ans.* 2 qr. 19 lb. 11 oz. +
7. How many furlongs, &c. in .425 of a mile? *Ans.* 3 fur. 16 rods.
8. How many quarters, &c. in .66 of a yard? *Ans.* 2 qr. 2 nails, 1.26 inches.
9. How many roods, &c. in .321 of an acre? *Ans.* 1 rood, 11 rods, 10 yd. 8 feet.
10. What is the value of .875 of a hogshead of wine? *Ans.* 55 gal. 1 pint.
11. What is the value of .875 of a yard? *Ans.* 3 qr. 2 nails.
12. What is the value of .9 of an acre? *Ans.* 3 roods, 24 rods.

**QUESTIONS.**—How have we contemplated the unit as divided in the preceding rule? How are we to regard it as divided in this rule? In what ratio do the several divisions in decimals decrease? What are they called? What relation is there between integers and decimals? What is the only important consideration which has not already been explained? How many terms are required to express decimals? How may that term be regarded? Repeat the table of whole numbers and decimals. Of what does the denominator of decimal fractions always consist? How do cyphers added to the right of a decimal, affect its value? Give the illustration. How do cyphers placed on the left of a decimal affect its value? Give the illustration. How do decimals and the denominations of federal money correspond? How do we reduce dollars to cents? How do we reduce them to mills? How do we reduce cents and mills to dollars? What is the rule for the addition of decimals? What is it for subtracting decimals? What is the rule for multiplying decimals? How does division stand related to multiplication? What is the rule for division of decimals? What is the rule for pointing off decimals in addition of decimals? What in subtraction? What in multiplication? And what in division? On what does the value of a fraction depend? What is the rule for reducing vulgar fractions to decimals? What is Case 2d? What is the rule for it? What is Case 3d? What is the rule? What note follows the rule? Explain the nature of the rule. What is the second rule? What is Case 4th? What is the rule for it? What note follows the rule?

## REDUCTION OF CURRENCIES.

The currency of the United States was originally pounds, shillings, and pence, the same as in England. This, however, was abolished by an act of Congress, in 1786, and Federal Money, consisting of dollars, cents, and mills, was adopted.

The following table gives the value of the dollar in the old currency of several states :

In the New England, Virginia, Kentucky, and Tennessee currency,  $\$1 = 6 \text{ s.}$  and  $1 \text{ £.} = 20 \text{ s.}$  ; therefore,  $1 \text{ £.} = \frac{20}{6}$  or  $\frac{10}{3}$  of  $\$1$  ; or  $\$1 = \frac{3}{10}$  of  $1 \text{ £.}$

In New York and North Carolina currency,  $\$1 = 8 \text{ s.}$  and  $1 \text{ £.} = 20 \text{ s.}$  ; therefore,  $1 \text{ £.} = \frac{20}{8} = \frac{5}{2}$  of  $\$1$ , or  $\$1 = \frac{2}{5}$  of  $1 \text{ £.}$

In New Jersey, Pennsylvania, Delaware, and Maryland currency,  $\$1 = 7 \text{ s. } 6 \text{ d.} = 90 \text{ d.}$   $1 \text{ £.} = 20 \text{ s.} = 240 \text{ d.}$  ; therefore,  $1 \text{ £.} = \frac{240}{90} = \frac{8}{3}$  of  $\$1$ . Or,  $\$1 = \frac{3}{8}$  of  $1 \text{ £.}$

In South Carolina and Georgia currency,  $\$1 = 4 \text{ s. } 8 \text{ d.} = 56 \text{ d.}$  and  $1 \text{ £.} = 20 \text{ s.} = 240 \text{ d.}$  ; therefore,  $1 \text{ £.} = \frac{240}{56} = \frac{15}{7}$  of  $\$1$ , or  $\$1 = \frac{7}{15}$  of  $1 \text{ £.}$

In Canada currency,  $\$1 = 5 \text{ s.}$  ; therefore,  $1 \text{ £.} = \frac{20}{5}$  of  $\$1$ , or  $\$1 = \frac{1}{4}$  of  $1 \text{ £.}$

That the scholar may understand why the dollar is composed of a different number of shillings and pence in the different states, it is necessary only to say, that these states (or colonies, as they were at first called) originally issued each their own money, in pounds, shillings, and pence, the value of which soon depreciated. This depreciation was greater in some states than in others ; and hence, when federal money was adopted, more of the old currency was required in some states than in others, to equal the dollar of the new currency. The value of this dollar was 4 s. 6 d. sterling money, or English currency ; while 6 s. New England currency, 8 s. New York, 7 s. 6 d. New Jersey, and 4 s. 8 d. Georgia currency, were required to equal the same value.

To understand changing these several currencies to dollars, cents, and mills, the scholar needs to examine carefully the

preceding table. He will there observe that in New England currency 1 £. =  $\frac{1}{3}$  of \$1. Therefore, if pounds of that currency be multiplied by 10, and the product divided by 3, they will be changed to federal money, or dollars, cents, and mills. A similar explanation is applicable to other currencies.

*Note.*—If the given money consist of pounds, shillings, and pence, reduce the shillings and pence to the decimal of a pound, (see Case 3d of the Reduction of Vulgar and Decimal Fractions,) and then proceed according to the following rule:

*RULE.*—Notice from the preceding table what fraction of one dollar makes 1 £. of the given currency, and multiply the given sum by that fraction; that is, multiply it by the numerator and divide the product by the denominator.

**Ex. 1.** Reduce 72 £. New England currency to federal money. In N. E. currency, the pound =  $\frac{1}{3}$  of one dollar; therefore,

$$\begin{array}{r} 72 \\ 10 \\ \hline 3 \overline{) 720} \\ \hline \$ 240 \text{ Ans.} \end{array}$$

**2.** Reduce 75 £. 6 s. 8 d. N. E. currency to federal money.

$$\begin{array}{r} 12 \overline{) 8.0} \\ 20 \overline{) 6.6666+} \\ \hline .3333 = \text{the decimal value of 6 s.} \\ \quad \quad \quad 8 \text{ d. ; therefore,} \\ 75.3333 \\ 10 \\ \hline 3 \overline{) 753.3330} \\ \hline \$ 251.111 \text{ Ans.} \end{array}$$

**3.** Reduce 15 £. 6 s. 6 d. New York currency to federal money.



$$\begin{array}{r|l} 12 & 6. \\ 20 & 6.5 \end{array}$$

.3 2 5 = the decimal value of  
6s. 6 d.; therefore,

$$\begin{array}{r} 1\ 5.3\ 2\ 5 \\ \phantom{1\ 5.}5 \\ \hline 2\ )\ 7\ 6.6\ 2\ 5 \\ \hline \$\ 3\ 8.3\ 1\ 2\ 5\ \text{Ans.} \end{array}$$

To abbreviate the given operation by canceling, the following rule may be adopted :

**RULE FOR CANCELING.**—Place the given sum above a horizontal line, and, noticing as before what fraction of a dollar makes 1 £. of the given currency, write its numerator above the line, and its denominator below; then cancel, multiply, and divide.

4. Reduce 48 £. 10 s. 6 d. New England currency to federal money.

$$\begin{array}{r|l} 12 & 6 \\ 20 & 1\ 0.5 \end{array}$$

.5 2 5 = decimal value of 10 s. 6 d.

Therefore,  $\frac{16.175}{48.525} \times 10$ , and  $16.175 \times 10 = \$161.75$ .

5. Reduce 240 £. New Jersey currency to federal money.

$$\text{Statement, } \frac{240. \ 8}{3}$$

80  
Canceled,  $\frac{240. \ 8}{3}$ , and  $80 \times 8 = \$640$ , Ans.

6. Reduce 243 £. New Jersey currency to federal money.  
Ans. \$648.

7. Reduce 150 £. South Carolina currency to federal money. Ans. \$642.857. +

8. Reduce 27 £. New England currency to federal money.  
Ans. \$90.

9. Reduce 304 £. 12 s. 9 d. New England currency to federal money. *Ans.* \$1015.458.
10. Reduce 80 £. New York currency to federal money. *Ans.* \$200.
11. Reduce 84 £. 10 s. 6 d. New Jersey currency to federal money. *Ans.* \$225.40.
12. Reduce 100 £. New Jersey currency to federal money. *Ans.* \$266.666.+
13. Reduce 16 £. Canada currency to federal money. *Ans.* \$64.
14. Reduce 96 £. Nova Scotia currency to federal money. *Ans.* \$384.
15. Reduce 112 £. Georgia currency to federal money. *Ans.* 480.
16. Reduce 365 £. 10 s. 6 d. New York currency to federal money. *Ans.* \$913.812.+
17. Reduce 29 £. 12 s. 8 d. Canada currency to federal money. *Ans.* \$118.533.+
18. Reduce 276 £. 10 s. 3 d. South Carolina currency to federal money. *Ans.* \$1185.053.+
19. Reduce 45 £. 12 s. New Jersey currency to federal money. *Ans.* \$121.60.
20. Reduce 42 £. 6 s. New England currency to federal money. *Ans.* \$141.

The scholar will now reverse the preceding, that is, he will change any sum of federal money to either of the preceding currencies. To do this he must adopt the opposite mode of operation. Therefore,

**RULE.**—*Notice from the preceding table what fraction of 1 £. of the required currency makes \$1; then multiply the given sum by the numerator of that fraction, and divide the product by the denominator.*

**Ex. 1.** Reduce \$161.75 to pounds, shillings, and pence, New England currency.

PERFORMED.

1 6 1.7 5  
3

---

4 8 5.2 5 ÷ 10 = 48.525 £.

Now to find the value of the decimal .525, (see Case 4th, Decimal Fractions,)  $.525 \times 20 = 10.500$  s. and  $.500 \times 12 = 6.000$  d.; therefore the required pounds, shillings, &c. are 48 £. 10 s. 6 d.

2. Reduce \$38.3125 to pounds, shillings, and pence, New York currency.

$$\begin{array}{r}
 38.3125 \quad \text{then,} \quad .325 \quad \text{and } 500 \\
 \underline{\phantom{38.3125}2} \quad \quad \quad \underline{\phantom{.325}20} \quad \quad \quad \underline{\phantom{500}12} \\
 5) \underline{76.6250} \quad \quad \quad \underline{6.500} \text{ shillings, } \underline{6.000} \text{ d.} \\
 15.325 \quad \text{Therefore, } 15 \text{ £. } 6 \text{ s. } 6 \text{ d. is the answer.}
 \end{array}$$

These sums may also be solved by canceling. Take the following rule :

**RULE FOR CANCELING.**—Place the given dollars, cents, and mills, above a horizontal line, and, noticing as before what fraction of 1 £. of the required currency makes \$1, place the numerator of the fraction above the line, and the denominator below. Cancel, multiply, and divide for the answer.

3. Reduce \$95.75 to pounds, shillings, &c. in New England currency. *Ans.* 28 £. 14 s. 6 d.

$$\text{Statement, } \frac{95.75. 3}{10}.$$

4. Reduce \$648 to pounds, New Jersey currency. *Ans.* 243 £.

$$\text{Statement, } \frac{648. 3}{8}.$$

5. Reduce \$642.857 to pounds, South Carolina currency. *Ans.* 150 £.

6. Reduce \$578 to pounds, &c. New York currency. *Ans.* 231 £. 4 s.

7. Reduce \$580 to pounds, &c. Canada currency. *Ans.* 145 £.

8. Reduce \$742.50 to pounds, &c. New England currency. *Ans.* 222 £. 15 s.

9. Reduce \$21.758 to pounds, &c. New England currency. *Ans.* 6 £. 10 s. 6 d. 2 qr. +

10. Reduce \$141 to pounds, &c. New England currency.  
*Ans.* 42 £. 6 s.

11. Reduce \$250 to pounds, &c. Canada currency. *Ans.*  
 62 £. 10 s.

12. Reduce \$121.60 to pounds, &c. New Jersey currency.  
*Ans.* 45 £. 12 s.

13. Reduce \$475.75 to pounds, &c. New York currency.  
*Ans.* 190 £. 6 s.

14. Reduce \$89.54 to South Carolina currency. *Ans.* 20 £.  
 17 s. 10 d.

15. Reduce \$75 to pounds, &c. New England currency.  
*Ans.* 22 £. 10 s.

16. Reduce \$384 to pounds, &c. Nova Scotia currency.  
*Ans.* 96 £.

**QUESTIONS.**—What was the original currency of the United States? When this was abolished, what currency was substituted? What are the denominations of federal money? What is the value of the dollar, New England currency? What fraction of a dollar does the pound of that currency equal? What fraction of a pound does the dollar equal? Similar questions should be asked relative to the currencies of the other states. The scholar may explain why it is that the dollar is composed of a different number of shillings and pence in the different states. What note precedes the rule? What is the rule for bringing pounds, &c. into dollars? How are the terms arranged for canceling? What is the rule for bringing dollars into pounds, &c.? What is the rule for canceling?

## SIMPLE INTEREST.

Interest is an allowance made for the use of money.

It is computed at a certain rate per cent.; that is, at a *certain number of dollars for the use of a hundred for one year.*

If the sum on interest be more or less than \$100, or the time during which it draws interest, more or less than one year, the *sums paid as interest* must be in proportion both to the time and the sum lent.

For illustration: if \$100 be borrowed for one year, and if the interest allowed be 6 per cent., the interest for one year

will be \$6. Now, if twice that sum, viz. \$200, be borrowed for the same time, or if the same sum be borrowed for twice that time, viz. for two years, the interest will in either case be twice that sum, viz. \$12. Again, if twice the sum be borrowed for twice the time, that is, if \$200 be borrowed for two years, the sum to be paid as interest will be increased four-fold; or it would be  $6 \times 4 = \$24$ .

The scholar will carefully notice the following particulars :

1st. The *principal* is the money lent, or the sum on which interest is paid.

2d. The *interest* is the money paid for the use of the principal. *Legal interest* is that established by law. In the New England states, it is fixed at 6 per cent.; in New York, at 7, and in Louisiana, at 8 per cent.

3d. The *amount* is the sum obtained by adding the interest to the principal.

4th. The *rate per cent.* is always a decimal of two places, when expressed by cents, and of three, when expressed by cents and mills.

CASE 1st.—TO CAST INTEREST ON ANY SUM FOR ONE OR MORE YEARS.

*RULE.*—Multiply the principal by the rate per cent. written as a decimal, the product will be the interest for one year; which, being repeated as many times as there are years given, will be the required interest.

*Note 1st.*—In all cases where no rate per cent. is mentioned, six per cent. is always implied.

Ex. 1. What is the interest of \$215 for one year, at 6 per cent.?

It is obvious that the required interest will be 6 times 215 cents, for it is 6 cents on each dollar. Therefore,

$$\begin{array}{r} 215 \\ .06 \\ \hline 1290 \text{ cents} = \$12.90, \text{ Ans.} \end{array}$$

On a note of \$215, which had been on interest two years, there would then be due \$227.90; that is, \$215 principal +

\$12.90 interest = \$227.90, the amount. Therefore, the amount is found by adding the principal and interest together.

2. What is the interest of \$47.86 for 3 years, at 6 per cent. per annum?

OPERATION.

$$\begin{array}{r} 47.86 \\ .06 \\ \hline \end{array}$$

$$2.8716 = \text{interest for one year.}$$

3

$$\$8.6148 = \text{interest for three years.}$$

To understand why 4 figures are cut off in this sum, the scholar should remember, that 1 cent is  $\frac{1}{100}$  of a dollar; and consequently, 6 cents are  $\frac{6}{100}$  of a dollar, which is the same as .06. (See introductory remarks to Decimal Fractions.)

3. What is the interest of \$72.72 for 4 years, at 6 per cent. ?  
Ans. \$17.45. +

4. What is the amount of \$456 for two years? Ans.  
\$510.72.

5. What is the interest of \$146.31 for five years, at 6 per cent. ? Ans. 43.893.

6. What is the interest of \$24.91 for 6 years, at 5 per cent. ?  
Ans. \$7.473.

7. What is the interest of \$222.46 for three years, at 3 per cent. ? Ans. \$26.695. +

8. What is the amount of \$42 for six years, at 3 per cent. ?  
Ans. \$49.56.

9. What is the amount of \$566.33 for one year, at 5 per cent. ? Ans. \$594.646. +

10. What is the amount of \$1567 for 9 years, at 2 per cent. ?  
Ans. \$1849.06.

#### CASE 2d.—WHEN THE TIME CONSISTS OF YEARS AND MONTHS.

Lawful interest in the New England states is 6 per cent. per annum; that is, it is 6 cents for 12 months, or  $\frac{1}{2}$  cent per month on a single dollar. Therefore,

**RULE.**—Reduce the given years to months, and add the given months; then with half this number of months as a multiplier, multiply the given sum. The product will be the interest for the whole time.

*Note 2d.*—It is obvious that half the whole number of months in the given time, is the same as the number of cents on a dollar for the whole time, when the interest is at 6 per cent.; for, from the above remark, the interest of one dollar for one month is evidently  $\frac{1}{2}$  per cent., therefore, the whole number of months divided by 2, must determine the number of cents on each dollar for the whole time. If, in taking half the number of months, there be an odd one, the interest for that odd month will be  $\frac{1}{2}$  cent, or 5 mills.

**Ex. 1.** What is the interest of \$220 for 2 years and 6 months, at 6 per cent.?

2 yr. 6 months = 30 months, and  $30 \div 2 = 15$ . The interest on each dollar for the whole time is, therefore, 15 cents. Consequently, \$33 is the interest of the given sum.

$$\begin{array}{r}
 220 \\
 .15 \\
 \hline
 1100 \\
 220 \\
 \hline
 \$33.00 \text{ Ans.}
 \end{array}$$

**2.** What is the interest of \$756.20 for 1 year and 3 months, at 6 per cent.?

1 yr. 3 mo. = 15 months; therefore,  $7\frac{1}{2}$  cents, or 7 cents, 5 mills, is the interest on each dollar for the whole time. Therefore,

$$\begin{array}{r}
 756.20 \\
 .075 \\
 \hline
 378100 \\
 529340 \\
 \hline
 5671500 \text{ Ans. } \$56.715.
 \end{array}$$

*Note 3d.*—If the interest be required at some other than 6 per cent., first cast it at 6 per cent. by the preceding rule. Then divide the interest so found by 6, and the quotient will be the interest of the given sum at one per cent. for the time specified; and this multiplied by the given per cent., will be the required interest.

3. What is the interest of \$656 at 8 per cent. per annum, for 3 years and 4 months ?

3 yr. and 4 mo. = 40 months, and  $40 \div 2 = 20$ , the per cent. for the whole time. Therefore,

$$\begin{array}{r}
 656 \\
 .20 \\
 \hline
 6) 131.20 = \text{interest at 6 per cent.} \\
 \hline
 21.8666 = \text{interest at 1 per cent.} \\
 8 \\
 \hline
 174.9328 = \text{interest at 8 per cent., viz.} \\
 \$174.932. +
 \end{array}$$

4. What is the interest of \$37.50 for 3 years and 6 months, at 6 per cent. ? *Ans.* \$7.875.

5. What is the interest of \$672 for 3 years and 8 months, at 6 per cent. ? *Ans.* 147.84.

6. What is the interest of \$372 for 2 years and 11 months, at 5 per cent. ? *Ans.* \$54.25.

7. What is the interest of \$215.34 for 4 years and 6 months, at  $3\frac{1}{2}$  per cent. ? *Ans.* 33.916. +

8. What is the interest of \$350 for 2 years and 6 months, at 6 per cent. ? *Ans.* \$52.50.

**CASE 3d.—WHEN THE GIVEN TIME CONSISTS OF YEARS, MONTHS, AND DAYS.**

It was shown in the preceding case, that the interest of \$1 for one month was  $\frac{1}{2}$  cent = 5 mills, whenever the rate is 6 per cent. Now since 30 days are always allowed for a month in computing interest, 5 mills also equals the interest for 30 days; and  $30 \div 5 = 6$ , the number of days required for one dollar at 6 per cent. per annum, to gain one mill. If, therefore, the number of days given, be divided by 6, the quotient will be the number of mills to which the interest of one dollar will amount during that time. Therefore,

**RULE.**—Find the interest of one dollar for the given time, by allowing half a cent for every month, and one mill for every six days; the sum thus obtained will be the per cent. for the whole time. Multiply the principal by this, and the product will be the interest.



*Note 4th.*—If the given days be less than 6, or if they be more, and in taking a sixth part of them, a number remain, (which in all cases will be less than 6,) the interest for those days will be a fraction of a mill; that is, it will be as many sixths of a mill as there are days.

*Note 5th.*—This rule always gives the interest at 6 per cent. per annum. If, therefore, any other per cent. be required, proceed as directed in Note 3d.

**Ex. 1.** What is the interest of \$150 for 1 year, 3 months, and 15 days, at 6 per cent. per annum?

*Solution:* 1 year and 3 months = 15 months; the interest, therefore, for that time is 15 half cents =  $7\frac{1}{2}$  cents, or 7 cents, 5 mills; and the interest for 15 days is  $15 \div 6 = 2\frac{1}{2}$  mills; therefore, 7 cents, 5 mills +  $2\frac{1}{2}$  mills = 7 cents,  $7\frac{1}{2}$  mills, which is the interest of one dollar at 6 per cent. for the whole time. Consequently,

$$\begin{array}{r}
 150 \\
 .077\frac{1}{2} \\
 \hline
 1050 \\
 1050 \\
 75 \\
 \hline
 \end{array}$$

\$11.625 = *Ans.* or interest required.

**2.** What is the interest of \$320 for 3 years, 4 months, and 18 days, at 6 per cent. per annum?

*Solution:* 3 yr. 4 mo. = 40 months. The interest for this time is .20 per cent, and for 18 days it is  $18 \div 6 = 3$  mills; therefore, 20 cents and 3 mills is the interest of one dollar for the whole time. Hence,

$$\begin{array}{r}
 320 \\
 .203 \\
 \hline
 960 \\
 6400 \\
 \hline
 \end{array}$$

\$64.960 = interest required, viz. \$64.96.

**3.** What is the interest of \$75 for 1 year, 6 months, and 12 days, at 6 per cent. per annum? *Ans.* \$6.90.

**4.** What is the interest of \$162 for 1 year, 8 months, and 15 days, at 6 per cent.? *Ans.* \$16.605.

5. What is the interest of \$350 for 2 years, 3 months, and 5 days, at 6 per cent.? *Ans.* \$47.54.+

6. What is the interest of \$275 for 3 years, 6 months, and 12 days, at 4 per cent.? *Ans.* \$38.866.+

7. What is the interest of \$560 for 9 months and 20 days, at 6 per cent.? *Ans.* \$27.066.+

8. What is the interest of \$420 for 2 years, 1 month, and 27 days, at 6 per cent.? *Ans.* \$54.39.

*Note 6th.*—If the interest of \$100 for one year be \$6, the interest of \$200 for the same time, is obviously \$12. Again, if the interest of \$100 for one year be \$6, for 2 years it must be twice as much, viz. \$12; and for 3 years, three times as much, viz. \$18. In casting interest, we are therefore to regard not only the principal, but also the time during which it is on interest. This may help the scholar to understand the following rule for casting interest by canceling.

**RULE FOR CANCELING**—AND FIRST, WHEN YEARS ONLY ARE GIVEN.—*Draw a horizontal line, and write the sum on which the interest is to be cast, above it. On the right of this, also above the line, place the given time in years, and the rate per cent. Then place \$100 below the line. Proceed to cancel, &c.; the sum obtained will be the required interest.*

**Ex. 1.** What is the interest of \$300 for 3 years at 6 per cent.?

$$\text{Statement, } \frac{300. \quad 3. \quad 6}{100}$$

It is obvious, that the interest of \$300 for one year, will be three times as much as the interest of \$100 for the same time; and that for 3 years, it will be three times as great as for 1 year. The above statement canceled:  $\frac{300. \quad 3. \quad 6}{100}$ ; and  $3 \times 3 \times 6 = \$54$ , *Ans.*

2. What is the interest of \$350 for 4 years, at 5 per cent. per annum?

$$\text{Statement, } \frac{350. \quad 4. \quad 5}{100}$$

$$\text{Canceled, } \frac{350. \quad 4. \quad 5}{100}, \text{ and } 35 \times 2 = \$70, \text{ Ans.}$$

3. What is the interest of \$500 for 5 years, at 9 per cent. per annum. *Ans.* \$225.

4. What is the interest of \$240 for 6 years, at 4 per cent. per annum? *Ans.* \$57.60.

AGAIN, WHEN THE GIVEN TIME CONSISTS OF YEARS AND MONTHS.

**RULE.**—*Having reduced the given time to months, arrange the terms as in the preceding rule, and in addition to the terms there introduced, place 12, the number of months in one year, below the line; cancel, &c.*

*Note 7th.*—If the interest of \$100 for one year be \$6, for two years it would be \$12, &c. (See preceding note.) Consequently, for one year and six months, it would be  $\frac{3}{2}$  of \$6 =  $\frac{9}{2}$ , and for two years and nine months,  $1\frac{3}{4}$  =  $\frac{7}{4}$  of \$6, &c. These fractional expressions are obtained by reducing the given time to months, for the numerator, and making 12, the number of months in a year, the denominator. Hence we see the reason of the above rule.

**Ex. 5.** What is the interest of \$360 for 2 years and 6 months, at 6 per cent. per annum? *Solution:* 2 years, 6 months = 30 months; therefore,  $\frac{360. 30. 6}{100. 12}$ . It will be observed that the whole time is  $\frac{3}{2}$  of a year.

Canceled,  $\frac{360. 30. 6}{100. 12}$ , and  $3 \times 3 \times 6 = \$54$ , *Ans.*

6. What is the interest of \$440 for 2 years and 4 months, at 6 per cent. per annum?

The time =  $\frac{2\frac{2}{3}}{12}$  of a year, therefore,  $\frac{440. 28. 6}{100. 12}$ . *Ans.* \$61.60.

7. What is the interest of \$150 for 1 year and 6 months, at 6 per cent. per annum?

Statement,  $\frac{150. 18. 6}{100. 12}$ , *Ans.* \$13.50.

8. What is the interest of \$25.32, for 9 months, at 4 per cent. per annum?

Statement,  $\frac{25.32. 9. 4}{100. 12}$ , *Ans.* \$0.759.

9. What is the interest of \$375.75 for 4 years and 9 months, at 7 per cent.? *Ans.* \$124.936.+

10. What is the interest for \$1500 for 3 years and 4 months, at 6 per cent.? *Ans.* \$300.

11. What is the interest of \$175 for 4 years and 2 months, at 8 per cent.? *Ans.* \$58.333.+

LASTLY, WHEN THE TIME CONSISTS OF YEARS, MONTHS, AND DAYS.

**RULE.**—Reduce the years and months to months, and the days to the fraction of a month; then placing this fraction on the right of the months, reduce the whole to an improper fraction. This expression will represent the whole time in months; therefore, having arranged the other terms as before, write its numerator above the horizontal line, and its denominator below, and proceed as before directed.

**Note 8th.**—As the fractional expression of the whole time obtained by the preceding rule represents that time in months, 12, the number of months in one year, must be placed below the line, as before directed, to reduce those months to years.

**Ex. 1.** What is the interest of \$150 for 1 year, 3 months, and 15 days, at 6 per cent.? 1 year and 3 months = 15 months; and 15 days =  $\frac{1}{2}$  of a month; therefore,  $15\frac{1}{2}$  months is the whole time given, and this reduced to an improper fraction, equals  $\frac{31}{2}$  of a month. Agreeably to the rule, we then have the following

Statement, viz.  $\frac{150. \ 31. \ 6}{100. \ 2. \ 12}$

Canceled,  $\frac{150. \ 31. \ 6}{100. \ 2. \ 12}$ ; then,  $31 \times 3 = 93$ , and  $2 \times 2 \times 2 = 8$ .

Therefore,  $93 \div 8 = \$11.625$ , *Ans.*

2. What is the interest of \$320 for 3 years, 4 months, and 18 days, at 6 per cent. per annum? 3 years and 4 months = 40 months, and 18 days =  $\frac{3}{4}$  of a month. The whole time is therefore  $40\frac{3}{4} = 29\frac{1}{2}$  of one month.

Therefore,  $\frac{320. \ 203. \ 6}{100. \ 5. \ 12}$  is the statement.

The same canceled, &c. is  $\frac{168}{100} \times \frac{203.6}{5.12}$ ;  $203 \times 8 = 1624$ ; and  $5 \times 5 = 25$ .

Hence,  $1624 \div 25 = \$64.96$ , *Ans.*

3. What is the interest of \$75 for 1 year, 6 months, and 10 days, at 6 per cent.? *Ans.* \$6.875.

The time =  $\frac{75}{100} \times \frac{55}{3} \times \frac{6}{12}$ .

*Note 9th.*—In interest, 30 days are always allowed for a month.

In solving the following sums, the scholar can apply either of the preceding rules.

4. What is the interest of \$4.20 for 2 years, 1 month, and 27 days, at 6 per cent.? *Ans.* \$54.39.

5. What is the interest of \$80 for 1 year, 5 months, and 12 days, at 6 per cent.? *Ans.* \$6.96.

6. What is the amount of \$275 for 3 years, 6 months, and 12 days, at 4 per cent.? \$313.866.+

7. What is the amount of \$560 for 9 months and 20 days, at 6 per cent.? *Ans.* \$587.066.+

8. What is the interest of \$50.11 for 1 year and 21 days, at 6 per cent.? *Ans.* \$3.18.+

9. What is the interest of \$340.50 for 3 months and 1 day, at 6 per cent.? *Ans.* \$5.16.+

10. What is the interest of \$90 for 1 year, 2 months, and 6 days, at 6 per cent.? *Ans.* \$6.39.

11. What is the interest of \$4119.20 for 1 year and 5 days, at 6 per cent.? *Ans.* \$250.584.+

12. What is the interest of \$23.08 for 3 years, 6 months, and 22 days, at 6 per cent.? *Ans.* \$4.93.+

13. What is the interest of \$439.50 for 1 year, 11 months, and 9 days, at 6 per cent.? *Ans.* \$51.20.

14. What is the amount of \$28 for 9 years, 8 months, and 3 days, at 6 per cent.? *Ans.* \$44.254.

15. What is the amount of \$42 for 5 years, 5 months, and 9 days, at 5 per cent.? *Ans.* \$53.427.

16. What is the amount of \$50.50 for 1 year and 3 months, at 3 per cent.? *Ans.* \$53.393.+

17. What is the amount of \$300 for 16 years and 8 months, at 6 per cent.? *Ans.* \$600.

18. What is the interest of \$375.75 for 4 years and 9 months, at 7 per cent.? *Ans.* 124.936.+

19. What is the interest of \$3.75 for 12 years, at 6 per cent. per annum? *Ans.* \$2.70.
20. What is the amount of \$75 for 6 years and 3 months, at 6 per cent.? *Ans.* \$103.125.
21. What is the interest of \$63 for 1 year and 3 months, at 6 per cent.? *Ans.* \$4.725.
22. What is the interest of \$156 for 2 years and 4 months, at 8 per cent.? *Ans.* \$29.12.
23. What is the amount of \$650 for 3 years and 2 months, at 6 per cent.? *Ans.* \$773.50.
24. What is the amount of \$128.30 for 1 year and 9 months, at 9 per cent.? *Ans.* \$148.507.
25. What is the interest of \$33.50 for 2 years and 6 months, at 5 per cent.? *Ans.* \$4.1875.
26. What is the interest of \$150 for 2 years, 7 months, and 7 days, at 6 per cent.? *Ans.* \$23.425.
27. What is the interest of \$730 for 5 years, 9 months, and 12 days, at 6 per cent.? *Ans.* \$253.31.
28. What is the interest of \$875.49 for 5 years, 8 months, and 20 days, at 6 per cent.? *Ans.* \$300.58.+
29. What is the amount of \$630 for 8 months, at 6 per cent.? *Ans.* \$655.20.
30. What is the amount of \$7342 for 1 year and 4 months, at 6 per cent.? *Ans.* \$7929.36.
31. What is the amount of \$750 for 9 months, at 7 per cent.? *Ans.* \$789.375.
32. What is the amount of \$375 for 5 months and 15 days, at 6 per cent.? *Ans.* \$385.31.
33. What is the amount of \$460.50 for 4 months, at 6 per cent.? *Ans.* \$469.71.
34. What is the amount of \$230.25, for 8 months, at 7 per cent.? *Ans.* \$240.995.
35. What is the amount of \$764.50 for 3 years and 10 months, at 6 per cent.? *Ans.* \$940.335.

**CASE 4th.—TO FIND THE INTEREST AT SIX PER CENT. FOR ANY NUMBER OF DAYS.**

In computing interest, the month is reckoned at 30 days, and the interest of one dollar for that time at 6 per cent., as has already been shown, is  $\frac{1}{2}$  cent or 5 mills; hence for twice that time, or 60 days, it would be just one cent for every dollar on interest. We therefore have the following rule:

**RULE.**—Consider the number of dollars given so many cents, and reduce these cents to dollars again by removing the point of separation two places to the left; the result will be the interest of the given sum for 60 days. Then, if the given days be more or less than 60, add to, or subtract from, the interest of 60 days, such parts of itself as the given days require.

**Ex. 1.** What is the interest of \$450.82 for 93 days, at 6 per cent. per annum?

**SOLUTION.**

4.5082 = interest for 60 days. (See the rule.)

2.2541 = interest for 30 days, being half the int. for 60 days.

22541 = interest for 3 days, being  $\frac{1}{10}$  of the int. for 30 days.

\$6.98771 = interest for 93 days.

**2.** What is the interest of \$4562 for 45 days, at 6 per cent. per annum?

**SOLUTION.**

45.62 = interest for 60 days.

22.81 = interest of 30 days, or one half the int. of 60 days.

11.405 = interest of 15 days, or one half the int. of 30 days.

\$34.215 = interest of  $30 + 15 = 45$  days.

In computing by this rule, 12 months of only 30 days each, are allowed for the year, equal to 360 days. It consequently gives  $\frac{1}{12}$  part more than 6 per cent. interest. On small sums, and for short intervals, however, the difference is trifling.

**3.** What is the interest of \$420.72 for 120 days, at 6 per cent. per annum? *Ans.* \$8.414. +

**4.** What is the interest of \$56.74 for 25 days, at 6 per cent. per annum? *Ans.* \$0.236. +

**5.** What is the interest of \$156.36 for 96 days, at 6 per cent. per annum? *Ans.* \$2.50. +

**6.** What is the interest of \$1000 for 29 days, at 6 per cent. per annum? *Ans.* \$4.833. +

**7.** What is the interest of \$204 for 40 days, at 6 per cent. per annum? *Ans.* \$1.36.

**8.** What is the interest of \$472 for 18 days, at 6 per cent. per annum? *Ans.* \$1.416.

**Banking.**—When a promissory note is presented at a banking institution, if properly endorsed or otherwise secured, it is received by the officers of the bank as security, and so much money in their own notes is given in return as is equal to the face of the note after the interest is deducted for 3 days more than the whole time till payment is promised. Hence, if the time specified be 60 days, the interest on the face of the note for 63 days is deducted, and the balance drawn from the bank; then at the expiration of the 63 days, the whole face of the note is due. The 3 days added to the specified time of payment, are called "*days of grace*."

The preceding rule is, therefore, a convenient one for banking institutions.

In solving the following sums, the scholar will allow "*3 days of grace*," that is, he will find *bank discount* for 3 days more than are specified in the sum.

Ex. 1. What is the bank discount on \$256 for 30 days, and grace?

SOLUTION.

2.56 = discount for 60 days.

1.28 = discount for 30 days.

.128 = discount for 3 days grace.

\$1.408 = required discount.

Therefore, \$256 — \$1.408 = \$254.592, the sum to be drawn from the bank.

2. How much money would be drawn from the bank on a note of \$650, payable in 90 days?

6.50 = discount for sixty days.

3.25 = discount for thirty days.

.325 = discount for three days.

10.075 = discount for ninety-three days.

Therefore, \$650 — \$10.075 = \$639.925, the money to be drawn.

3. What is the bank discount on \$1056.29 for 30 days, and grace? *Ans.* \$5.81.

4. What is the bank discount on \$756 for 90 days, and grace? *Ans.* \$11.718.



5. What is the bank discount on \$676.19 for 25 days, and grace? *Ans.* \$3.155.+

6. How much money may be drawn on a note of \$692, payable 70 days after date, if discounted at a bank? *Ans.* \$683.581.

7. How much money may be drawn on a note of \$1567.89, payable 120 days from date? *Ans.* \$1535.748.

8. What is the bank discount on \$542.78 for 90 days, and grace? *Ans.* \$8.413.+

9. What is the bank discount on \$195.77 for 60 days, and grace? *Ans.* \$2.055.+

10. How much money may be drawn on a note of \$726, payable 80 days from date? *Ans.* \$715.957.

**CASE 5th.**—WHEN THE MONEY ON WHICH THE INTEREST IS TO BE CAST, IS IN POUNDS, SHILLINGS, AND PENCE.

**RULE.**—Reduce the shillings and pence to the decimal of a pound, (see Case 3d, Decimal Fractions,) and cast the interest in the same manner as when the principal consists of dollars, cents, and mills. The decimal part of the answer may then be reduced to shillings and pence by Case 4th, Decimal Fractions.

**Ex. 1.** What is the interest on 42 £. 16 s. 6 d. for 1 year and 6 months, at 6 per cent. per annum?

**SOLUTION.**

$$\begin{array}{r|l} 12 & 6 \\ 20 & 16.5 \end{array}$$

.825 = the decimal value of 16 s. 6 d.

1 yr. 6 mo. = 18 months, and  $18 \div 2 = 9$ , the per cent. for the whole time. Therefore,

$$42.825 \text{ £.}$$

$$.09$$

3.85425 £. = interest in pounds and decimals.

To find the value of the decimal:

$$\begin{array}{r} .85425 \\ 20 \\ \hline 17.08500 \\ 12 \\ \hline 1.02000 \end{array}$$

The answer, therefore, is 3 £. 17 s. 1 d.

2. What is the interest of 55 £. 15 s. 6 d. for 2 years, 6 months, at 6 per cent.?

55 £. 15 s. 6 d. = 55.775 £. And 2 yr. 6 mo. = 30 months = 15 per cent. for the whole time. Therefore,

$$\begin{array}{r}
 55.775 \\
 .15 \\
 \hline
 278875 \\
 55775 \\
 \hline
 836625 \text{ £.}
 \end{array}$$

To find the value of the decimal:

$$\begin{array}{r}
 36625 \\
 20 \\
 \hline
 732500 = \text{shillings.} \\
 12 \\
 \hline
 390000 = \text{pence.} \\
 4 \\
 \hline
 360000 = \text{farthings.}
 \end{array}$$

Therefore, 8 £. 7 s. 3 d. 3 qr. *Ans.*

3. What is the interest of 21 £. 18 s. 4 d., for 3 years and 4 months, at 6 per cent.? *Ans.* 4 £. 7 s. 8 d.

4. What is the amount of 156 £. 9 s. 3 d. for 1 year and 9 months, at 6 per cent.? *Ans.* 172 £. 17 s. 9 d. 3 qr. +

5. What is the amount of 27 £. 2 s. 6 d. 3 qr. for 1 year and 10 months, at 6 per cent.? *Ans.* 30 £. 2 s. 2 d. 3 qr. +

6. What is the interest of 36 £. 15 s. for 2 years and 3 months, at 6 per cent.? *Ans.* 4 £. 19 s. 2 d. 2.8 qr.

7. What is the interest of 45 £. 10 s. for 8 months, at 6 per cent.? *Ans.* 1 £. 16 s. 4 d. 3.2 qr.

8. What is the interest of 9 £. 12 s. 6 d. for 2 years, 4 months, and 12 days, at 6 per cent.? *Ans.* 1 £. 7 s. 4 d. +

CASE 6th.—WHEN INTEREST IS REQUIRED ON NOTES OR BONDS ON WHICH PARTIAL PAYMENTS HAVE BEEN MADE.

*RULE.*—Cast the interest on the principal at the given rate per cent. up to the time of the first payment; then, if the payment exceed the interest, deduct the excess from the principal;

but if it be less, set both payment and interest aside, and cast the interest on the same principal to the time of the next payment, or to the time of some payment, which, when added to the preceding payments, will exceed the sum of interest then due, and deduct the sum of these payments from the amount of the principal. The remainder will form a new principal, with which proceed as before, till the time of settlement.

1. For value received, I promise to pay A. B. & Co., or order, fifteen hundred dollars on demand, with interest.

Jan. 1, 1825.

JOHN JAMES.

On this note are the following endorsements: Oct. 1, 1825, three hundred dollars. July 1, 1827, four hundred and fifty dollars. Sept. 1, 1828, six hundred and fifty dollars. What was due on settlement, July 1, 1830?

The principal on interest from Jan. 1, 1825, . . . . . \$1500.00  
Interest from Jan. 1, 1825, to Oct. 1, of the same year, 9 mo. 67.50

1567.50

The payment being more than the interest, deduct it; . . . 300.00

Remainder, forming a new principal, . . . . . 1267.50

The interest from Oct. 1, 1825, to July 1, 1827, 1 yr. and 9 mo. 133.87

1400.587

Deduct second payment, because it is more than the interest, 450.000

Remainder, forming a new principal, . . . . . 950.587

The interest from July 1, 1827, to Sept. 1, 1828, 1 yr. and 2 mo. 66.541

1017.128

Deduct the third payment, . . . . . 650.000

Remainder, forming a new principal, . . . . . 367.128

The interest from Sept. 1, 1828, to July 1, 1830, the time of settlement, . . . . . 40.394

Ans. \$407.512

#### SOLUTION BY CANCELING.

| yr.  | m. | d. |
|------|----|----|
| 1825 | 10 | 1  |
| 1825 | 1  | 1  |

9 0 time till first payment.

Therefore,  $\frac{1500.9.6}{100.12}$ . Canceled,  $\frac{1500.9.6}{100.12}$ ;

and  $15 \times 9 \div 2 = \$67.50$ , the interest till first payment; and  $1500 + 67.50 = \$1567.50$ , amount; and  $1567.50 - 300 = \$1267.50$ , the new principal.

| yr.  | mo. | d. |
|------|-----|----|
| 1827 | 7   | 1  |
| 1825 | 10  | 1  |

1 9 0 = time from 1st to 2d payment.

Statement,  $\frac{1267.50. 21. 6}{100. 12}$ . Canceled,  $\frac{1267.50. 21. 6}{100. 12}$ ;  
2

$1267.50 \times 21 \div 200 = 133.087$ , the interest till 2d payment; then,  $1267.50 + 133.087 = 1400.587$ , and  $1400.587 - 450 = 950.587$ , the new principal.

| yr.  | m. | d. |
|------|----|----|
| 1828 | 9  | 1  |
| 1827 | 7  | 1  |

1 2 0 = time from 2d to 3d payment.

Statement,  $\frac{950.587. 14. 6}{100. 12}$ . Canceled,  $\frac{950.587. 14. 6}{100. 12}$ ;  
7  
2

and  $950.587 \times 7 \div 100 = 66.541$ , interest till the 3d payment, and  $950.587 + 66.541 = 1017.128$ , and  $1017.128 - 650 = 367.128$ , the new principal.

| yr.  | m. | d. |
|------|----|----|
| 1830 | 7  | 1  |
| 1828 | 9  | 1  |

1 10 0 = time from 3d payment to settlement.

Statement,  $\frac{367.128. 22. 6}{100. 12}$ . Canceled,  $\frac{367.128. 22. 6}{100. 12}$ ;  
11  
2

and  $376.128 \times 11 \div 100 = 40.384$ , interest till time of settlement, and  $367.128 + 40.384 = \$407.512$ , answer, or sum due on settlement.

2. I have in my possession a note, dated April 15, 1833, for \$2150.25, on which are the following endorsements: Nov. 8, 1834, \$500.00; Sept. 1, 1835, \$723.64; January 1, 1837, \$378.295; and Oct. 29, 1837, \$850.00. What amount was due on this note, April 15, 1838? Ans. \$138.337.

3. On a note of \$767.95, given Dec. 25, 1827, and drawing interest after ninety days, were the following endorsements: Jan. 1, 1830, \$75.00; March 25, 1831, \$565.25. What was due Jan. 1, 1833? *Ans.* \$294.118.

BOSTON, Jan. 13, 1809.

4. On demand, I promise to pay J. Anderson, or order, one thousand five hundred eighty-five and  $\frac{23}{100}$  dollars, with interest, for value received.

Received May 5, 1812, \$863.12.

Received May 7, 1814, \$221.

Received July 21, 1815, \$1009.03.

Will the scholar determine whether the note is fully paid?

5. What was due on a note of \$2100, dated June 15, 1820, on settlement, June 15, 1830, the following sums being endorsed on the back of it, viz. June 30, 1824, \$750, and Sept. 30, 1828, \$1200, on interest at 6 per cent.? *Ans.* \$1249.527.

6. For value received of A. B., I promise to pay him, or order, seven hundred and fifty dollars, with interest at 6 per cent.?

Jan. 1, 1824.

M. S.

On the above were the following payments endorsed: April 1, 1826, one hundred and fifty dollars; July 1, 1829, four hundred and fifty dollars. What was due on settlement, Sept. 1, 1832? *Ans.* \$461.71. +

## COMPOUND INTEREST.

Compound Interest is that which is computed annually, and immediately added to the principal. The amount of each year, is made the principal for the succeeding year.

**RULE.**—Cast the interest at the given rate per cent. for the first year, by multiplying by that rate per cent., and make the amount the principal for the second year. Make the amount of the second year, principal for the third; and the amount of the third, the principal for the fourth, &c., through the whole number of years. From the amount thus obtained, subtract the principal; the remainder will be the Compound Interest.

Ex. 1. What is the compound interest of \$256 for 3 years, at 6 per cent.?

$$\begin{array}{r}
 256 \\
 .06 \\
 \hline
 15.36 \text{ interest.} \\
 256.00 \text{ principal added.} \\
 \hline
 271.36 \text{ principal for 2d year.} \\
 .06 \\
 \hline
 16.2816 = \text{interest of 2d year.} \\
 271.36 = \text{principal of 3d year added.} \\
 \hline
 287.6416 \text{ principal for 3d year.} \\
 .06 \\
 \hline
 17.258496 \text{ interest of 3d year.} \\
 287.6416 \text{ principal.} \\
 \hline
 304.900096 \text{ amount of 3d year.} \\
 256.00 = \text{the original principal subtracted.} \\
 \hline
 48.90 \text{ compound interest on } \$256, \text{ for 3 years.}
 \end{array}$$

2. What is the compound interest of \$450 for 3 years, at 6 per cent. ? *Ans.* \$85.957. +

3. What is the compound interest of \$50 for 3 years, at 5 per cent. ? *Ans.* \$7.881. +

4. What is the compound interest of \$400, at 6 per cent. for 4 years ? *Ans.* \$104.99. +

5. What will \$675 amount to at compound interest, in 3 years and 6 months, 6 at per cent. per annum ? *Ans.* \$828.053. +

6. What is the amount of \$40.20 at 6 per cent. compound interest, for 4 years ? *Ans.* \$50.75. +

7. What is the amount of \$63 at 6 per cent. compound interest, for 2 years ? *Ans.* \$70.786. +

8. What is the compound interest of \$127.85 for 3 years, at 6 per cent. ? *Ans.* 24.421. +

## COMMISSION.

Commission is an allowance made by merchants and others to an agent for buying and selling goods. This allowance is usually a certain per cent. on the amount of money received for the sales effected, or on that expended in making purchases. The only respect in which it differs from interest, is, that in computing commission no regard is paid to *time*; hence,

**RULE.**—*Multiply by the rate per cent.*

Ex. 1. An agent sold for his employer goods to the value of \$1800, for which he received 5 per cent.; what was the amount of his commission? *Ans.* \$90.

2. What is the amount of my commission for selling goods to the value of \$975, it being 8 per cent.? *Ans.* \$78.

3., My agent sends me word, that he has purchased goods on my account to the value of \$2768; what will his commission amount to at 6 per cent.? *Ans.* \$166.08.

4. The commission of \$1250 at 10 per cent. is required; what is it? *Ans.* \$125.

5. An agent sells 750 bales of cotton, at \$52 per bale, and is to receive  $2\frac{1}{2}$  per cent. commission. How much money will he receive, and how much will he pay over to his employer? *Ans.* He will receive \$975, and pay over \$38025.

6. What will my commission amount to at 3 per cent. in purchasing goods to the value of \$7846.90? *Ans.* \$235.407.

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## INSURANCE.

Insurance against the loss of buildings and goods by fire, and also of ships and their cargoes by storm, is obtained by paying a certain per cent. on the estimated value of the property insured.

The instrument which binds the contracting parties is called the *policy*, and the sum paid by the party insured to the insuring party, is called the *premium*.

**RULE.**—*Multiply the estimated value of the property insured, by the per cent.*

**Ex. 1.** What is the premium for the insurance of buildings and appurtenances, valued at \$3758.50, at  $\frac{1}{2}$  per cent.? *Ans.* \$18.79.+

2. What is the premium for insuring property, valued at \$3600, against loss by fire, at  $\frac{3}{4}$  per cent.? *Ans.* \$27.

3. What is the premium for insuring property valued at \$845, at  $\frac{1}{2}$  per cent.? *Ans.* \$1.69.

4. What is the premium for the insurance of a ship and cargo valued at \$20500, at  $\frac{1}{2}$  per cent.? *Ans.* \$68.333.+

5. What would be the premium for insuring a ship and cargo valued at \$18000, at  $\frac{3}{8}$  per cent.? *Ans.* \$67.50.

6. Insured my house and out buildings, valued at \$21560.38, at  $\frac{3}{4}$  per cent.; what was the amount of the premium? *Ans.* \$86.24.+

**QUESTIONS.**—What is interest? How is it computed? Suppose the sum on interest be more or less than \$100, or the time more or less than one year, how must the sum paid as interest compare? Give the illustration. What is the principal? What is interest? What is legal interest? What is the legal rate per cent. in New England? What in New York? What in Louisiana? What do you understand by the amount? The rate per cent. is a decimal of how many places does it consist when expressed by cents? And when expressed by mills? What is Case 1st? What is the rule for it? What is note first? What is Case 2d? What is the rule? Give the reason of the rule. What is note 2d? Note 3d? What is Case 3d? What is the rule? Give the explanation which precedes the rule. What is note 4th? What is note 5th? What is note 6th? What is the rule for canceling? What is the rule for canceling, when the time consists of years and months? What is Note 7th? What is the rule for canceling, when the time consists of years, months, and days? What is Note 8th? What is Note 9th? What is Case 4th? What is the rule? What explanation precedes the rule? What is said relative to banking institutions? What is Case 5th? What is the rule for it? What is Case 3d of Decimal Fractions? What is Case 4th of Decimal Fractions? What is Case 6th? What is the rule for it? Will the scholar now inform me why it is correct to multiply by one half of the even number of months in the given time, in casting interest at 6 per cent.? What is Compound Interest? What is the rule for it? What is Commission? In what respect does it differ from simple interest? What is the rule? How is insurance obtained? What is to be understood by the policy? What by the premium? What is the rule?



## RATIO.

Ratio is the relation which one quantity or number has to another quantity or number of the same kind.

The former of the two numbers, between which the ratio exists, is called the antecedent, and the latter, the consequent.

The direct ratio of any two numbers is obtained by dividing the consequent of any couplet by the antecedent, and the inverse ratio, by dividing the antecedent by the consequent.

Thus, the direct ratio of 2 to 4, is 2, because the antecedent 2, is contained in the consequent 4, two times; and the inverse ratio is  $\frac{2}{4}$ , because 4, the consequent, is contained in 2, the antecedent,  $\frac{2}{4} = \frac{1}{2}$  of a time. Both these expressions establish the same general fact, viz. that one of the given numbers is twice as large as the other.

From the above, it is obvious that ratio can exist *only* between quantities of the same kind. It would be absurd to inquire how many times 3 acres must be repeated to equal 12 tons.

Any simple ratio is expressed by two dots placed between the antecedent and the consequent; thus, 2 : 8, or 5 : 10.

The following four propositions require to be carefully studied.

1. *The ratio of any couplet is not affected either by multiplying or by dividing its antecedent and consequent by the same number*; for the ratio of 9 : 18 is 2; and the ratio of  $9:18 \times 2 = 18:36$ , is also 2. The same result is obtained by dividing these terms by any number whatever; thus,  $9:18 \div 3 = 3:6$ , and this equals 2.

2. *The ratio of any couplet is multiplied by any number, by multiplying the consequent, or by dividing the antecedent by that number*. The ratio of 12 to 36, is 3; if, however, the consequent be multiplied by 3, the ratio 3 will be multiplied by the same number; thus,  $12:36 \times 3 = 12:108 = 9$ . The same result is obtained by dividing the antecedent by the same number; thus, 12, the antecedent, divided by 3, equals 4, and the ratio of 4 to 36 is 9, the same as before.

3. *The ratio of any couplet is divided by any number, by dividing the consequent, or by multiplying the antecedent by that number*. Take the ratio 12 : 36, as before, and let it be divided by 2. If the consequent be divided, we obtain the ratio  $12:18 = 1\frac{1}{2}$ ; but if the antecedent be multiplied, we obtain the ratio  $24:36 = 1\frac{1}{2}$  also.

4. If two or more ratios be multiplied together, that is, if the antecedents be multiplied into the antecedents, and the consequents into the consequents, the resulting ratio is called COMPOUND RATIO, and is equal to the product of the simple ratios. The ratio compounded of the simple ratios, 2 : 4, 3 : 6, and 4 : 8, is the ratio,  $24 : 192 = 8$ ; but the ratio,  $2 : 4 = 2$ , also,  $3 : 6 = 2$ , and  $4 : 8 = 2$ ; and  $2 \times 2 \times 2 = 8$ .

Ex. 1. What is the ratio of 6 to 36? *Ans.* 6.

2. What is the inverse ratio of 7 to 12? *Ans.*  $\frac{7}{12}$ .

3. What is the ratio of 7 to 42?

4. What is the inverse ratio of 5 to 20?

5. What is the ratio of 1 £. sterling to 7 s. 6 d.? It is obvious that the terms here given must be reduced to the same denomination, in order to compare them, therefore, 1 £. = 240 d. and 7 s. 6 d. = 90 d. The direct ratio, therefore, is  $\frac{240}{90} = \frac{8}{3}$ .

6. What is the ratio of 16 lb. to 3 cwt.? *Ans.* 21.

7. Multiply the terms 3 : 7 by 9, and see how the given ratio is affected.

8. Divide the terms 9 : 15 by 3, and compare ratios. (See proposition 1st.)

9. Multiply the ratio 11 : 16 by 5. *Ans.* 11 : 80.

10. Multiply the ratio 9 : 11 by 3. *Ans.* 3 : 11. (See proposition 2d.)

11. Divide the ratio 3 : 12 by 6. *Ans.* 3 : 2 or  $\frac{1}{2}$ .

12. Divide the ratio 3 : 19 by 4. *Ans.* 12 : 19. (See proposition 3d.)

13. What is the ratio compounded of the ratios 2 : 3 ; 3 : 4, and 4 : 5? *Ans.* 24 : 60, or 2 : 5.

*Note.*—Whenever the antecedent of any couplet is the same as the consequent of any other couplet, the several ratios may be reduced to one, by rejecting such antecedents and consequents. Hence, the preceding simple ratios may be reduced to a compound one, by rejecting their similar terms. The ratio thus obtained is 2 : 5.

14. What is the ratio compounded of the simple ratios 3 : 5 ; 4 : 5 ; and 2 : 3? (See proposition 4th.) *Ans.* 24 : 75, or 8 : 25.

15. Multiply the ratio 5 : 9 by 4. *Ans.* 5 : 36.

16. Divide the same ratio by 3. *Ans.* 5 : 3.

17. Multiply the ratio 12 : 21 by 6. *Ans.* 2 : 21.

18. Divide the same by 7. *Ans.* 84 : 21, or 12 : 7.

19. Reduce the ratios 4 : 7 ; 3 : 1 ; and 6 : 2, to a compound ratio. *Ans.* 36 : 7.

## PROPORTION.

Equality of ratios constitutes proportion.

Each simple statement in proportion requires at least two equal ratios, or four terms, the first and second of which are of one kind, and constitute one of the ratios; and the third and fourth are of another kind, and constitute the other ratio.

If 6 men earn 12 dollars in a given time, 36 men will earn 72 dollars in the same time. In this proposition, the two ratios required to constitute a proportion, are, the ratio of 6 men to 36 men, and of 12 dollars to 72 dollars; and the proposition can be true only on the condition that these two ratios are equal; and that is actually the case, for  $36 \div 6 = 6$ , and  $72 \div 12 = 6$ . Therefore, 6 is the ratio of each couplet.

Proportion is usually expressed by dots, thus:  $6 : 36 :: 12 : 72$ , and thus read: 6 is to 36 as 12 is to 72, or 6 men is to 36 men as 12 dollars is to 72 dollars.

When any four numbers are proportionals, the first and fourth are called *extremes*, and the second and third, the *means*; and the product of the extremes must always equal the product of the means. This is true with regard to the statement made above, for  $6 \times 72 = 432$ , and  $12 \times 36 = 432$ .

Since, therefore, these products are always equal, if any three terms are given, two of which bear a given ratio to each other, a fourth term may be found, to which the third given term shall bear the same ratio; for the required term must be either one of the extremes or one of the means. If it be one of the extremes, the product of the means divided by the given extreme, will give the required extreme; and if it be one of the means, the product of the extremes divided by the given mean, will give the required mean.

Suppose, in the proposition stated above, it be required to find how many dollars 36 men would earn in a given time, if 6 men earn 12 dollars in the same time.

Let the three given numbers stand as before, thus: 6 men : 36 men :: 12 dollars : what number of dollars?

Now, since 6, the left hand term, multiplied by the required term, must produce the same number as 36 multiplied by 12, it follows that 36 multiplied by 12, and the product divided by 6, will give the required number. Thus,  $36 \times 12 = 432$ , and

$432 \div 6 = 72$ , the fourth term in the preceding proposition. Or, suppose it required to find what number bears the same ratio to 36 that 12 does to 72. The statement would be, what number : 36 :: 12 : 72. The other extreme is here required; hence,  $36 \times 12 = 432$ , and  $432 \div 72 = 6$ , the required extreme. Again, let one of the means be required, thus: 6 is to what number as 12 is to 72. The statement may stand thus: 6 : — :: 12 : 72. One of the means is here required, and the extremes are given; therefore,  $72 \times 6 = 432$ , and  $432 \div 12 = 36$ , the required mean. Lastly, let the other mean be required. We then have the statement as follows: 6 : 36 :: — : 72. The third term of the statement is here wanting, or the other mean. Therefore,  $72 \times 6 = 432$ , and  $432 \div 36 = 12$ , the required mean.

From the preceding remarks we learn that operations in simple proportion consist in having three of the terms of a proportion given and a fourth term required. For finding this fourth term, we have the following rule:

**RULE.**—*Notice which of the three terms given is of the same name or kind as the required term or answer, and give it the right hand place. Notice again whether the term required must be greater or less than this; and, if it is to be greater, place the greater of the two remaining terms next it, on the left, for the second term of the proportion, and the less number for the first; but if it is to be less, place the less of the two remaining numbers for the second term, and the greater for the first term; then multiply the second and third terms together for a dividend, and divide their product by the first: the quotient will be the fourth term, or answer, and of the same denomination as the third term.*

**Note 1st.**—If the third term consists of different denominations, it must be reduced to the lowest mentioned before stating, and the fourth term will be of the same denomination; but if either the first or second terms be of different denominations, they must both be reduced to the lowest mentioned, before stating.

**Ex. 1.** If 8 yards of cloth cost \$3.20, what will 96 yards cost?

Since 8 yards cost money, 96 yards will cost money; the required term must, therefore, be money; and must be a number to which \$3.20 will bear the same ratio that 8 yards

bears to 96 yards. \$3.20 is, therefore, to be made the right hand term.

The next inquiry is, which will cost most, 8 yards or 96 yards? The answer to this inquiry places (in accordance with the rule) the 96 in the second place, and the 8 in the first, thus : 8 : 96 :: 3.20 : what number dollars?

PERFORMED.

$$\begin{array}{r}
 8 : 96 :: 3.20 \\
 \phantom{8 : 96 :: } 96 \\
 \hline
 \phantom{8 : 96 :: } 1920 \\
 \phantom{8 : 96 :: } \overline{) 2880} \\
 \phantom{8 : 96 :: } 8) 307.20
 \end{array}$$

\$ 38.40 Ans.

Therefore, 8 yards : 96 yards :: \$3.20 : \$38.40.

2. If 9 men earn 72 dollars in a given time, how much will 24 men earn in the same time? Statement: 9 men : 24 men :: \$72 : Ans. \$192.00.

Sums of this description may be solved with peculiar ease by canceling.

**RULE FOR CANCELING.**—*Notice which of the given terms is of the same kind or name as the required answer, and place it above a horizontal line, towards the left. Notice again whether the required term must be greater or less than this; and, if greater, place the greater of the two remaining terms on the right of the preceding term, and also above the line, and the less of the two terms below the line; but if less, place the less of the remaining terms above the line, and the greater below it; then cancel, multiply, and divide as before directed.*

**Note 2d.**—In arranging the terms as directed for canceling, the number placed first above the line, is the third term of the proportion, and that standing on the same side, on the right of this, is the second, and the number standing below the line, the first.

3. If 12 horses consume 42 bushels of oats in a given time, how much will 20 horses consume in the same time?

Statement,  $\frac{42. 20}{12}$ .

The answer required is obviously the oats that 20 horses would consume. It is also evident that 20 horses would consume more than 12. Hence the reason of the above statement.

The same canceled :  $\frac{7. 10}{12. 2}$

and  $7 \times 10 = 70$ , the bushels required.

It will be recollected that the numbers above the horizontal line form a dividend, and those below the line, a divisor. Hence, 42, the number of bushels consumed by 12 horses, may be regarded as divided by 12. This division would give the quantity of oats which one horse would consume in the given time, and this quantity multiplied by 20 would give the quantity 20 horses would consume in the same time.

4. If 72 yards of cambric cost \$119.44, what will 9 yards cost?

If \$119.44 be divided by 72, the quotient will be the price of one yard, and this price multiplied by 9, will be the required answer.

Therefore,  $\frac{119.44. 9}{72}$ .

Canceled,  $\frac{119.44. 9}{8. 72}$ ; and  $119.44 \div 8 = 14.93$ , Ans.

5. If 10 shillings pay for 20 pounds of beef, how many pounds may be bought for 5 shillings? Ans. 10.

Statement,  $\frac{20. 5}{10}$ .

6. If 3 lb. of sugar cost 4 s. how much will 6 lb. cost? Ans. 8 s.

Statement,  $\frac{4. 6}{3}$ .

7. If 12 bushels of wheat are worth \$16, what is the value of 48 bushels? Ans. \$64.  $12 \times 16 = 192$

8. Sold 12 yards of cloth for \$72; what is the value of 5 yards, at the same rate? Ans. \$30.  $12 \times 72 = 864$

9. What is the value of 16 cords of wood, if 48 cords are worth \$120? Ans. \$40.  $48 \times 120 = 5760$

10. If 16 cords of wood are worth \$40, what is the value of 48 cords? Ans. \$120.  $16 \times 40 = 640$

11. What is the value of 4 gallons of wine, if 108 gallons of the same kind are worth \$324? *Ans.* \$12.

12. If 8 yards of cloth cost \$3.20, how many yards of the same kind may be bought for \$38.40? *Ans.* 96 yards.

13. Paid 75 cents for 7 lb. of sugar; how many pounds of the same kind may be bought for \$6? *Ans.* 56 pounds.

14. If 7 men can accomplish a piece of work in 12 days, how many men are required to accomplish the same in 3 days? *Ans.* 28 men.

15. If a ship sail 24 miles in 4 hours, in how many hours will she sail 150 miles, if she continue at the same rate? *Ans.* 25 hours.

16. If 17 men perform a piece of work in 25 days, in how many days would 5 men perform the same? *Ans.* 85 days.

17. A staff, 4 feet long, casts a shadow 6 feet; another staff, placed in the same situation, casts a shadow 58 feet; what is its length? *Ans.*  $38\frac{1}{2}$  feet.

18. A garrison has provision for 8 months, at the rate of 15 ounces per day; what must be each man's daily allowance, in order that the same provision may last them 11 months? *Ans.*  $10\frac{1}{9}$  ounces.

19. When a quarter of wheat affords 60 ten-penny loaves, how many eight-penny loaves may be made from the same? *Ans.* 75.

20. If \$10 worth of provision serve 7 men 4 days, how many days will the same provision serve 9 men? *Ans.*  $3\frac{1}{2}$  days.

21. If 12 gallons of wine cost \$30, what is the value of 56 gallons, at the same price per gallon? *Ans.* \$140.

22. If 15 pounds of sugar cost \$1.20, how many pounds may be bought for \$38? *Ans.* 475 pounds.

23. If a staff, 4 feet long, casts a shadow 7 feet long, what is the height of a steeple, whose shadow at the same time measures 198 feet? *Ans.*  $113\frac{1}{2}$  feet.

24. If a pole, 6 feet long, casts a shadow 10 feet on level ground, what would be the length of a shadow from a steeple 72 feet high, at the same time? *Ans.* 120 feet.

25. If 12 men build a house in 48 days, in how many days can 36 men do the same work? *Ans.* 16 days.

26. If 100 men can finish a piece of work in 12 days, how many men will be required to do the same in 4 days? *Ans.* 300 men.

27. How many men must be employed to complete in 15 days what 5 men can do in 24 days? *Ans.* 8 men.

28. If a man perform a journey in 3 days, when the days are 16 hours long, how many days of 12 hours each, will he require to perform the same, if he continue to travel at the same rate? *Ans.* 4 days.

29. If 48 men can build a wall in 24 days, how many men can do the same in 192 days? *Ans.* 6 men.

30. In how many days will 8 men finish a piece of work which 5 men can do in 24 days? *Ans.* 15 days.

31. In what time will \$600 gain \$50 interest, if \$80 gain it in 15 years? *Ans.* 2 years.

32. When \$100 principal will gain \$6 in 12 months, what principal will gain the same in 8 months? *Ans.* \$150.

33. How many yards of cloth 3 qr. wide, are equal to 30 yards, 5 qr. wide? *Ans.* 50 yards.

34. How many yards of paper  $1\frac{1}{4}$  yard wide, will be sufficient to hang a room 20 yards square, and 4 yards high? *Ans.* 256 yards.

35. If a board be 9 inches wide, how much in length will make a square foot? *Ans.* 16 inches.

36. What quantity of shalloon 3 qr. wide, will line  $7\frac{1}{2}$  yards of cloth,  $1\frac{1}{4}$  yards wide? *Ans.* 15 yards.

37. Lent a friend \$100 for 6 months; afterwards he lent me \$75; how long ought I to retain it to balance my favor, allowing to each the same rate per cent. of interest? *Ans.* 8 months.

38. In what time will \$858 gain as much as \$286 will gain in 12 months? *Ans.* 4 months.

39. If 375 cwt. may be carried 660 miles for a given sum, how many cwt. may be carried 60 miles for the same money? *Ans.* 4125.

40. If 10 s. worth of wine will suffice for 46 men, when a tun is worth \$240, for how many will the same 10 s. suffice, when a tun costs \$160? *Ans.* 69 men.

41. If  $5\frac{1}{2}$  yards of muslin that is  $1\frac{1}{4}$  yards wide, will make a dress, how many yards of lining will be required, that is but 3 qr. wide? *Ans.* 11 yards.

42. If 40 rods in length and 4 in breadth make an acre, what is the width of a piece of ground containing the same quantity, that is 24 rods in length? *Ans.* 6 rods, 3 yards, 2 feet.

43. An insurance company, consisting of 82 persons, sustains a loss, of which each man's share was \$12; what would their shares have been, had the company consisted of only 32 persons? *Ans.* \$30.75.



44. If a hogshead of wine which cost \$180, afford a handsome profit, when retailed at \$4 per gallon, how must another be retailed, which cost \$196, to gain the same per cent.?

*Ans.* \$4.355. +

45. A lot of ground was walled in by 16 men in 6 days; the same being demolished, is required to be rebuilt in 4 days; how many men must be employed? *Ans.* 24 men.

46. A person by traveling 12 hours per day, performs a journey of 800 miles in 32 days; how many days will he require to perform the same journey, if he travel 15 hours per day? *Ans.*  $25\frac{2}{3}$  days.

47. If 1800 cwt. may be carried 64 miles for a given sum, how far may 225 cwt. be carried for the same money? *Ans.* 512 miles.

48. If 50 gallons of water fall into a cistern of sufficient capacity to contain 230 gallons, in one hour, and by a pipe 35 gallons be drawn off in the same time, how long will it take to fill the cistern? *Ans.* 15 hours, 20 minutes.

49. If 150 lb. of soap cost \$15.60, what would 15 lb. cost? *Ans.* \$1.56.

50. How many yards of cloth 3 qr. wide are equal to 39 yards 5 qr. wide? *Ans.* 65.

51. A cistern has a pipe by which it may be emptied in 10 hours; how many pipes of the same capacity will empty it in 30 minutes? *Ans.* 20 pipes.

It has already been said, that if the given terms (see Note 1st) are of different denominations, they must be reduced before stating the sum. This labor is, for the most part, saved, whenever the question is solved by canceling. Take the following sums, in which it is required to find the value of a quantity in one denomination, the price of some other denomination being given.

**RULE.**—Write the quantity the price of which is required, above a horizontal line; then (if the price of a lower denomination be given) on the right of this, also above the line, place the numbers required to reduce the given quantity to that denomination, together with the price of the same denomination; then below the line, write such numbers as will reduce the given price to the required denomination. But if the price of a higher denomination be given, and that of a lower denomination be required, place the quantity the price of which is required, as

*before, and write the numbers necessary to reduce that quantity to the denomination of which the price is given, below the line; then, lastly, place the quantity the price of which is given below, and its price above the line. Solve the statement by canceling.*

*Note 3d.*—If either the given quantity or price be of different denominations, they may be reduced to the lowest given, before stating; or, if preferred, the lower denominations may be reduced to a decimal.

Ex. 1. How many pounds sterling will 3 cwt. of sugar cost, at 20 pence per pound?

The price of 3 cwt. is required, and that of one pound is given. The given price is also in pence, and pounds sterling are required. Hence,  $\frac{3. \ 4. \ 28. \ 20}{12. \ 20}$ , is the required statement. The numbers 4 and 28 above the line, are required to reduce the 3 cwt. to pounds, and these pounds multiplied by 20, will give the price of the whole quantity in pence. If, then, these pence be divided by 12 and 20, they will be reduced to pounds sterling.

$$\text{Statement solved, } \frac{3. \ 4. \ 28. \ 20}{12. \ 20}$$

Therefore, 28 £. is the required answer.

2. How many pounds sterling will 3 pipes of wine cost, at 10 s. a gallon?

$$\text{Statement, } \frac{3. \ 2. \ 63. \ 10}{20}$$

In this statement, the 3 and 63 above the line are the numbers required to reduce pipes to gallons; then, the gallons multiplied by 10, will give the cost in shillings; and, lastly, the shillings divided by 20, (the number below the line,) will be reduced to pounds.

$$\text{Statement canceled, } \frac{3. \ 2. \ 63. \ 10}{20}, 63 \times 3 = 189 \text{ £. Ans.}$$

3. How many dollars, New York currency, will 12 cwt. of sugar cost, at 10 d. a pound? *Ans.* \$140.

$$\text{Statement, } \frac{12. \ 4. \ 28. \ 10}{12. \ 8}. \quad (8 \text{ s.} = \$1, \text{ New York currency.})$$

4. How many dollars, New England currency, will 2 hogs-heads of wine cost, at 6 d. a pint?

Statement,  $\frac{2. 63. 4. 2. 6}{12. 6}$ . Ans. \$84. (\$6 s. = 1 N. E. currency.)

5. At 15 pence a pound, what will 1 cwt. of loaf sugar cost in dollars, New England currency? Ans. \$23.333. +

If it is preferred to solve sums of this kind without canceling, it may be done by the following rule :

**RULE.**—Reduce the given quantity to that denomination, the price of which is given, and multiply it by the price ; then divide by such numbers as are required to reduce the value obtained to the required denomination.

6. How many dollars, New York currency, will 6 cwt. of sugar cost at 10 d. a pound ?

SOLUTION.

6

4 = qr. in cwt.

24

28 = lbs. in qr.

192

48

672

10 = price of 1 lb.

12 ) 6720 = cost in pence.

8 ) 560 = cost in shillings.

70 = dollars, answer.

7

Canceled,  $\frac{8. 4. 28. 10}{12. 8}$ .  $10 \times 7 = 70$ , answer.

7. How many dollars will 53 ells English cost, at 8 s. New York currency, per yard? Ans. \$66.25.

8. If I purchase molasses at 1 s. 3 d. per quart, how much in pounds, shillings, and pence, will 12 hogsheads of the same kind cost? Ans. 189 £.

9. If I purchase 16 cwt. of steel for \$156, what will 1 qr. of a cwt. cost, at the same rate?

Statement,  $\frac{1. 156}{4. 16}$ . Canceled,  $\frac{1. 156}{4. 16}$ ;  $39 \div 16 = \$2.437. +$

10. What cost 9 cwt. of sugar at 10 pence per pound? *Ans.* 42 £.

11. What cost 12 cwt. of sugar at 9 pence per pound? *Ans.* 50 £. 8 s.

12. What cost 42 cwt. of sugar at 3 s. 8 d. per pound? *Ans.* 862 £. 8 s.

13. What would 480 yards cost, in federal money, at 2 pence, New York currency, per yard?

Statement,  $\frac{480. 2.}{12. 8}$ . *Ans.* \$10. +

For the solution of the following sums, see the table of Currencies, given in Reduction of Currencies.

14. What would 862 yards cost, in federal money, at 3 pence per yard, New England currency?

Statement,  $\frac{862. 3.}{12. 6}$ . *Ans.* \$35.916. +

15. What would 920 yards cost, in federal money, at 4½ pence per yard, New Jersey currency? *Ans.* \$46.

16. What would 988 pounds of rice cost, in federal money, at 6 d. per pound, South Carolina currency? *Ans.* \$105.857. +

17. What would 899 gallons of vinegar cost, in federal money, at 8 d. per gallon, New York currency? *Ans.* \$74.916. +

18. How much will 672 yards cost, in federal money, at 6 s. 6 d. New York currency, per yard? *Ans.* \$546.

19. What will 1000 yards of ribbon cost, in federal money, at 3 s. 4 d. New England currency, per yard? *Ans.* \$555.555. +

20. How many dollars will 123 yards of cloth cost, at 10 s. New Jersey currency, per yard? *Ans.* \$164.

21. What will 687 yards of cloth cost in federal money, at 5 s. South Carolina currency, per yard? *Ans.* \$736.071. +

22. How many dollars and cents will 127 gallons of wine cost, at 3 s. 4 d. New England currency, per gallon? *Ans.* \$70.555. +

23. How many dollars will pay for 14 cwt. 3 qr. of hay, at 15 s. 8 d. New York currency, per cwt.? *Ans.* \$28.885. +

24. How many dollars will pay for 12 cwt. 2 qr. of cheese, at 2 £. 15 s. New Jersey currency, per cwt.? *Ans.* \$91.666. +

25. What will 12 cwt. 7 lb. of brown sugar cost, at 6d. New Jersey currency, per pound? *Ans.* \$90.066.+

26. A. owes B. 1138 £. but can pay only 15 s. on a pound; what will B. receive? *Ans.* 853 £. 10 s.

27. C. has a journey of 75 leagues to perform. In what time will he complete it, if he travel 30 miles a day. *Ans.*  $7\frac{1}{2}$  days.

28. How long will it take to travel one fourth round the earth, at the rate of 36 miles a day; the whole circumference being 360 degrees, and one degree  $69\frac{1}{2}$  miles? *Ans.*  $173\frac{1}{2}$  days.

29. If 9 lb. of coffee cost 27 s. how many dollars, at 6 s. each, will 45 lb. cost? *Ans.* \$22.50.

30. If I buy 20 pieces of cloth, each 20 ells, for 12 s. 6 d. per ell, how many dollars, at 8 s. will pay for the same? *Ans.* \$625.

31. How many dollars will 7 casks of prunes cost, each weighing 4 cwt. 2 qr., at 2 £. 19 s. 8 d. New York currency, per cwt.? *Ans.* \$234.937.+

32. A vessel at sea discharges a cannon, the report of which reaches me in 1 minute, 30 seconds. How far distant is she, allowing sound to travel 1142 feet in a second? *Ans.* 19 miles, 3 furlongs, and  $29\frac{1}{11}$  rods.

33. How many yards of flannel, 1 yard wide, will line 125 yards of broadcloth, 1 ell English wide? *Ans.*  $156\frac{1}{2}$ .

34. How many yards of cloth may be bought for \$37.62, if  $\frac{3}{4}$  of a yard cost 66 cents? *Ans.* \$42 yd. 3 qr.

35. How many dollars will 28 yards of linen cost, at 5 s. 6 d. New England currency, per yard? *Ans.* \$25.666.+

36. Bought 32 yards of muslin, at 6 s. 8 d. New York currency, per yard. What was the cost, in federal money? *Ans.* \$26.666.+

37. How many gallons of wine may be purchased for \$45, at 6 s. New York currency, per gallon? *Ans.* 60 gallons.

**QUESTIONS.**—What is ratio? What is the former of the two numbers between which the ratio exists called? What is the latter? How is the direct ratio of any two numbers obtained? How is the inverse ratio obtained? Between what quantities only, does ratio exist? How is simple ratio expressed? How is the ratio of any couplet affected by multiplying or dividing both the antecedent and the consequent by the same number? In what two ways may the ratio of any two numbers be multiplied? In what two ways is the ratio of any couplet divided by any number? When two or more ratios are multiplied together, what is the resulting ratio called? What does it equal?

What is Note 1st? What constitutes proportion? How many equal ratios are required for a statement of proportion? What terms must be of the same kind? When any four terms are proportional, what are the first and fourth called? And what are the second and third called? How does the product of the extremes compare with the product of the means? When any of the four terms are wanting, explain how it may be found. In what do operations in Simple Proportion consist? What is the rule? What note follows the rule? What is the rule for canceling? What is Note 2d? What is the rule, when it is required to find the value of a quantity in one denomination, the price of some other denomination being given? What is Note 3d? What is the rule for solving sums of this kind without canceling?

## COMPOUND PROPORTION.

Simple Proportion consists of two equal ratios.

Compound Proportion is that in which the relation of one of the given quantities to a required quantity of the same name, is traced through two or more simple proportions.

The smallest number of terms of which a statement in compound proportion can consist, is five. Of these terms, one is always of the same name as the answer required; and the others are always two of a kind. The following sum will serve as an illustration:

If 3 men, in 4 days, spend \$5, how many dollars will 6 men spend in 12 days?

In the above sum, there are five terms given, viz. two of men, two of days, and one of dollars; and dollars are also required for the answer; so that when the sixth term is found, the sum may be resolved into three simple ratios, the third of which is a compound of the preceding two. These ratios are 3 men: 6 men, and 4 days: 12 days, and \$5: the required number of dollars. Now it is obvious, that the ratio of \$5 to the required number of dollars, is not the same as the ratio of 3 men to 6 men; for then no regard would be paid to the time, and the solution would be effected on the supposition that one man or a number of men would spend as much in one day, as in any given number of days. Nor is it the same as the ratio of 4

days to 12 days, for then the supposition would be, that 3 men would spend as much as 6, or any number of men. But it is a ratio compounded of the two ratios, viz. the ratios of 3 men : 6 men, and of 4 days : 12 days. The ratio of 3 : 6 = 2, and  $\$5 \times 2 = \$10$ . This would double the given quantity of money. Again, the ratio of 4 : 12, is 3; this would treble the sum last obtained, viz.  $\$10$ , and would give  $10 \times 3 = \$30$ , which is the answer to the above question. Now, it will be observed, that the  $\$5$  in the above operation was multiplied by the product of the two simple ratios; for  $3 : 6 = 2$ , and  $4 : 12 = 3$ ; therefore,  $2 \times 3 = 6$ , the product of the simple ratios, and  $\$5 \times 6 = \$30$ , *Ans.*

The same result would have been obtained, by multiplying the  $\$5$  by the consequents, or latter terms of each ratio, and dividing their product by the product of the antecedents, or former terms of the same ratios. Thus, 3 : 6 and 4 : 12, are the given ratios of which 6 and 12 are consequents; therefore,  $6 \times 12 \times 5 = 360$ , and  $3 \times 4 = 12$ , the product of the antecedents; hence,  $360 \div 12 = 30$  dollars, the same answer as before. Hence, we have the following rule:

**RULE.**—Write the number which is of the same kind as the answer required, for the third term. Of the remaining terms, take any two of the same name, and arrange them as directed in Simple Proportion; then any other two of a kind, and so on till the terms are all taken. Lastly, multiply the product of the second terms by the third term, and divide the last product by the product of the first terms, and the quotient will be the required term or answer.

**Ex. 1.** If 4 men build a wall ten feet long, 3 feet high, and 2 feet thick, in 6 days, in what time will 12 men build one 100 feet long, 4 feet high, and 3 feet thick? The question asked is, in what time will the work be done; therefore, by the rule, 6 days is the third term.

$$\left. \begin{array}{l} 12 \text{ men} : 4 \text{ men,} \\ 10 \text{ feet length} : 100 \text{ feet length,} \\ 3 \text{ feet high} : 4 \text{ feet high,} \\ 2 \text{ feet thick} : 3 \text{ feet thick,} \end{array} \right\} :: 6 \text{ days.}$$

It is obvious that 12 men would require less time to execute a given piece of work than 4 men, and also that a wall 100

feet long, 4 feet high, and 3 feet thick, would require more time than a wall 10 feet long, 3 feet high, and 2 feet thick.

The same solved :

$$\left. \begin{array}{l} 12:4 \\ 10:100 \\ 3:4 \\ 2:3 \end{array} \right\} :: 6; 4 \times 100 \times 4 \times 3 \times 6 = 28800; \text{ and } 12 \times 10 \times 3 \\ \times 2 = 720; \text{ and } 28800 \div 720 = 40 \text{ days, the} \\ \text{number required.}$$

The work of this sum may be much abbreviated by canceling.

$$\text{The statement would be, } \frac{6. 4. 100. 4. 3}{12. 10. 3. 2}.$$

$$\text{Canceled, } \frac{6. 4. 100. 4. 3}{12. 10. 3. 2}, 10 \times 4 = 40, \text{ Ans.}$$

For the preceding mode of operation, we have the following rule :

**RULE FOR CANCELING.**—Write the number, which is of the same name as the answer required, above a horizontal line, towards the left. Then take each two terms of the same name, and compare them with this number, and notice whether more or less be required, as in Simple Proportion : if more be required, place the greater of the two numbers above the line, and the less below ; but if less be required, place the less of the two above the line, and the greater below. Cancel, &c.

Ex. 2. If 4 men in 12 days mow 48 acres of grass, how many acres will 8 men mow in 16 days ?

$$\text{Common statement, } \frac{4:8}{12:16} \} :: 48.$$

$$\text{Statement for canceling, } \frac{48. 8. 16}{4. 12}.$$

$$\text{Canceled, } \frac{48. 8. 16}{4. 12}, 4 \times 2 \times 16 = 128, \text{ the number of acres} \\ \text{required.}$$

*Note 1st.*—If the sixth term, or answer, be placed below the horizontal line, the other term remaining as before, the product of the terms standing above the line, will just equal the product of those standing below, thus :

$$\begin{array}{r} 48 \times 8 \times 16 = 6144 \\ 128 \times 4 \times 12 = 6144 \end{array}$$



3. If 12 horses eat 20 bushels of oats in 16 days, how many bushels will 24 horses eat in 48 days?

Statement,  $\frac{20 \cdot 24 \cdot 48}{12 \cdot 16}$  Ans. 120 bushels.

4. If 18 men, in 32 days, consume 128 lbs. of bread, how many pounds will 12 men consume in 64 days, their daily allowance being the same?

Statement,  $\frac{128 \cdot 12 \cdot 64}{18 \cdot 32}$  Ans. 170 $\frac{1}{2}$ .

5. If 8 men receive 4 dollars for 3 days work, how many days must 20 men work, to earn 40 dollars, if they receive the same daily allowance? Ans. 12.

Statement,  $\frac{3 \cdot 8 \cdot 40}{20 \cdot 4}$

To understand this statement, it should be remembered, that 20 men would require less time for the accomplishment of an object, than 8 men; and also that 40 dollars would employ, for a given time, more men than 4 dollars.

6. If 4 men receive \$3.20 for 3 days work, how many men, if they receive the same per day, will earn \$12.80, in 16 days? Ans. 3 men.

Statement,  $\frac{4 \cdot 12.80 \cdot 3}{3.20 \cdot 16}$

7. How much ought 60 men to receive for 25 days work, if 12 men, under the same circumstances, receive 50 dollars for 4 days work? Ans. \$1562.50.

8. If 16 men cut 112 cords of wood in 7 days, how many cords will 24 men cut in 19 days? Ans. 456 cords.

9. If the transportation of 12 cwt. 150 miles, cost 75 shillings, for how many dollars, at 8 s. each, may 6 cwt. be transported 45 miles? Ans. \$1.406. +

10. If the freight of 9 hhds. of sugar, each weighing 12 cwt. 20 leagues, cost \$38.40, what will be the expense of transporting 50 casks of the same, each weighing 2 cwt. 2 qr., 100 leagues? Ans. \$222.22. +

11. If 100 dollars, in 1 year, gain 6 dollars interest, how much will 200 dollars gain in 26 weeks. Ans. 6 dollars.

12. If 350 dollars, in 9 months, gain 15 dollars; what principal will gain 6 dollars in 12 months? Ans. 105 dollars.

13. If 8 men can build a wall, 20 feet long, 6 feet high, and 4 feet thick, in 12 days, in what time can 24 men build one, 200 feet long, 8 feet high, and 6 feet thick? Ans. 80 days.

14. A wall, 32 feet high, and 40 feet long, was built in 8

days, by 145 men; in how many days would 68 men build another wall, 28 feet high, and of the same length, allowing each man to perform an equal portion of labor, in the same time? *Ans.*  $14\frac{83}{88}$  days.

15. What must be paid for the transportation of 56 bags of coffee, each weighing 3 qr. 16 lb., 66 miles, if 14 bags, weighing each 125 lb., be carried 6 miles for \$6.25. *Ans.* 220 dollars.

16. If 6 persons can earn 120 £. in 21 weeks, how much will 14 persons earn, in 46 weeks, if they receive the same per week? *Ans.* 613 £. 6 s. 8 d.

17. If, when the days are 14 hours long, a person perform a journey of 276 miles in 16 days, in what time will he travel 860 miles, when the days are 12 hours long? *Ans.*  $58\frac{24}{107}$  days.

18. If 960 dollars defray the expenses of 20 men 88 weeks, for how many weeks will \$1440 defray the expenses of 48 men, if they spend at the same rate? *Ans.* 55 weeks.

19. If 100 £. gain 6 £. in 12 months, what principal, at the same rate per cent., will gain 3 £. 7 s. 6 d. in 9 months? *Ans.* 75 £.

20. If a man travel 240 miles in 12 days, when the days are 12 hours long, in how many days, of 16 hours, will he travel 720 miles, if he travel at the same rate? *Ans.* 27 days.

21. What is the interest of \$650 for 36 weeks, at 6 per cent. per annum? *Ans.* \$27.

22. If \$150, in 12 months, gain \$8 interest, what will \$400 gain in 4 months? *Ans.* \$7.11.+

23. If 3 men lay 144 square yards of pavement in 7 days, how many square yards will 12 men lay in 49 days? *Ans.* 4032 square yards.

24. If 200 lb. be carried 40 miles for 40 cents, how far may 202000 lb. be carried for \$60.60? *Ans.* 60 miles.

25. In what time will 200 £. gain 6 £. if 100 £. gain 6 £. in 52 weeks? *Ans.* 26 weeks.

26. In what time will 400 £. gain 96 £. interest, if 350 £. gain 10 £. 10 s. interest in 6 months? *Ans.* 4 years.

27. If 4 men mow 48 acres of grass in 12 days, in what time will 8 men mow 128 acres? *Ans.* 16 days.

28. If I receive 88 £. 17 s. 4 d. for the principal and interest of 86 £. for 8 months, what is the rate per cent. of interest? *Ans.* 5 per cent.

29. What will the transportation of 7 cwt. 2 qr. 25 lb. 64

miles cost, if 5 cwt. 3 qr. be carried 150 miles for \$24.58 ?

*Ans.* \$14.086.+

30. If I pay \$50.25 for the tuition of 2 boys, 3 quarters each, what will the tuition of 100 boys amount to, in  $7\frac{1}{2}$  years, at the same rate? *Ans.* \$25125.

31. Purchased goods to the amount of 750 £. and sold the same, six months after, for 825 £.; what did I gain per cent. ? *Ans.* 20 per cent.

32. If 56 lb. of bread be sufficient for 7 men 14 days, how long will 36 lb. suffice for 21 men ? *Ans.* 3 days.

33. A person having engaged to carry 8000 cwt. a certain distance in 9 days; removed 4500 cwt. with 18 horses, in 6 days; how many horses were required to remove the remainder, in the 3 remaining days ? *Ans.* 28 horses.

34. If 3 men perform a piece of work in 20 days, how many men will accomplish 4 times as much work in 4 days ? *Ans.* 60 men.

35. If 4 men, in 5 days, eat 6 lb. of bread, how many pounds of the same will be sufficient for 16 men 15 days ? *Ans.* 72 lb.

36. If it take 15 men 20 days to make 300 pairs of shoes, how many men will be required to make 1200 pairs in 60 days ? *Ans.* 20 men.

37. A wall, which was to be raised to the height of 27 feet, was raised 9 feet by 12 men, in 6 days; how many men were required to complete the work in 4 days, allowing each man to do the same amount of work per day ? *Ans.* 36 men.

38. If 6 men, in 21 weeks, earn \$120, how much will 14 men earn in 46 weeks ? *Ans.* \$613.33.+

39. If 48 bushels of corn produce 576 bushels in one year, what will be the product of 240 bushels in 6 successive years ? *Ans.* 17280 bushels.

40. In how many days will 25 men reap 200 acres of grain, if 12 men reap 80 acres in 6 days ? *Ans.*  $7\frac{1}{2}$ .

41. If 20 men mow 208 acres, 1 rood, and 24 rods of grass, in 24 days, how many men will cut down 8 times as much grass in twice the time ? *Ans.* 80.

QUESTIONS.—Of what does simple proportion consist ? What is compound proportion ? What is the smallest number of terms of which a statement in compound proportion can consist ? How must these terms compare with each other, in kind ? How many simple ratios will there be in the example given for illustration, when the sixth term is found ? Of what is the third ratio a compound ? What are the three ratios in the illustration given ? Give the entire illustration. What is the

rule first given? What is to be the denomination of the third term? How are the other terms to be arranged? What is the rule for cancelling? What term is to be placed first above the line? How are the other terms to be arranged? What is the given note?

## SOLUTION OF ARITHMETICAL PROBLEMS BY ANALYSIS.

An arithmetical question is solved analytically, when the operation is guided entirely by the conditions embraced in the question itself.

Take the following illustration:

Ex. 1. If 3 men perform a piece of work in 6 days, in what time will 9 men perform the same labor?

It is obvious that if it take 3 men 6 days to perform the proposed labor, it will take one man 3 times 6, or 18 days, to perform the same. But 9 men operating together, will perform 9 days work in one day; consequently, they will do the whole in 2 days; for  $18 \div 9 = 2$ , *Ans.*

2. If 21 men earn 63 dollars in a given time, how much will 42 men earn in the same time?

$63 \div 21 = 3$ , the number of dollars one man will earn in the given time. Therefore,  $\$42 \times 3 = \$126$ , the answer required. Or the ratio of 21 men to 42 is 2; and,  $\$63 \times 2 = \$126$ , the same as before.

Solutions of this kind may, therefore, be effected by the following general principle:—*Find the ratio of the two given terms which are of the same kind, and by this ratio multiply the term corresponding in kind with the one required.*

3. If 42 men can make 3 rods of wall in a given time, how much can 8 men make in the same time?

The ratio of 42 men to 8 men is  $42 : 8 = \frac{3}{1} = \frac{4}{1}$ ; therefore,  $\frac{4}{3} \times 3 = \frac{4}{1} = 4$  of one rod, which is the distance required.

4. If a staff 4 feet long, cast a shadow 6 feet, how high is that steeple, whose shadow measures 75 feet?

The ratio of  $6 : 75 = 12\frac{1}{2}$ , and  $12\frac{1}{2} \times 4 = 50$  feet, *Ans.* Or, the shadow is  $1\frac{1}{2} = \frac{3}{2}$  as long as the staff; hence,  $75 \div 3 = 25$ , and  $25 \times 2 = 50$  feet, the same answer as before.

5. An express traveling at the rate of 60 miles per day, had been absent 5 days, when a second express was dispatched on the same rout, traveling 75 miles per day. How many miles must the second travel to overtake the first?

$60 \times 5 = 300$ , the whole number of miles traveled by the first express before the second started, and consequently, the number of miles the second had to gain. But the first travels 60, and the second 75 miles per day; hence,  $75 - 60 = 15$ , the number of miles gained daily, by the second express. 15 miles are, therefore, gained in traveling 75 miles, consequently, one mile is gained in traveling 5 miles; and since 300 miles are to be gained,  $300 \times 5 = 1500$  miles, answer.

6. If 6 men in 14 days earn 84 dollars, how much will 9 men earn in 11 days?

$\$84 \div 6 = \$14$ , the money one man will earn in 14 days, and  $\$14 \div 14 = \$1$ , the wages of one man for one day; therefore,  $\$1 \times 9 = \$9$ , the money 9 men will earn in one day, and  $\$9 \times 11 = \$99$ , the money 9 men will earn in 11 days.

7. If 6 persons spend \$300 in 8 months, how much will be sufficient for a family of 15 persons 20 months?

$300 \div 6 = 50$ , and  $50 \div 8 = \$6\frac{1}{4}$ , the money spent by one person in one month; then,  $\$6\frac{1}{4} \times 15 \times 20 = \$1875$ , answer.

8. If 12 men build 36 feet of wall in 9 days, how many men would build 108 feet in 16 days?

$36 \div 12 = 3$ , and  $3 \div 9 = \frac{1}{3}$ , the distance built by one man in one day; and  $\frac{1}{3} \times 16 = 5\frac{1}{3}$ , the distance one man would build in 16 days; therefore,  $108 \div 5\frac{1}{3} = 20\frac{1}{4}$ , the number of men required.

9. A merchant owning  $\frac{1}{8}$  of a vessel, sold  $\frac{2}{3}$  of his share for \$1456. What was the value of the whole vessel?

$\frac{1}{8}$  of  $\frac{2}{3} = \frac{2}{12}$ . If then,  $\frac{2}{12}$  cost \$1456,  $1456 \div 8 = \$182$ , the value of  $\frac{1}{6}$  of the vessel; hence,  $182 \times 15 = \$2730$ , answer.

10. If  $\frac{3}{4}$  of a yard cost  $\frac{1}{4}$  of a dollar, what will 40 yards cost?

If  $\frac{3}{4}$  of a yard cost  $\frac{1}{4}$  of a dollar,  $\frac{1}{4}$  of a yard will cost  $\frac{1}{3}$  of that sum, or  $\frac{1}{3}$  of  $\frac{1}{4} = \frac{1}{12}$ , and one yard or  $\frac{4}{4}$  will cost 5 times that sum, or  $\frac{5}{12}$ ; therefore,  $\frac{5}{12} \times 40 = \$16\frac{2}{3} = \$16.333, +$  *Ans.*

11. If 240 men perform a piece of work in 8 months, how many men must be employed to finish the same work in 2 months?

The ratio of  $2 : 8 = 4$ , and  $240 \times 4 = 960$  men, answer.

## APPLICATION OF CANCELING TO ANALYTICAL SOLUTIONS.

12. If 8 pounds of tea cost \$12, what will 32 pounds cost?

$$\text{Statement, } \frac{12. \cancel{32}}{8}$$

The two terms of the same name here given are 8 and 32, and their ratio is 4, and is obtained by

$$\text{Canceling, } \frac{12. \cancel{32}}{8 \cdot 4}$$

Therefore,  $\$12 \times 4 = \$48$ , answer. The third term is here multiplied by the ratio of the first and second, as required for analytical solution. The terms are also canceled and multiplied as directed by the rule for canceling.

13. If 16 horses consume 84 bushels of grain in 24 days, how many bushels will suffice 32 horses 48 days?

In the preceding sum, it is evident that the given quantity of grain is to be increased by the ratios of 16 horses to 32 horses, and of 24 days to 48 days. Hence,

$$\frac{84. \cancel{32}. \cancel{48}}{16. \cancel{24}} \text{ is the statement expressing these ratios.}$$

$$\text{Canceled, } \frac{84. \cancel{32}. \cancel{48}}{16. \cancel{24}} \cdot 2 \cdot 2 = 336,$$

the number of bushels required.

We therefore see by the above example, that the effect of the operation is to increase the quantity of the same name as the required quantity, by all the given ratios. The same is true in all cases, that is, every statement for canceling is a complete analysis of the question under consideration.

14. If 8 men build 9 feet of wall in 12 days, how many men must be employed to build 36 feet in 4 days?

$$\text{Statement, } \frac{6. \cancel{36}. \cancel{12}}{9. \cancel{4}}$$

The number of men required will obviously depend on the ratios 9 : 36, and 4 : 12, the former of which is 4, and the latter, 3. Therefore,  $8 \text{ men} \times 4 \times 3 = 96 \text{ men}$ , the number required. The above statement canceled, gives the same result, thus:

$$\frac{8. \cancel{36}. \cancel{12}}{9. \cancel{4}} \cdot 4 \cdot 3 = 96 \text{ men.}$$

The same number would have been obtained, had the numbers been canceled in any other order; thus,

$$\frac{8 \cdot 36 \cdot 12}{9 \cdot 4}, \text{ and } 8 \times 12 = 96.$$

Hence we perceive, that in a correct solution of any sum by canceling, a complete analysis of that sum is given.

15. If 10 men make 300 pairs of boots in 20 days, how many men must be employed to make 450 pairs in 30 days? *Ans.* 10 men.

$$\text{Statement, } \frac{10 \cdot 450 \cdot 20}{300 \cdot 30}.$$

If 10 men make 300 pairs in 20 days, they would make 15 pairs in one day; and if 10 men make 15 pairs in one day, one man would make one and a half pairs per day; and in 30 days, he would make 45 pairs; therefore,  $450 \div 45 = 10$ , *Ans.*

16. If  $\frac{1}{2}$  of a yard cost  $\frac{3}{4}$  of a pound sterling, what will  $\frac{5}{8}$  of a yard of the same cloth cost?

If  $\frac{1}{2}$  yard cost  $\frac{3}{4}$  of a pound, the whole yard would cost  $\frac{3}{2}$  of a pound, and  $\frac{1}{2}$  of the same would cost  $\frac{1}{2}$  of  $\frac{3}{2}$  of a pound =  $\frac{3}{4}$  of a pound; consequently,  $\frac{5}{8}$  would cost 5 times that sum, or  $\frac{15}{8}$  =  $\frac{3}{2}$  of a pound, or 15 s. *Ans.*

17. If  $\frac{1}{6}$  of a house cost 49 pounds, what will be the value of  $\frac{3}{2}$  of the same? *Ans.* 10 £. 10 s.

18. A merchant bought a number of bales of velvet, each containing  $129\frac{1}{2}$  yards, at the rate of \$7 for 5 yards, and sold the same at the rate of \$11 for 7 yards, and gained \$200 by the transaction. How many bales were there?

He paid  $\frac{7}{5}$  of a dollar per yard, and received  $\frac{11}{7}$  of a dollar for the same. Hence,  $\frac{11}{7} - \frac{7}{5} = \frac{55}{35} - \frac{49}{35} = \frac{6}{35}$  of one dollar, the amount gained on one yard. Therefore,  $\$200 \div \frac{6}{35} = \frac{7000}{6}$ , the whole number of yards; and  $\frac{7000}{6} \div 129\frac{1}{2} = \frac{7000}{6} \div \frac{259}{2} = \frac{3500}{359.5} = 9$  bales, *Ans.*

19. If 7 horses consume  $2\frac{1}{2}$  tons of hay in 6 weeks, how many tons will 12 horses consume in 8 weeks? *Ans.*  $6\frac{2}{3}$  tons.

20. If 14 men finish a piece of work in 42 days, how long will it take 21 men to do it? *Ans.* 28 days.

21. If  $\frac{2}{5}$  of a farm be valued at \$895, what is the whole farm worth? *Ans.* \$1611.

22. If 7 horses consume 29 bushels of oats in 5 weeks, how many will 12 horses consume in 6 weeks? *Ans.*  $59\frac{2}{3}$  bushels.

23. A merchant owning  $\frac{1}{4}$  of a vessel, sold  $\frac{5}{8}$  of his share

for \$1200; what was the value of the whole vessel, at the same rate? *Ans.* \$1645.714.+

24. There is a pole  $\frac{1}{4}$  in the mud,  $\frac{2}{3}$  in the water, and 8 feet out of the water. What is its length? *Ans.*  $53\frac{1}{3}$  feet.

25. In a certain orchard  $\frac{1}{2}$  of the trees bear apples,  $\frac{1}{4}$  pears,  $\frac{1}{8}$  plums, 30 of them peaches, and 20 cherries. How many trees does the orchard contain? *Ans.* 600.

26. A certain school is classified as follows:  $\frac{1}{16}$  study grammar,  $\frac{1}{8}$  study geography,  $\frac{3}{10}$  arithmetic,  $\frac{3}{20}$  write, and 9 learn to read. How many are there in all, and how many in each study? *Ans.* Whole number 80. In grammar 5, geography 30, arithmetic 24, writing 12, and 9 read.

## SIMPLE AND COMPOUND PROPORTION IN FRACTIONS.

In stating such sums in Simple or Compound Proportion as consist of fractions, it is only necessary to compare terms as already directed, and then, if they are solved without canceling, having inverted the divisor, to divide the product of the numerators by the product of the denominators. If, however, they are to be solved by canceling, arrange the numerators of the several fractions as directed to arrange whole numbers, when whole numbers only are given, and place each denominator opposite its own numerator.

*Note.*—Before stating the sum, mixed numbers, if any are given, must be reduced to improper fractions.

Ex 1. If  $\frac{3}{8}$  of a yard cost  $\frac{7}{12}$  of a pound, what will  $\frac{3}{4}$  of a yard cost?

Statement,  $\frac{7}{15} \cdot \frac{3}{14} \cdot \frac{5}{3}$

The  $\frac{3}{8}$  is inverted, that its numerator may stand below the line as the same term would stand if it were a whole number.



$$\text{Solution, } \frac{\begin{smallmatrix} \text{7. 2. 5} \\ \text{15. 14. 3} \end{smallmatrix}}{2}$$

Therefore,  $\frac{1}{6}$  of a pound, or 3 s. 4 d., is the answer required.

2. If  $\frac{3}{4}$  of a pound of sugar cost  $\frac{8}{9}$  of a shilling, what will  $\frac{9}{10}$  of a pound cost?

$$\text{Statement, } \frac{8. 9. 4}{9. 10. 3} \quad \text{Ans. 1 s. 0 d. } 3\frac{1}{2} \text{ qr.}$$

3. A person who owned  $\frac{3}{5}$  of a vessel sold  $\frac{1}{5}$  of his share for 375 £. What was the value of the whole vessel, at the same rate? *Ans.* 1000 £.

These sums may all be solved analytically, if preferred. The following is the solution of the last:  $\frac{3}{5}$  of  $\frac{1}{5} = \frac{1}{25}$ , and 375 £.  $\div 15 = 25$  £., or  $\frac{1}{10}$  of the whole value; therefore, 25 £.  $\times 40 = 1000$  £. *Ans.*

4. If  $\frac{3}{5}$  of a ship be worth 3740 £., what is the value of the whole? *Ans.* 9973 £. 6 s. 8 d.

5. If  $1\frac{1}{4}$  yards cost 9 shillings, what is the value of 16 $\frac{1}{2}$  yards? *Ans.* 5 £. 17 s.

6. What is the value of  $\frac{2}{3}$  of a pound of lard, if  $1\frac{1}{6}$  of a pound cost  $\frac{1}{4}$  of a shilling? *Ans.* 2 $\frac{1}{2}$  pence.

7. A person who owned  $\frac{3}{8}$  of a lot of land, sold  $\frac{1}{8}$  of his share for \$3024. What was the value of the whole lot, at the same rate? *Ans.* \$12096.

8. A certain vessel is valued at \$1562.50. What is the value of  $\frac{2}{3}$  of  $\frac{1}{3}$  of the same? *Ans.* \$500.

9. I owned  $\frac{2}{3}$  of a ship, and sold  $\frac{1}{3}$  of my share for \$780. What was the value of the whole, at the same rate? *Ans.* \$3120.

10. A merchant bought 5 $\frac{1}{2}$  pieces of cloth, each containing 24 $\frac{3}{4}$  yards, for 6 $\frac{3}{4}$  shillings per yard. How many dollars did the whole cost, in New York currency? *Ans.* \$111.

11. A merchant had 4 $\frac{3}{4}$  cwt. of sugar, at 6 $\frac{1}{2}$  pence per lb., which he exchanged for tea, at 8 $\frac{1}{2}$  shillings per pound. How many pounds of tea did he receive? *Ans.* 29 $\frac{1}{4}$ .

12. How many pounds sterling will 150 yards of cloth cost, at 1 $\frac{1}{2}$  shilling per yard? *Ans.* 9 £.

13. If 3 $\frac{1}{2}$  times 3 $\frac{1}{2}$  yards cost 1 $\frac{1}{2}$  times 1 $\frac{1}{2}$  pounds sterling, how many shillings and pence will  $\frac{1}{2}$  of  $\frac{1}{3}$  of 12 $\frac{1}{2}$  yards cost? *Ans.* 7 s. 6 d.

14. What is the value of  $\frac{3}{4}$  of an ounce of silver, if 2 oz. be valued at 12 $\frac{3}{4}$  shillings? *Ans.* 4 s. 9 d.

15. What quantity of shalloon, that is  $\frac{3}{4}$  of a yard wide, will be sufficient to line  $7\frac{1}{2}$  yards of cloth,  $1\frac{1}{2}$  yards wide? *Ans.* 15 yards.

16. If  $2\frac{1}{2}$  yards,  $1\frac{1}{2}$  yard wide, be sufficient to make a coat, how much will it require of cloth that is  $\frac{3}{4}$  of a yard wide to make the same kind of garment? *Ans.* 4 yd.  $3\frac{1}{4}$  qr.

17. How many pieces of cloth, at  $\$18\frac{1}{2}$  per piece, are equal in value to  $224\frac{3}{4}$  pieces, at  $\$12\frac{1}{2}$  per piece? *Ans.*  $\$150\frac{1}{2}$ .

18. A merchant exchanged  $7\frac{1}{2}$  cwt. of sugar, at  $7\frac{1}{2}$  pence per pound, for tea at  $9\frac{1}{2}$  shillings per pound; how many pounds of tea did he receive? *Ans.*  $60\frac{3}{4}$  lb.

19. If 8 men can perform a piece of work in  $6\frac{3}{4}$  hours, in what time will 20 men do the same? *Ans.* 2 hours, 40 minutes.

20. How many yards of cloth,  $\frac{3}{4}$  of a yard wide, will line 20 yards,  $\frac{1}{2}$  of a yard wide? *Ans.* 12 yards.

21. How many pieces of cloth, at  $18\frac{1}{2}$  shillings per yard, are equal in value to  $350\frac{1}{2}$  pieces, at  $12\frac{1}{2}$  shillings per yard? *Ans.*  $241\frac{3}{4}$  pieces.

22. Lent a friend  $\$72\frac{1}{2}$  for  $8\frac{1}{2}$  months; what sum must he lend me for  $2\frac{1}{2}$  years, to balance the favor? *Ans.*  $\$21.233.+$

The following sums properly belong to Compound Proportion. They may be solved either by canceling, by analysis, or by the common rule of Compound Proportion.

Ex. 23. If  $\frac{3}{4}$  of a yard of cloth, which is  $\frac{1}{2}$  of a yard wide, cost  $\frac{2}{3}$  of a pound sterling, what is the value of  $\frac{1}{2}$  of a yard, that is  $1\frac{1}{2}$  yard wide?

Analysis:  $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$ , the fraction of a square yard purchased, which cost  $\frac{2}{3}$  of a pound sterling. Therefore,  $\frac{2}{3} \div 21 = \frac{2}{105}$ , the value of  $\frac{1}{8}$  part of a square yard, and  $\frac{2}{105} \times 32 = \frac{64}{105}$ , the price of 1 yard.  $\frac{64}{105} \times \frac{1}{2} = \frac{32}{105}$ , the quantity of which the price is required. Therefore,  $\frac{64}{105} \times \frac{32}{105} = \frac{2048}{11025} = \frac{2}{3}$  of 1 £. *Ans.* = 13 s. 4 d.

The same canceled,  $\frac{2. 5. 7. 4. 8}{5. 8. 4. 3. 7}$ .

To understand why the fractions  $\frac{3}{4}$  and  $\frac{1}{2}$  are inverted, it must be remembered that a fraction is divided by a fraction, by inverting the divisor and then multiplying numerators and denominators together. (See Sec. 7th, Introduction to Fractions.)

The above solved,  $\frac{2. 5. 7. 4. 8}{5. 8. 4. 3. 7}$   $\frac{3}{4}$  of a £. = 13 s. 4 d. *Ans.*

24. If 9 men spend  $12\frac{1}{2}$  £. in 27 days, what sum will 25 men spend in 40 days?

Analysis:  $12\frac{1}{2} \text{ £.} = 25 \text{ £.}$ , and  $25 \div 9 = 2\frac{5}{9} \text{ £.}$  the money one man spends in 27 days; and  $2\frac{5}{9} \text{ £.} \div 27 = \frac{25}{243} \text{ £.}$  the money spent by one man daily. Therefore,  $\frac{25}{243} \text{ £.} \times 25 = \frac{625}{243} \text{ £.}$  the money 25 men spend daily; and  $\frac{625}{243} \text{ £.} \times 40 = \frac{25000}{243} \text{ £.}$  the sum of money required, which reduced gives 51 £. 8 s.  $9\frac{4}{9}$  pence, *Ans.*

The same stated for canceling,  $\frac{25.25.40}{2.9.27}$

Cancelled,  $\frac{25.25.40}{2.9.27}$ , and  $25 \times 25 \times 20 = 12500$ ; and  $27 \times 9 = 243$ ; and  $12500 \div 243 = 51 \text{ £.} 8 \text{ s.} 9\frac{4}{9} \text{ d.}$  *Ans.*

25. If 18 persons consume  $2\frac{1}{2}$  lb. of tea in one month, how much will 8 persons consume in six months? *Ans.*  $4\frac{1}{2}$  lb.

26. If the tuition of 2 boys for  $\frac{1}{2}$  of a year be  $56\frac{1}{2} \text{ £.}$ , how much will be the tuition of 3 boys for  $5\frac{1}{2}$  years? *Ans.* 600 £.

27. If 90 cwt. be carried 30 miles for \$29, how many cwt. may be carried 45 miles for  $54\frac{1}{2}$ ? *Ans.* 12 cwt.

28. If 10 persons drink  $15\frac{1}{2}$  gallons of wine in one week, how much will 16 persons drink in 43 weeks? *Ans.*  $1073\frac{1}{2}$  gallons.

29. If  $\frac{1}{2}$  cwt. be carried 600 $\frac{1}{2}$  miles for  $12\frac{1}{2}$ , how far may  $\frac{3}{4}$  of a cwt. be carried for  $30\frac{1}{2}$ ? *Ans.*  $988\frac{1}{2}$  miles.

QUESTIONS.—When is an arithmetical question solved analytically? What is the general principle by which sums may be solved analytically? How are sums in Simple or Compound Proportion solved without canceling? How are they solved by canceling? What is the note?

## CONJOINED PROPORTION.

Conjoined Proportion consists of a comparison instituted between a series of terms bearing a certain relation to each other, as the coins, weights, and measures of different countries.

The principle involved in this rule is the same as in Single and Compound Proportion. No farther explanation is therefore needed.

**RULE.**—Above a horizontal line near the left, place the demanding term; then below the line, place the term of the same name as the demand, with the term which it equals in value directly above it. Next, seek another term of the same name as the one last placed, and set it also below the line, with the one it equals in value also above it. Thus proceed to arrange the terms, making each term standing below the line of the same name as the preceding term standing above it. The product of the numbers standing above the line divided by the product of those standing below it, will give the required number.

The numbers may of course be canceled as far as practicable, before multiplying and dividing.

Ex. 1. If 100 lb. English make 90 lb. Flemish, and 22 lb. Flemish make 28 lb. Bologna, how many pounds English are equal to 56 lb. Bologna?

The demand obviously lies on the 56 lb. Bologna; therefore,

$$\begin{array}{r} 56. 22. 100 \\ 28. 90 \end{array}$$

Canceled,  $\frac{56. 22. 100}{28. 90}$   $2 \times 22 \times 10 = 440 \div 9 = 48\frac{4}{9}$  lb. English, Ans.

2. If 40 lb. at New York make 48 lb. at Antwerp, and 30 lb. at Antwerp make 36 at Leghorn, how many pounds at New York are equal to 144 at Leghorn?

$$\begin{array}{r} \text{Statement, } 144. 30. 40 \\ 36. 48 \end{array}$$

Canceled,  $\frac{144. 30. 40}{36. 48. 12. 8}$  and  $5 \times 20 = 100$  lb. New York, Ans.

3. If 70 braces at Venice make 84 braces at Leghorn, and 12 at Leghorn make 7 American yards, how many braces at Venice are equal to 96 American yards? Ans. 1374.

4. If 24 lb. at New London make 20 lb. at Amsterdam, and 50 lb. at Amsterdam make 60 lb. at Paris, how many lb. at Paris are equal to 40 lb. New London? Ans. 40 lb.

5. If 50 lb. at New York make 45 lb. at Amsterdam, and 80 lb. at Amsterdam, 103 lb. at Dantzic, how many lb. at Dantzic are equal to 240 lb. New York? Ans. 278  $\frac{1}{16}$ .

6. If 24 braces at Leghorn be equal to 15 vares at Lisbon, and 45 vares at Lisbon be equal to 90 braces at Lucca, how

many braces at Lucca are equal to 120 braces at Leghorn?  
*Ans.* 150 braces.

**QUESTIONS.**—In what does Conjoined Proportion consist? How does the principle involved, compare with Simple and Compound Proportion? What is the rule for Conjoined Proportion?

## DISCOUNT.

Discount is an allowance made for the payment of money before it becomes due.

The present worth of any sum of money, payable at some future time without interest, is that sum which, if put at interest, would in the given time and rate per cent. amount to the whole debt.

Discount is not, therefore, a deduction of the given per cent. from a hundred cents or a hundred dollars. If I have a claim upon an individual for \$100, payable a year hence, and propose to allow him 6 per cent. discount for present payment, I must receive more than \$100 —  $\$6 = \$94$ ; since \$94 put on interest at 6 per cent. will not amount to \$100 in the given time. The interest on \$94 one year at 6 per cent. is \$5.64; and  $\$94 + \$5.64 = \$99.64$ , which is 36 cents less than the required sum, or \$100. If, however, a person owe me \$106, payable in one year without interest, and I propose to allow him the same discount for immediate payment, he must obviously pay me \$100, since \$100 in one year at six per cent. will amount to precisely \$106.

Hence, we learn that the ratio which any sum due a year hence without interest bears to its present worth, is as 106 to 100; or, what is the same thing, as \$1.06 to \$1.00, whenever the discount is at 6 per cent. If the rate per cent. be any other than 6, or the time more or less than one year, the ratio varies accordingly. Therefore, as the amount of \$1 for the given time and rate per cent. is to \$1, so is the given sum to its present worth.

**Ex. 1.** What is the present worth of \$450, due 2 years hence, 6 per cent. discount being allowed?

The interest of \$1 for 2 years at 6 per cent. is 12 cents, and consequently the amount of \$1, for the same time, is \$1.12. Therefore,  $1.12 : 1 :: 450 : \text{the required sum}$ . And, since nothing is effected by multiplying by 1, the required sum is obtained by dividing \$450 by 1.12. Hence,  $\$450.00 \div 1.12 = \$401.785 + \text{Ans.}$

From the above we derive the following rule :

**RULE.**—*Divide the sum on which the discount is to be made, by the amount of one dollar for the given time and rate per cent.*

2. What is the present worth of \$700, due 3 years hence, at 5 per cent. discount?

The amount of \$1 for 3 years at 5 per cent., is 1.15. Therefore,  $\$700.00 \div 1.15 = \$608.695 + \text{Ans.}$

3. Sold goods to the amount of \$1200, on 6 months' credit. What is the present worth, allowing 8 per cent. discount?  
*Ans. \$1159.846. +*

4. What is the present value of a legacy of \$2000, due 2 years hence, discounting at 5 per cent. per annum? *Ans. \$1818.18. +*

5. What is the difference between the interest and discount on \$600 for 12 years, at 5 per cent.? *Ans. Interest \$360; discount, \$225; difference, \$135.*

*Note.*—To obtain the discount, *subtract the present value from the sum due.*

6. What is the discount on \$300 for 60 days, at 6 per cent. per annum? *Ans. \$2.97.*

7. What is the present value of \$750, due  $3\frac{1}{4}$  years hence, discounting at 4 per cent. per annum? *Ans. \$657.894. +*

8. What is the discount on \$500 for 2 years, at 9 per cent. per annum? *Ans. \$76.272. +*

9. What is the present value of 350 £., due 4 years hence, discounting at 4 per cent. per annum? *Ans. 301 £. 14 s. 5 d.  $3\frac{1}{2}$  qr.*

10. What is the present worth of 672 £., due 2 years hence, discounting at the rate of 6 per cent. per annum? *Ans. 600 £.*

11. Bought goods to the amount of \$820, on 6 months' credit. What ought I to have paid, if I had advanced the money on

the receipt of the goods, and had been allowed 4 per cent. discount? *Ans.* \$803.92.+

12. Sold goods to the amount of \$1200, one half of which is to be paid in 6 months, and the other half in 8 months. What is the discount for the present payment of the whole, discounting at 6 per cent. per annum? *Ans.* \$40.553.

13. A person having a legacy of \$1450 left him, payable in 6 years, requests present payment, and proposes to allow 6 per cent. discount. What must he receive? *Ans.* \$1066.176.+

14. What is the discount of \$458 for 8 months, discounting at 8 per cent. per annum? *Ans.* \$23.188.+

QUESTIONS.—What is discount? What is the present worth of any sum of money, payable at some future time, without interest? Is discount a deduction of a given per cent. from a hundred cents, or a hundred dollars? Why? What numbers express the ratio which any sum due a year hence, at 6 per cent. bears to its present worth? What is the rule for discount? How is the discount obtained? How is discount proved? *Ans.* *Cast the interest on the present worth, for the time and rate per cent. of discount, and add it to the present worth.*

## PROFIT AND LOSS.

Profit and Loss is the rule by which merchants and others engaged in trade, determine how much is gained or lost by any transaction. It also enables them so to regulate the price of their goods as to gain or loose a certain per cent. on the first cost.

1st. TO FIND HOW MUCH IS GAINED OR LOST ON A QUANTITY OF GOODS SOLD AT RETAIL, THE PURCHASE PRICE OF THE WHOLE QUANTITY BEING GIVEN.

RULE.—*Find the value of the whole quantity at the retail price; then, if there be a gain, subtract the purchase price from the same, and the remainder will be the sum gained; but if there be a loss, subtract the amount received from the purchase price, and the remainder will be the sum lost.*

Ex. 1. Bought 40 yards of cloth for \$160, and sold the same for \$5.20 per yard? How much did I gain?  $\$5.20 \times 40 = \$208.00$ ; and  $\$208.00 - \$160 = \$48.00$ , *Ans.*

2. Bought a hogshead of molasses for \$25, and sold the same for 8 cents a pint. How much did I gain? *Ans.* \$15.32.

3. Bought 12 cwt. of sugar at 8 d. per pound, and sold it at 3 £. New York currency, per cwt. Did I gain or lose, and how much? *Ans.* Lost \$22.

4. Purchased 2 hogsheads of wine for \$94.50, and retailed the same at 2 s. New York currency, per quart. How much did I gain? *Ans.* \$31.50.

5. Paid \$57 for 456 yards of cloth, and sold the same at the rate of 4 s. 6 d. New York currency, for 3 yards. What did I gain? *Ans.* \$28.50.

6. Bought 12 rolls of ribbon, each containing 50 yards, for \$18.75, and sold the same at 6 d. New York currency, per yard. How much did I gain by the operation? *Ans.* \$18.75.

7. Bought 44 lb. of tea for \$16.50, and sold it for 3 s. 6 d. New England currency, per pound. What did I gain? *Ans.* \$9.166.+

8. What do I gain on 15 cwt. of rice, which cost me \$50, by retailing the same at 4 d. New York currency, per pound? *Ans.* \$20.

## 2d. TO FIND WHAT IS GAINED OR LOST PER CENT

*RULE.*—Find the whole gain, or loss by the preceding rule, and, having multiplied it by one hundred, divide the product by the first cost. Or say, as the first cost is to the whole gain or loss, so is \$100 or 100 £. to the gain per cent.

Ex. 1. If I buy broadcloth at \$5.50 per yard, and sell the same for \$6.00 per yard, what do I gain per cent.?

\$6.00—\$5.50=\$0.50, the gain on \$5.50. Therefore,  $.50 \times 100 = 50.00$ , and  $50.00 \div \$5.50 = 9.09+$ , the gain on \$100. Or,  $\$5.50 : .50 :: 100 : \text{the gain on } \$100$ .

Or, the operation may be canceled, by placing *first above the horizontal line, the whole gain or loss found by subtraction, and \$100 or 100 £. at the right of this, on the same side, and the whole cost below the same. Cancel, &c.*

The above sum thus stated,  $\frac{.50. 100}{\$5.50}$ , and  $100 \div 11 = \$9.09+$ ,  
11

as before.



2. What do I gain per cent. if I buy wheat at 12 s. a bushel, and sell the same for 15 s. a bushel? *Ans.* 25 per cent.

3. Purchased pepper for 8 d. per pound, and sold the same for 9 d. per pound. What per cent. did I gain? *Ans.* 12½.

4. Bought 650 lb. of sugar for 10 cents a lb., and sold the same for 12 cents per lb. What was my whole gain, and what my gain per cent.? *Ans.* Whole gain \$13.00; gain per cent. \$20.

5. Bought goods to the amount of \$325, and sold the same for \$370. What was the per cent. gained? *Ans.* \$13.846. +

6. If I lose \$2 on \$25, at what rate per cent. do I lose? *Ans.* 8 per cent.

7. Purchased a hogshead of wine for \$50, and sold the same for \$75, on six months' credit. What was my gain per cent., allowing 4 per cent. discount for the 6 months' credit? *Ans.* \$44.23.

8. Bought 6 cwt. of cheese for \$48, but it being damaged, I am willing to sell it for the same on a year's credit. What is my loss per cent. discounting at 6 per cent. per annum? *Ans.* \$5.66. +

3d. TO FIND HOW A COMMODITY MUST BE SOLD TO GAIN OR LOSE A CERTAIN PER CENT. ON THE WHOLE COST.

*RULE.*—If the purchase price of the quantity for which the retail price is required, be not given, it must first be found, and then multiplied by \$100 increased by the per cent. to be gained, or diminished by the per cent. to be lost. The last product divided by 100, or, what is the same thing, with two figures cut off from the right, will be the answer required. Or the operation may be reduced to the following statement: As \$100 is to \$100 increased by the per cent. to be gained, or diminished by the per cent. to be lost, so is the purchase price of the commodity to the retail price.

*Ex. 1.* Bought 300 yards of cloth for \$550; how must I sell the same per yard, to gain 25 per cent.?

$\$550 \div 300 = \$1.83\frac{1}{3}$ , the price of one yard; and  $\$1.83\frac{1}{3} \times 125 = \$229.16$ ; and  $\$229.16 \div 100 = \$2.29$ , + *Ans.* Or  $100 : 125 :: 1.83\frac{1}{3} : \text{Ans. or } \$2.29$ . +

Or the operation may be canceled by the following rule:

*RULE.*—Write the given price above a horizontal line, and the quantity which cost that price directly below it. Then place

*\$100 increased by the per cent. to be gained or diminished by the per cent. to be lost, above the same line, and 100 below, and proceed to cancel, &c.*

The above sum stated,  $\frac{550.125}{300.100}$

Canceled,  $\frac{550.125}{288.188}$ , and  $55 \div 6 \times 4 = \$2.29. +$   
 $\begin{array}{r} 5 \\ 288 \cdot 188 \\ 6 \quad 4 \end{array}$

2. How must I retail melasses by the gallon, for which I paid \$30 per hogshead, to gain  $12\frac{1}{2}$  per cent.? *Ans.* \$0.535. +

3. Purchased 100 gallons of wine for \$130, but by accident 15 gallons leaked out. How must I sell the remainder per gallon, to gain 15 per cent.? *Ans.* \$1.758. +

4. Paid \$1.10 per gallon for melasses; how must I sell the same per quart, to gain 30 per cent.? *Ans.*  $35\frac{1}{2}$  cents.

5. Received from Lisbon 180 casks of raisins, which cost me \$2.13 per cask. How shall I sell them per cask, to gain 25 per cent.? *Ans.* \$2.66.

Statement,  $\frac{2.13.125}{100}$

6. Bought 2 cwt. of pepper at 1 s. New York currency, per pound, but it being damaged, I am willing to lose 10 per cent.; how must I sell it by the lb.? *Ans.* 11 cents  $2\frac{1}{2}$  mills.

7. Bought one ton of steel for \$184; how must I sell the same per pound, to gain 5 per cent.? *Ans.* 8 cents 6 mills.

8. A merchant bought 160 yards of cloth for \$240; how must he sell the same per yard to gain 12 per cent.? *Ans.* \$1.68.

9. Bought 8 pieces of cloth, each containing 15 yards, at 3 s. New England currency, per yard; how must I retail the same, to gain 8 per cent., and how much must I receive for the whole? *Ans.* 54 cents retail price; whole value, \$64.80.

10. If I buy 6 cwt. of sugar at 10 d. New York currency, per pound, and am allowed 4 per cent. discount for ready money, and sell the same so as to gain 15 per cent. on the money advanced, how much money do I receive? *Ans.* \$77.28.

11. Bought 12 chests of tea, each weighing 56 pounds, at 4 s. 6 d. New England currency, per lb. For ready money was allowed 2 per cent. discount; how much must I receive

for the whole to realize a profit of 10 per cent. on the money paid out? *Ans.* \$543.312.

Statement,  $\frac{12. 56. 54. 98. 110}{12. 6. 100. 100}$

12. Bought 700 yards of ribbon, at 6 d. New York currency, per yard; how must I sell the same to gain  $12\frac{1}{2}$  per cent. and what shall I receive for the whole quantity? *Ans.*  $6\frac{1}{2}$  d. per yard, nearly; and for the whole, \$49.218.+

13. How must cloth which costs 13 s. 4 d. be sold to gain 20 per cent.? *Ans.* At 16 s. per yard.

QUESTIONS.—What is profit and loss? What is Case 1st? What is the rule? What is Case 2d? What is the rule? How is the statement made for canceling? What is Case 3d? What is the rule? What is the rule for canceling?

## BARTER.

Barter is a rule by which those engaged in trade exchange commodities so that neither party suffer loss.

Take the following illustration:

Ex. 1. How much sugar at 10 d. per lb. must be given in exchange for 12 cwt. of butter at 15 d. per pound?

It is obvious that the 12 cwt. of butter bears the same ratio to the number of cwt. of sugar required, as 10 d. bears to 15 d.

Therefore, as 10:15::12:the answer, viz. 18 cwt. Or,  $\frac{12. 15}{10}$  is the statement for canceling:

$$\begin{array}{r} 6 \quad 3 \\ 12. \quad 15 \\ \hline 18 \end{array} \quad 6 \times 3 = 18, \text{ Ans.}$$

2

The scholar will perceive that the principle here involved is the same as in the rule of proportion, which has already been fully explained.

2. How much tea at 8 s. per lb. must be given for 2 cwt. of chocolate at 4 s. 6 d. per lb.? *Ans.* 1 cwt. 14 lb.

Statement, 8 s. : 4 s. 6 d. :: 2 cwt. : Ans.

For canceling,  $\frac{2.54}{12.8}$

In this statement, the price of the chocolate is reduced to pence. The 12 below the line reduces the pence to shillings.

3. How much rice at 32 s. per cwt. must be given for 4 cwt. 2 qr. of raisins at 6 d. per pound? *Ans.* 7 cwt. 3 qr. 14 lb.

4. B. has 90 yards of linen, worth 4 s. 6 d. per yard, which he wishes to exchange with C. for muslin at 2 s. per yard; how many yards of muslin must B. receive? *Ans.* 202½ yards.

5. Bought 75 yards of broadcloth at \$3 per yard, and paid for it in muslin at 1 s. 3 d. New York currency, per yard; how many yards of muslin did it take? *Ans.* 1440 yards.

6. Bought 7 tuns of wine at 10 d. a pint, and paid for it with melasses at 3 s. per gallon; how many hogsheads of melasses did it take? *Ans.* 62½ hhd.

7. Sold 5 cwt. 1 qr. of sugar at \$8.50 per cwt. and received in pay 24 yards of cloth; what was the value of the cloth per yard? *Ans.* \$1.859.

8. How many gallons of melasses at 8 cents a quart must be given for 24 cwt. of rice at \$8 per cwt. *Ans.* 600 gallons.

9. I have nuts worth \$4 per bushel, and B. has sugar worth 10 cents per pound; now if I charge him \$4.50 per bushel for my nuts, what ought he to charge me for his sugar? *Ans.* 11¼ cents.

10. What quantity of butter worth 12½ cents a pound must be given in exchange for 12 lb. of indigo worth \$2.25 per pound? *Ans.* 216 lb.

11. Sold 5 cwt. of cheese at 8 d. per pound, and received in pay muslin worth 1 s. 6 d. per yard; how many yards did I receive? *Ans.* 248⅘ yds.

12. Bought 90 bushels of wheat at 15 s. and gave in pay 115 yards of Irish linen, worth 6 s. per yard, and the remainder in cash; how many dollars did I pay, the currency being New York? *Ans.* \$82.50

13. Received of B. a quantity of broadcloth which he had sold at \$4.50 per yard, but charged me \$5.00; in pay I let him have a quantity of wheat which I had retailed at \$1.50 per bushel; what ought I to have charged him, to meet the advance he made on his broadcloth? *Ans.* \$1.66⅔.

14. Bought 12 gross of pen-knives at 9 dollars per gross, and

paid for them in cloth at 3 s. 6 d. New York currency, per yard; how many yards will be required to pay for the knives? *Ans.* 246½ yards.

15. Sold 48 cwt. of hops at 4 d. per pound, and received pay in prunes at 6 d. per pound; how many cwt. did I receive? *Ans.* 32 cwt.

16. Received 460 yards of linen worth 6 s. 6 d. New York currency, per yard, in exchange for 84 yd. of broadcloth; what was the value of the broadcloth per yard? *Ans.* \$4.449.+

17. Exchanged 320 lb. of chocolate valued at 4 s. 6 d. New York currency, per pound, for a quantity of cotton at 8 d. per lb.; but there not being cotton enough to pay me, I received the balance, viz. 50 dollars, in cash; how many lb. of cotton did I receive? *Ans.* 1560 lb.

18. How many cwt. of sugar, valued at 10 d. per lb. New England currency, must be given in exchange for 24 cwt. of pepper at 15 dollars per cwt.? *Ans.* 23½ cwt.

19. What deduction ought to be made on wheat, which has been valued at 9 s. per bushel, in exchange for cloth, when cloth valued at 14 s. per yard is sold for 12 s. per yard? *Ans.* 1 s. 3¾ d. per bushel.

20. How many pounds of cinnamon at 10 s. per lb. must be given in exchange for 5 cwt. of saleratus at 9 pence per lb.? *Ans.* 42 lb.

**QUESTIONS.**—What is Barter? What is the principle involved in this rule?

## PARTNERSHIP.

Partnership is the rule by which individuals trading with a joint stock, determine their several shares of the profit or loss realized.

The individuals thus trading are called *Stockholders*. The sum of money advanced by the company forms a *Capital Stock*; and the money from time to time received on the several shares is called *Dividend*. When the whole stock owned by the com-

pany is employed for the same period of time, the gain or loss which results from the adventure is divided among the individuals composing the company, in proportion to their several shares of the stock.

We have then the following rule :

**RULE.**—*As the whole stock is to each man's share of the stock, so is the whole gain or loss to each man's share of the gain or loss.*

Ex. 1. Three persons traded in company. A's stock was \$1200; B's stock was \$4800, and C's, \$2000: they gained \$800; what was each man's share of the gain?

$\$1200 + \$4800 + \$2000 = 8000$ , amount of the stock; therefore,  $8000 : 1200 :: 800$ : A's share, viz. \$120. Then to obtain B's share,  $8000 : 4800 :: 800$ : B's share, or \$480. And lastly, for C's share,  $8000 : 2000 :: 800$ : C's share, or \$200.

To state sums for canceling :

**RULE.**—*Write the whole gain or loss above the horizontal line, with the stock of one of the company standing at the right of it; then place the whole stock below the line, and proceed to cancel, &c.*

The above sum thus stated:  $\frac{800. 1200}{8000} = \$120$ , A's share.  
 $\frac{800. 4800}{8000} = \$480$ , B's share; and  $\frac{800. 2000}{8000} = \$200$ , C's share.

**Note.**—The operations in this rule are proved by adding together the several shares. Their sum must equal the whole gain.  $\$120 + \$480 + \$200 = \$800$ .

2. Three men, A., B., and C., traded in company. A. put in \$560; B., \$700; and C., \$640. At the expiration of their partnership, they found that they had gained only \$420; what was each man's share? *Ans.* A's, \$123.789; B's, \$154.737; and C's, \$141.473.

3. Three merchants trading in company suffered a loss of \$600; their several stocks were \$800, \$1000, and \$1200; what was each man's loss? *Ans.* \$160, \$200, and \$240.

4. A., B., and C., freighted a ship. A. put on board 36 tons;

B., 40 tons ; and C., 60 tons. In a storm, 30 tons were thrown overboard. What was each man's share of the loss? *Ans.* A's,  $7\frac{1}{7}$  tons ; B's,  $8\frac{1}{7}$  tons ; and C's,  $13\frac{1}{7}$  tons.

5. A bankrupt, who has property valued at \$2500, owes A. \$900 ; B., 1000 ; C., \$800, and D., \$500. How much will each man receive, if the whole property be given up to them? *Ans.* A., \$703.125 ; B., \$781.25 ; C., \$625 ; and D., \$390.625.

6. Three individuals unite in purchasing 600 acres of land. A. pays \$3000 ; B., \$5000 ; and C, \$7000. What are their several proportions of the whole? *Ans.* A. has 120 acres ; B., 200 ; and C., 280.

7. Three individuals, L., M., and N., freighted a ship with 324 tons burden. L. put on board 72 tons ; M., 108 tons ; and N., 144 tons. They cleared \$1200 by the adventure. What was each man's share of the gain? *Ans.* L's, \$266.666+ ; M's, \$400 ; and N's, \$533.333.+

8. Three persons agreed to pay \$150 for the use of a pasture, and that each should pay in proportion to the number of creatures he put in. What would be their several shares, provided the first put in 100 oxen ; the second, 150 oxen, and the third, 200 oxen? *Ans.* \$33.333+ ; \$50 ; and \$66.666.+

9. A person at his death owes A. 70 £. ; E., 400 £. ; and H., 140 £. ; and his property is valued at only 400 £. How much ought each to receive? *Ans.* A., 45 £.  $18\frac{2}{3}$  s. ; E., 262 £.  $5\frac{2}{3}$  s. ; and H., 91 £.  $16\frac{4}{5}$  s.

10. Four individuals purchased a horse for \$96. A. paid \$20 ; B., \$30 ; C., 25 ; D., \$21. They sold the horse for \$150. What was each man's share of the gain? *Ans.* A's, \$11.25 ; B's, \$16.875 ; C's, \$14.06 $\frac{1}{4}$  ; and D's, \$12.81 $\frac{1}{4}$ .

11. A man, whose property was worth only \$500, owed G., \$150 ; H., \$180 ; and M., \$300. He therefore concluded to give his property up to his creditors. What did each receive? *Ans.* G., \$119.047+ ; H., \$142.857 ; M., \$238.095.

12. L. has a farm of 90 acres ; M. has one of 120 acres ; and N. one of 150 acres. Their farms all joining, they agree to unite them for the purpose of renting. This they do, and receive for the whole the annual rent of \$820 ; what is each man's share of the rent? *Ans.* L's, \$205 ; M's, \$273.333+ ; N's, \$341.666.+

Whenever the different shares are not continued through the same period of time, it is obvious that each man's share of the profit and loss will depend not entirely on his portion of the

whole stock, but also on the time during which his stock was invested. Take the following sum as an illustration :

13. Three merchants traded in company. A. put in \$120 for 10 months; B., \$100 for 18 months; and C., \$150 for 5 months. They gained \$100. What was each man's share? A's, \$120 for 10 months =  $\$120 \times 10 = \$1200$  for one month; B's, \$100 for 18 months =  $\$100 \times 18 = \$1800$  for one month; and C's, \$150 for 5 months =  $\$150 \times 5 = \$750$  for one month. Therefore,  $\$1200 + \$1800 + \$750 = \$3750$ , and  $3750 : 1200 :: 100 : \text{A's share, or } \$32$ .  $3750 : 1800 :: 100 : \text{B's share, or } \$48$ ; and  $3750 : 750 :: 100 : \text{C's, share, or } \$20$ .

Hence, when the time of investment of the several shares differs :

*RULE.*—Multiply each man's stock by the time during which he continued in trade, and use the several products as directed in the preceding rule to use the several shares of the whole stock.

14. A., B., and C., hold a pasture in common, for which they pay 30 £. per annum. In this pasture, A. has 40 oxen for 75 days; B., 45 oxen for 50 days; and C., 50 oxen for 90 days. What part of the 30 £. ought each to pay? *Ans.* A., 9 £. 4 s.  $7\frac{1}{3}$  d.; B., 6 £. 18 s.  $5\frac{1}{3}$  d.; and C., 13 £. 16 s.  $11\frac{1}{3}$  d.

15. Three men enter into partnership on the following terms : A. invests \$1500 for 5 months; B., \$1800 for 6 months; and C., \$2000 for 8 months. During the continuance of their partnership they sustain a loss of \$1000. What is each man's share of the loss? *Ans.* A's loss is \$218.658; B's, \$314.868; and C's, \$466.472.

16. E. and S. enter into partnership for one year. E. at first advances \$480, and B. puts in his share 3 months after. How much must he advance to be entitled to an equal share of the gain at the expiration of one year? *Ans.* \$640.

17. Two merchants trading in company gain \$200. A's stock was \$220 for 6 months, and B's, \$380 for 9 months. How ought they to share the gain? *Ans.* A's portion is \$55.696; B's, \$144.304.

18. Two men commenced trading in company on Jan. 1st, 1825. A. advanced \$1000 at the time specified; but B., finding it inconvenient, did not advance his share till the first of May following. At the end of the year they shared the profits equally. What capital did B. advance? *Ans.* \$1500.



19. A. and B. traded in company. A. put in \$1200; B. advanced his share 3 months after. What sum was it necessary for him to advance so as to be entitled to one half of the profit at the expiration of one year? *Ans.* \$1600.

20. L., M., and N., entered into partnership; L. advanced \$300 for 3 years; 6 months after, M. put in \$450; and 6 months after M. put in his share, N. put in \$520. At the expiration of 3 years, they found they had cleared \$900. What was each man's share? *Ans.* L's, \$264.274; M's share, \$330.342; and N's, \$305.383.

**QUESTIONS.**—What is Partnership? What are the individuals trading in company called? What does the money advanced by the company form? What is the money they receive on their several shares called? What is the rule for operating when each man's stock is employed for the same period of time? What is the rule for canceling? When all the shares are not continued for the same period of time, on what will each man's share of the profit and loss depend? What is the rule?

## COMMERCIAL EXCHANGE.

Under this rule are included the operations of purchasing goods in one country, in the currency of that country, and selling them at wholesale or retail in the currency of another country, so as to give or lose some required per cent.

### TABLE OF GOLD COINS.

| NAMES OF COINS.               | VALUE.   | NAMES OF COINS.                     | VALUE.   |
|-------------------------------|----------|-------------------------------------|----------|
| <i>Austrian Dominions.</i>    |          | <i>Berne.</i>                       |          |
| Sovereign, - - -              | \$3.37 7 | Ducat, double in proportion, - - -  | \$1.98 6 |
| Double Ducat, - - -           | 4.58 9   | Pistole, - - -                      | 5.54 2   |
| Hungarian Ducat, - - -        | 2.29 6   |                                     |          |
| <i>Bavaria.</i>               |          | <i>Brazil.</i>                      |          |
| Carolus, - - -                | 4.95 7   | Johannes, half in proportion, - - - | 17.06 4  |
| Max d'or or Maximilian, - - - | 3.31 8   | Dobraon, - - -                      | 32.70 6  |
| Ducat, - - -                  | 2.27 5   |                                     |          |

| NAMES OF COINS.                      | VALUE.    | NAMES OF COINS.                          | VALUE.   |
|--------------------------------------|-----------|------------------------------------------|----------|
| Dobra, - - -                         | \$17.30 1 | <i>Genoa.</i>                            |          |
| Moidore, half in proportion, - - -   | 6.55 7    | Sequin, - - -                            | \$2.30 2 |
| Crusade, - - -                       | 63 5      | <i>Hanover.</i>                          |          |
| <i>Brunswick.</i>                    |           | Double George d'or, -                    | 7.87 9   |
| Pistole, double in proportion, - - - | 4.54 8    | Ducat, - - -                             | 2.29 6   |
| Ducat, - - -                         | 2.23      | Gold Florin, double in proportion, - - - | 1.67     |
| <i>Cologne.</i>                      |           | <i>Holland.</i>                          |          |
| Ducat, - - -                         | 2.26 7    | Double Ryder, - - -                      | 12.20 5  |
| <i>Colombia.</i>                     |           | Ryder, - - -                             | 6.04 3   |
| Doubloon, - - -                      | 15.53 5   | Ducat, - - -                             | 2.27 5   |
| <i>Denmark.</i>                      |           | Ten Guilder Piece, -                     | 4.03 4   |
| Ducat, Current, - -                  | 1.81 2    | <i>Malta.</i>                            |          |
| Ducat, Specie, - -                   | 2.26 7    | Double Louis, - - -                      | 9.27 8   |
| Christian d'or, - -                  | 4.02 1    | Louis, - - -                             | 4.65 2   |
| <i>East India.</i>                   |           | Demi-Louis, - - -                        | 2.33 6   |
| Rupee, Bombay, -                     | 7.09 6    | <i>Mexico.</i>                           |          |
| Rupee, Madras, -                     | 7.11      | Doubloons, - - -                         | 15.53 5  |
| Pagoda, Star, - -                    | 1.79 8    | <i>Milan.</i>                            |          |
| <i>England.</i>                      |           | Sequin, - - -                            | 2.29     |
| Guinea, half in proportion, - - -    | 5.07 5    | Dopia or Pistole, -                      | 3.80 7   |
| Sovereign, half in proportion, - - - | 4.84 6    | Forty Livre Piece, -                     | 7.74 2   |
| Seven shilling piece, -              | 1.69 8    | <i>Naples.</i>                           |          |
| <i>France.</i>                       |           | Six Ducat Piece, 1783, -                 | 5.24 9   |
| Louis, coined before 1786, -         | 4.84 6    | Two do. or Sequin, 1762, -               | 1.59 1   |
| Double Louis, before 1786, -         | 9.69 7    | Three do. or Oncetta, 1818, -            | 2.49     |
| Louis, coined since 1786, -          | 4.57 6    | <i>Netherlands.</i>                      |          |
| Double Louis, since 1786, -          | 9.15 3    | Gold Lion, or 14 Florin Piece, - - -     | 5.04 6   |
| Napoleon, or 20 francs, -            | 3.85 1    | Ten Florin Piece, -                      | 4.01 9   |
| Double Napoleon, or 40 francs, - - - | 7.70      | <i>Parma.</i>                            |          |
| <i>Frankfort on the Main.</i>        |           | Quadruple Pistole, -                     | 16.62 8  |
| Ducat, - - -                         | 2.27 9    | Pistole, or Dappia, 1787, -              | 4.19 4   |
| <i>Geneva.</i>                       |           | do. do. 1796, -                          | 4.13 5   |
| Pistole, old, - - -                  | 3.98 5    | Maria Theresa, 1818, -                   | 3.86 1   |
| Pistole, new, - - -                  | 3.44 4    | <i>Piedmont.</i>                         |          |
| <i>Hamburg.</i>                      |           | Pistole, coined since 1785, -            | 5.41 1   |
| Ducat, double in proportion, - - -   | 2.27 9    | Sequin, half in proportion, -            | 2.28     |
|                                      |           | Carlino, coined since 1785, -            | 27.34    |
|                                      |           | Piece of 20 Francs, or Marengo, - - -    | 3.56 4   |

| NAMES OF COINS.            | VALUE.  | NAMES OF COINS.             | VALUE.   |
|----------------------------|---------|-----------------------------|----------|
| <i>Poland.</i>             |         | <i>Sicily.</i>              |          |
| Ducat, - - -               | \$2.27  | Ounce of 1751, - -          | \$2.50 4 |
| <i>Portugal.</i>           |         | Double Ounce, - -           | 5.04 4   |
| Dobraon, - - -             | 32.70 8 | <i>Spain.</i>               |          |
| Dobra, - - -               | 17.30 1 | Doubloons of 1772, parts    |          |
| Johannes, - - -            | 17.06 4 | in proportion. - -          | 16.02 8  |
| Moidore, half in propor-   |         | Doubloon, - - -             | 15.53 5  |
| tion, - - -                | 6.55 7  | Pistole, - - -              | 3.88 4   |
| Piece of 16 Testoons, 1600 |         | Coronilla, Gold Dollar, or  |          |
| Rees, - - -                | 2.12 1  | Vintern, 1801, - -          | 98 3     |
| Old Crusado, or 400 Rees,  | 58 8    | <i>* Sweden.</i>            |          |
| New Crusado, or 480 Rees,  | 63 5    | Ducat, - - -                | 2.23 5   |
| Milree, coined in 1755,    | 73      | <i>Switzerland.</i>         |          |
| <i>Prussia.</i>            |         | Pistole of the Helvetic Re- |          |
| Ducat, 1748, - - -         | 2.27 9  | public, - - -               | 4.56     |
| Ducat, 1787, - - -         | 2.26 7  | <i>Treves.</i>              |          |
| Frederick, double, 1769,   | 7.97 5  | Ducat, - - -                | 2.26 7   |
| Frederick, double, 1800,   | 7.95 1  | <i>Turkey.</i>              |          |
| Frederick, single, 1778,   | 3.99 7  | Sequin fonduecli of Con-    |          |
| Frederick, single, 1800,   | 3.97 5  | stantinople, - - -          | 1.86 8   |
| <i>Rome.</i>               |         | Sequin fonduecli of Con-    |          |
| Sequin, coined since 1760, | 2.25 1  | stantinople, 1789, - -      | 1.84 8   |
| Scudo of Republic, - -     | 15.81 1 | Half Misseir, 1818, - -     | 52 1     |
| <i>Russia.</i>             |         | Sequin fonduecli, - -       | 1.83     |
| Ducat of 1796, - - -       | 2.29 7  | Yeermeeblekblek, - -        | 3.02 8   |
| Ducat of 1763, - - -       | 2.26 7  | <i>Tuscany.</i>             |          |
| Gold Ruble of 1756, - -    | 96 7    | Zechino or Sequin, - -      | 2.31 8   |
| Gold Ruble of 1799, - -    | 73 7    | Ruspone of the Kingdom      |          |
| Gold Polten of 1777, - -   | 35 5    | of Etruria, - - -           | 6.93 8   |
| Imperial of 1801, - - -    | 7.82 9  | <i>Venice.</i>              |          |
| Half Imperial of 1801,     | 3.91 8  | Zechino or Sequin, shares   |          |
| Half Imperial of 1818,     | 3.93 3  | in proportion, - -          | 2.31     |
| <i>Sardinia.</i>           |         | <i>Wirttemberg.</i>         |          |
| Carlino, half in propor-   |         | Carolin, - - -              | 4.89 8   |
| tion, - - -                | 9.47 2  | Ducat, - - -                | 2.23 5   |
| <i>Saxony.</i>             |         | <i>Zurich.</i>              |          |
| Ducat of 1784, - - -       | 2.26 7  | Ducat, double and half in   |          |
| Ducat of 1797, - - -       | 2.27 9  | proportion, - - -           | 2.26 7   |
| Augustus of 1754, - -      | 3.92 5  |                             |          |
| Augustus of 1784, - -      | 3.97 4  |                             |          |

## TABLE OF UNCOINED AND SILVER MONEY.

| <i>English Currency.</i>   |        | <i>Currencies of other Nations.</i> |        |
|----------------------------|--------|-------------------------------------|--------|
| 1 £. sterling, before 1832 |        | 1 millree of Portugal,              | \$1.24 |
| == \$4.444. Since 1832 ==  | \$4.80 | 1 Russian silver ruble,             | 75     |
| Or prior to 1832, 9 £. ==  |        | 1 rix dollar of Sweden,             | 1.00   |
| \$40. Since 1832, 5 £. ==  | 24.00  | 1 Russian rix dollar                | 66 6   |
| 1 English crown,           | 1.10   | Or 3 rix dollars ==                 | 2.00   |
| Or 10 crowns ==            | 11.00  | 1 Danish rigsbank dollar,           | 50     |
| 1 English shilling,        | 22 2   | 1 silver ducat of Naples,           | 80     |
| 1 pistareen,               | 20     | Or 5 ducats ==                      | 4.00   |
| Or 5 pistareens ==         | 1.00   | 1 scudo of Sicily,                  | 96     |
| 1 English penny,           | 0185   | 1 oncia of Sicily,                  | 2.40   |
| <i>Irish Currency.</i>     |        | 1 pezza of Leghorn,                 | 90     |
| 1 £. Irish, - - -          | 4.10   | 1 pezza of Genoa,                   | 89     |
| Or 10 £. == - - -          | 41.00  | 1 flourin of Trieste,               | 48     |
| 1 shilling, - - -          | 20 5   | 1 rix dollar of Trieste,            | 96     |
| 1 penny, - - -             | 01 7   | 1 Roman crown,                      | 1.00   |
| <i>French Currency.</i>    |        | 1 gold crown of Rome,               | 1.53   |
| 1 French crown,            | 1.10   | 1 Maltese scudo,                    | 40     |
| Or 10 crowns == - -        | 11.00  | 1 rupee of Bengal,                  | 55 5   |
| 1 five franc piece,        | 93     | 1 rupee of Bombay,                  | 50     |
| 1 franc, - - -             | 18 6   | 1 pagoda of Madras                  | 1.80   |
| 1 Decimes, - - -           | 0186   | 1 tale of Canton,                   | 1.48   |
| <i>Spanish Currency.</i>   |        | 1 Japanese tale,                    | 75     |
| 1 Spanish dollar,          | 1.00   | 1 dollar of Sumatra,                | 1.10   |
| 1 real, new plate,         | 10     | 1 tale of Sumatra,                  | 4.16   |
| 1 real vellon, - - -       | 05     | 1 florin of Java,                   | 40     |
|                            |        | 1 mark banco of Hamburg,            | 33 3   |
|                            |        | 1 guilder of Amsterdam,             | 40     |

To reduce a foreign currency to federal money, multiply the given money of that currency by the value of its unit in federal money, as given in the preceding tables.

If, however, it be required to find how goods, the value of which is given in a foreign currency, must be sold in federal money, to gain or lose a given per cent., first change the foreign currency to federal money, and calculate the gain or loss on this by the rules already given. The retail price of any denomination, as cwt., lb., yd., gal., &c., may then be found by division.

Ex. 1. Purchased in London 360 yards of broadcloth, which cost me, including transportation, 300 £. sterling; how must I sell the same per yard, in federal money, to make a profit of 20 per cent.?

Solution:  $300 \text{ £.} \times 40 = 12000$ ; and  $12000 \div 9 = \$1333\frac{1}{3}$ , the value of 300 £. in federal money.

Then,  $\$1333\frac{1}{3} \times 1.20 = \$1600$ , the value in federal money increased by the gain per cent.; therefore,  $\$1600 \div 360 = \$4.444$ , + the required price of one yard.

If the wholesale price only be required, it is only necessary to omit the last step. Thus the \$1600 is the wholesale price of the cloth given in the preceding sum.

To solve sums like the preceding by canceling:

**RULE.**—Place the whole cost in the given currency first above, and (if the retail price be required) the number expressing the quantity procured for that price, first below, a horizontal line. Write next above the line the value of a unit of the given currency, in federal money. And, lastly, to increase or diminish the price by the required per cent., place 100 below the line, and 100 increased by the per cent. to be gained, or diminished by the per cent. to be lost, above the same.

**Note 1st.**—If the wholesale price be required, the number expressing the whole quantity (by the preceding rule, placed below the line) must be rejected.

**Note 2d.**—Whenever in the preceding tables the value of a number of units of foreign currency is given in federal money, place that number below the line, and its federal value above the sum. (See £'s sterling.)

The preceding sum stated for canceling:

$$\begin{array}{r} 300. \quad 40. \quad 120 \\ \hline 360. \quad 9. \quad 100 \end{array}$$

To understand the reason of the middle terms, see Note 2d, and £'s sterling in the preceding tables.

$$\begin{array}{r} 40 \\ 300. \quad 40. \quad 120 \\ 360. \quad 9. \quad 100 \\ \hline 9 \quad 2 \end{array}$$

And  $40 \div 9 = \$4.444$ , + the answer, the same as before.

2. Purchased in London 350 yards of sheeting for 75 £., and paid 12 £. for its transportation to New York city; how must I retail the same in federal money, to gain 15 per cent. on the first cost? *Ans.* \$1.27. +

Statement,  $\frac{87. 40. 115}{350. 9. 100}$  The 87 in the statement = 75 £. + 12 £.

3. Received from London 470 yards of dimity, which, including transportation, cost me 65 £.; sold the same by the yard so as to gain 30 per cent. on the first cost; how did I sell it? *Ans.* \$0.799+ per yard.

4. Received from Dublin 600 yards of Irish linen, the whole cost of which was 75 £. Irish currency; how must I retail the same in federal money, to gain  $12\frac{1}{2}$  per cent.? *Ans.* \$0.576+ per yard.

5. My agent in Dublin has forwarded to me 900 yards of linen; whole cost 60 £. Irish currency; how must I retail the same in federal money, to gain 15 per cent.? *Ans.* \$0.314.+

6. I have in my store 120 yards of broadcloth, forwarded me by my agent in Paris, which cost me, including transportation, 325 crowns; how must I sell the same in federal money, to gain 16 per cent.? *Ans.* \$3.455+ per yard.

7. Received 680 yards of silk from Paris, for which I paid 560 crowns; expenses of transportation, 12 crowns; how must I sell the same in federal money, to gain 30 per cent.? *Ans.* \$1.20+ per yard.

8. Received from Madrid 6 hogsheads of wine, each containing 63 gallons, for which my agent paid 188 Spanish dollars; how must I sell the same per gallon, to gain  $12\frac{1}{2}$  per cent.? *Ans.* \$0.559.+

9. I have on hand a bale of silk, containing 174 yards, which I received from Cadiz, at a cost, including transportation, of 140 piastres or Spanish dollars; how must I sell the same per yard, to gain 5 per cent.? *Ans.* \$0.844.+

10. Received from Oporto 3 hogsheads of port wine, containing 63 gallons each; cost, including transportation, 30 milrees per hogshead; how must I retail the same by the gallon, to gain 25 per cent.? *Ans.* \$0.738.+

11. How must I sell broadcloth by the yard, in federal money, of which 3 pieces, each containing 35 yards, cost me 135 £. sterling, to gain 30 per cent.? *Ans.* \$7.428.+

12. Received from A. B., Dublin, 560 yards of linen; whole cost 90 £. Irish currency; what must be the retail price, in federal money, to gain 15 per cent.? *Ans.* \$0.757+ per yard.

13. Received from the same 600 yards of muslin, worth 56 £.; how must I sell the whole quantity in federal money, to gain 5 per cent.? *Ans.* \$241.08.

14. Consigned to my agent, J. Jones, of London, 300 barrels of flour, for which I paid \$1500; how many pounds sterling ought he to receive for the same, to gain 10 per cent., the expense of transportation being \$50? *Ans.* 383 £. 12 s. 6 d.

15. Received of my agent in London, J. Jones, 2510 gallons of Madeira wine, which cost me, per invoice, 1640 £. sterling; but it being of an inferior quality, I am willing to lose 5 per cent. on the cost; what must be the price per gallon, in federal money? *Ans.* \$2.758.+

16. Three men trading in company, received from France 1200 bottles of champagne, for which they paid 600 French guineas, each \$4.60; how must they sell the same per bottle, in federal money, to gain 40 per cent., and what will be each man's gain per bottle? *Ans.* \$3.22 per bottle; each man's gain per bottle, \$0.306.+

17. Received 360 ells of cloth from Hamburg, which cost me 1500 mark bancos; how must the same be sold in federal money, by the yard, to gain  $12\frac{1}{2}$  per cent., the ell Hamburg being  $2\frac{1}{2}$  qr.? *Ans.* \$1.17.+

18. New York, Jan. 6, 1838. This day received from Amsterdam, 600 yards of carpeting; whole cost, 2400 guilders. Required the retail price in federal money, to gain 20 per cent. *Ans.* \$1.92 per yard.

19. Shipped to London 380 barrels of flour, which cost me, including transportation, \$6 per barrel. How many English crowns must I receive for the whole quantity, to gain 10 per cent. *Ans.* 2280 crowns.

20. Shipped to Dublin 3000 bushels of flax-seed, which cost me \$2500. How many pounds, Irish currency, must I receive for the whole quantity, to gain 5 per cent.? *Ans.* 640 £. 4 s. 10 d. 2 qr.+

21. Boston, Jan. 16, 1835. This day received from my agent at Lisbon, 16 hogsheads of wine, each 65 gallons; whole cost, 640 milrees. The whole being of an inferior quality, I am willing to lose 6 per cent. on the cost. How much in federal money must I charge per gallon? *Ans.* \$0.72, nearly.

22. New York, Sept. 6, 1835. This day received from A. B., London, 1200 yards of superfine broadcloth; whole cost, 1500 £. How must I sell the same in federal money, at wholesale, to realize a profit of 20 per cent.? *Ans.* \$8000.

23. Received from Russia a quantity of fur; whole cost, 900 silver rubles. What must be the wholesale price in federal

money, to make an advance of 15 per cent. on the cost? *Ans.* \$776.25.

24. Boston, Feb. 26, 1836. This day received from my agent in Paris, 24 hogsheads French wine, each 60 gallons; whole cost, 288 guineas. How must the same be retailed by the quart, in federal money, to gain  $12\frac{1}{2}$  per cent.? *Ans.* \$0.258.+

25. Shipped to my agent in London, 650 barrels of flour; whole cost, \$3600. How many pounds sterling must I receive for the same, to gain 10 per cent. on the cost? *Ans.* 891 £.

QUESTIONS.—What operations are included under this rule? How is a foreign currency reduced to federal money? How is the operation performed, when it is required to find how goods, the value of which is given in a foreign currency, must be sold to gain or lose a certain per cent. in federal money? What is the rule for canceling? What is Note 1st? What is Note 2d?

## TARE AND TRET.

We come now to consider the allowances to be made in the purchase of goods by weight. The following particulars require to be first noticed:

*Gross Weight* is the whole weight of the goods purchased, including that of the box, barrel, bag, &c. containing them.

*Draft* is a deduction from the gross weight made in favor of the buyer.

*Tare* is an allowance made for cask, box, barrel, &c. containing the goods; and may be either a certain deduction from the whole quantity, or so much per box, &c.

*Tret* is an allowance of 4 lb. for every 104 lb. made for the dust, &c.

*Suttle* is what remains after some of the preceding allowances have been made.

*Net Weight* is what remains after all the deductions have been made.



**Ex. 1.** Bought a hogshead of sugar, weighing 7 cwt. 2 qr. 26 lb., tare on the whole, 3 qr. 24 lb. What is the net weight?

|      |     |     |
|------|-----|-----|
| cwt. | qr. | lb. |
| 7    | 2   | 26  |
| 3    | 24  |     |
|      |     |     |

6 3 2 Net weight.

2. What is the net weight of 12 casks of raisins, each weighing 2 cwt. 2 qr. 14 lb., tare per cask, 12 lb.?

2 cwt. 2 qr. 14 lb.  $\times 12 = 31$  cwt. 2 qr., the gross weight.  $12 \times 12 = 144$  lb.  $= 1$  cwt. 1 qr. 4 lb.; and 31 cwt. 2 qr.  $- 1$  cwt. 1 qr. 4 lb.  $= 30$  cwt. 0 qr. 24 lb. *Ans.*

3. What is the net weight of 6 casks of prunes, each weighing 3 cwt. 2 qr. 10 lb., tare 20 lb. per cask? *Ans.* 20 cwt. 1 qr. 24 lb.

4. What is the net weight of 44 cwt. gross, if 14 lb. per cwt. be allowed for tare?  $44 \times 14 = 616$  lb.  $= 5$  cwt. 2 qr., and 44 cwt.  $- 5$  cwt. 2 qr.  $= 38$  cwt. 2 qr. *Ans.*

Or the solution may be effected by canceling, by the following rule:

**RULE.**—Place the whole gross weight first above a horizontal line. Then place 112 lb. below the line, with 112 diminished by the tare per cwt., standing directly above it. Cancel, &c.

The above sum solved by this rule:

$$112 - 14 = 98. \quad \frac{44. 98}{112} \quad \text{Canceled, } \frac{11 \ 14 \ 7}{44. 98} \\ \frac{112 \ 4}{28 \ 2}$$

and  $11 \times 7 = 77$ , and  $77 \div 2 = 38$  cwt. 2 qr.

5. Bought 84 cwt. of sugar. What is the net weight, if 20 lb. per cwt. be allowed for tare? *Ans.* 69 cwt.

6. Bought 9 hogsheads of sugar, each weighing 8 cwt. 2 qr. From this, a deduction of 16 lb. per cwt. was made for tare. What was the net weight? *Ans.* 65 cwt. 2 qr. 8 lb.

**Note 1st.**—When the price per cwt. or per lb. is given, the reduction for tare and tret may be made, and the whole cost ascertained by a single statement, as may be seen from the following example:

7. What is the value of 8 hogsheads of sugar, each weighing 12 cwt. gross, tare 12 lb. per cwt., at \$8.50 per cwt.?

$$\begin{array}{r} \text{Statement, } \frac{8. \ 12. \ 100. \ 8.50}{112.} \quad \text{Canceled, } \frac{\overset{2}{8.} \ \overset{3}{12.} \ 100. \ 8.50}{112. \ 28.} \\ \phantom{\text{Statement, }} \phantom{\frac{8. \ 12. \ 100. \ 8.50}{112.}} \phantom{\text{Canceled, }} \phantom{\frac{\overset{2}{8.} \ \overset{3}{12.} \ 100. \ 8.50}{112. \ 28.}} \\ \phantom{\text{Statement, }} \phantom{\frac{8. \ 12. \ 100. \ 8.50}{112.}} \phantom{\text{Canceled, }} \phantom{\frac{\overset{2}{8.} \ \overset{3}{12.} \ 100. \ 8.50}{112. \ 28.}} \phantom{7} \end{array}$$

and  $8.50 \times 100 \times 3 \times 2 \div 7 = \$728.57$ , *Ans.*

8. Bought 32 casks of figs, each weighing 2 cwt. 2 qr., at a deduction of 18 lb. per cwt. for tare. What did the whole cost me, at \$4 per cwt. net weight? *Ans.* \$268.57.

9. Bought 15 cwt. of sugar at \$6.50 per cwt. net weight. Reduction for tare, 12 lb. per cwt.; tret, 4 lb. per 104 lb. How must I sell the whole, to gain 20 per cent. on the first cost, and how must I retail it, to gain the same per cent.? *Ans.* Wholesale price, \$100.446; retail price, \$0.059, nearly.

To effect all the reductions and the gain per cent. of the preceding sum, a single statement only is required; thus:

$$\frac{15. \ 6.50. \ 100. \ 100. \ 120}{112. \ 104. \ 100.}$$

10. Bought 32 chests of tea, each weighing 4 cwt. 2 qr. at \$49 per cwt. net weight; tare, 12 lb. per cwt.; tret, 4 lb. per 104 lb. How must I sell the whole quantity to gain 20 per cent., and how must the same be retailed, to gain the same? *Ans.* Wholesale price, \$7269.23; retail price, \$0.525.

11. Bought 742 lb. of wool, and was allowed a deduction of 5 per cent. from the gross weight for dust, &c. For the net weight I paid 9 s. New York currency, per pound, and was allowed a deduction of 6 per cent. on the whole cost for ready money. I then sold the same so as to realize a profit of 20 per cent. on the money I advanced. How much did I receive for the whole? *Ans.* \$900.

12. Purchased 5 cwt. of sugar; tare allowed, 8 lb. per cwt. For the net weight I paid 6 d. New York currency, per lb. How must I sell the whole quantity to gain 20 per cent., and how must the same be retailed, to gain the same per cent.? *Ans.* Wholesale price, \$39; retail price, \$0.07½ per pound.

13. Purchased 12 bags of coffee, each weighing 96 lb.; tare per bag, 6 lb. What was the whole cost at 30 cents per lb. and the retail price, to gain 25 per cent.? *Ans.* \$324 whole cost; \$0.37½ retail price.

14. How much will 8 hogsheads of sugar, each weighing

8 cwt. 3 qr., cost at \$9 per cwt. if a deduction of 12 lb. per cwt. be allowed for tare; and what will be received for the whole, if it be sold at an advance of 30 per cent.? *Ans.* \$562.50 cost, and \$731.25 received.

15. What is the net weight of 3 tierces of rice, each weighing 4 cwt. 3 qr. gross; the tare allowed, 16 lb. per cwt.; tret, 4 lb. per 104 lb.? *Ans.* 11 cwt. 2 qr. 27 lb. +

16. What is the cost of 15 chests of tea, each containing 140 lb. gross, at 5 s. New York currency, per lb.; tare, 16 lb. per cwt. *Ans.* \$1125.

17. Bought 12 hogsheads of sugar, each weighing 10 cwt. 2 qr. at \$9 per cwt. net weight. Deduction for tare, 12 lb. per cwt. How much must I receive for the whole quantity, to gain 10 per cent. on the cost? *Ans.* \$1113.75.

18. Bought 16 firkins of butter, each weighing 108 lb.; reduction for tare, 8 lb. per cwt. Paid 15 pence, New England currency, per pound. What did it cost me; and what must be the wholesale, and what the retail price, to gain 20 per cent. on the first cost? *Ans.* Whole cost, \$334.285; + wholesale price, \$401.142; retail price, \$0.25.

19. Bought 18 cwt. of sugar, at \$12 per cwt. net weight; tare, 16 lb. per cwt. How must I sell the same per lb. to gain 12 per cent. on the first cost? *Ans.* \$0.12.

20. Purchased in London, 16 cwt. of tea at 28 £. sterling per cwt. net weight; tare, 12 lb. per cwt. How much must I receive in federal money, for the whole quantity, to realize a profit of 12 per cent., and what retail price will allow the same profit? *Ans.* Wholesale price, \$1991.11; retail price, \$1.24.

**Questions.**—What is Tare and Tret? What is gross weight? What is draft? What is tare? What is tret? What is suttle? What is net weight? What is the rule? What is the note?

## EQUATION OF PAYMENTS.

Equation of Payments is the method of finding a mean time for the payment of several debts due at different periods of time.

Ex. 1. I owe a friend \$380, to be paid as follows, viz. \$100 in 6 months; \$120 in 7 months; and \$160 in 10 months. If I pay the whole at once, at what time must the payment be made, so that neither I nor my friend shall lose interest?

### SOLUTION.

The interest of \$100 for 6 mo. = the interest of \$1 for 600 mo.

The interest of \$120 for 7 mo. = the interest of \$1 for 840 mo.

The interest of \$160 for 10 mo. = the interest of \$1 for 1600 mo.

Amt. of pay'ts, \$380

3040 mo.

3040 = the months requisite for \$1 to gain as much interest as \$380 would gain in the required time. Therefore, \$380 : \$1 :: 3040 months : to the required time, viz. 8 months.

The 600, 840, and 1600 months are obviously obtained by multiplying the several payments by the time which must elapse before they severally become due.

We therefore have the following rule:

**RULE.**—*Multiply each payment by the time which must elapse before it becomes due, and divide the sum of the products by the sum of the payments.*

2. A. owes me \$50, payable in 4 months; \$100, payable in 10 months; and \$150, payable in 16 months. In what time must he pay the whole, so that neither shall lose interest?  
*Ans.* 12 months.

3. What is the equated time of payment for the three following sums, viz. \$500, payable in 3 years; \$400, payable in 4 years; and \$600, payable in 5 years? *Ans.*  $4\frac{1}{2}$  years.

4. What is the equated time of payment for the three following sums, viz. \$50, payable in 4 months; \$75, payable in 6 months; and \$100, payable in 7 months? *Ans.* 6 months.

5. A. owes B. \$400; of which \$80 is payable in 6 months; \$120 in ten months; and the remainder in 1 year and 8 months. What is the equated time of payment? *Ans.* 1 year, 2 months, and 6 days.

6. A merchant has a certain sum of money due him, of which  $\frac{1}{3}$  is payable in 2 months;  $\frac{1}{4}$  in 4 months; and the remainder in 6 months. What is the equated time for the payment of the whole? *Ans.*  $4\frac{1}{3}$  months.

7. I owe four sums of money, payable as follows, viz. \$60 payable in 9 months; \$80 in 10 months; \$50 in 11 months; and \$60 in 12 months. At what time may I pay the whole, without loss? *Ans.*  $10\frac{11}{13}$  months.

8. A person owes a debt of \$2000, payable in 7 months, of which he proposes to pay \$600 down, on condition that the remainder be allowed to remain unpaid an adequate term of time. In what time ought it to be paid? *Ans.* 10 months.

QUESTIONS.—What is Equation of Payments? What is the rule?

## DUODECIMALS.

*Duodecimals are fractions of a foot.* The unit or foot is first supposed to be divided into 12 equal parts, called inches or primes, and marked '. Each inch or prime is then divided into 12 equal parts, called seconds, and marked ". Each second is again divided in like manner, and each part obtained by this division is called a third, and marked "' . A foot is therefore divided duodecimally, when it is separated into 12 equal parts, or when the several divisions sustain a twelve-fold relation to each other. This relation may be thus illustrated:

1' inch or prime is  $\frac{1}{12}$  of a foot.  
 1" second is  $\frac{1}{12}$  of  $\frac{1}{12}$ ,  $\frac{1}{144}$  of a foot.  
 1''' third is  $\frac{1}{12}$  of  $\frac{1}{12}$  of  $\frac{1}{12}$ ,  $\frac{1}{1728}$  of a foot.  
 1'''' fourth is  $\frac{1}{12}$  of  $\frac{1}{12}$  of  $\frac{1}{12}$  of  $\frac{1}{12}$ ,  $\frac{1}{20736}$  of a foot.  
 1''''' fifth is  $\frac{1}{12}$  of  $\frac{1}{12}$  of  $\frac{1}{12}$  of  $\frac{1}{12}$  of  $\frac{1}{12}$ ,  $\frac{1}{248832}$  of a foot.  
 1'''''' sixth is  $\frac{1}{12}$  of  $\frac{1}{12}$  of  $\frac{1}{12}$  of  $\frac{1}{12}$  of  $\frac{1}{12}$  of  $\frac{1}{12}$ ,  $\frac{1}{2985984}$  of a foot, &c.

The marks ', " , ' , ' , &c. are the indices of the several denominations.

### TABLE OF DENOMINATIONS.

|                           |           |                   |
|---------------------------|-----------|-------------------|
| 12'''' sixths make        | - - - - - | 1'''' fifth.      |
| 12'''' fifths make        | - - - - - | 1'''' fourth.     |
| 12'''' fourths make       | - - - - - | 1'''' third.      |
| 12''' thirds make         | - - - - - | 1'' second.       |
| 12'' seconds make         | - - - - - | 1' prime or inch. |
| 12' inches or primes make | - - - - - | 1 foot.           |

Duodecimals may be added or subtracted in the same manner as compound numbers; the denominations decreasing or increasing in the constant ratio of 12. These operations are so simple that it is unnecessary to introduce any examples. The principle is the same as in Compound Addition and Subtraction.

## MULTIPLICATION OF DUODECIMALS.

The greatest difficulty the scholar will here encounter, will be to determine *the denomination of the product of any two denominations*. Suppose it be required to multiply 6' inches or primes by 4' inches, their product is obviously 24, but of what denomination is it? By recurring to the preceding tables, it will be seen that 6' inches =  $\frac{6}{12}$  of a foot, and 4' inches =  $\frac{4}{12}$  of a foot; and  $\frac{6}{12} \times \frac{4}{12} = \frac{24}{144}$ , that is, 24''. Hence, inches multiplied by inches produce seconds. Again, let it be required to multiply 9'' seconds by 3' inches. 9 seconds =  $\frac{9}{12}$ , and 3 inches =  $\frac{3}{12}$ ; therefore,  $\frac{9}{12} \times \frac{3}{12} = \frac{27}{144}$ , or 27'''. Lastly, what is the product of 7'' seconds multiplied by 9'' seconds. 7'' seconds =  $\frac{7}{12}$ , and 9'' seconds =  $\frac{9}{12}$ , and  $\frac{7}{12} \times \frac{9}{12} = \frac{63}{144}$ , or 63'''' fourths. From the above we draw the following conclusions, viz. *that feet multiplied by feet produce square feet; feet multiplied by inches produce inches; inches or primes multiplied by inches, give seconds; seconds multiplied by seconds,*

give fourths; seconds multiplied by thirds, give fifths, &c.; that is, the product of any two denominations will always be of the denomination expressed by the sum of their indices.

**RULE.**—Begin with the lowest denomination of the multiplicand, and multiply it by the highest denomination of the multiplier, and place each term of the product according to its respective value. Multiply in the same manner by each remaining denomination of the multiplier, and place the product of each succeeding multiplication one or more places farther to the right, according to the denomination. The several products thus obtained, when added together, will give the required product.

**Note.**—It will be remembered to carry by 12 in all cases.

**Ex. 1.** Multiply 4 feet 4 inches, by 4 feet 4 inches.

OPERATION.

| ft.    | in.   |
|--------|-------|
| 4      | 4'    |
| 4      | 4'    |
| <hr/>  |       |
| 17     | 4'    |
| 1      | 5' 4" |
| <hr/>  |       |
| 18 ft. | 9' 4" |

Beginning with the 4 feet in the multiplier, we say 4 times 4' are 16' = 1 ft. 4'. The 4' is set down and the 1 ft. carried to the next product, making it 17 feet. Then multiplying by 4", we say 4' × 4" = 16" = 1' and 4"; set down the 4" and carry the 1', and say 4' times 4 are 16' and 1' is 17' = 1 ft. and 5',

which being set down, and the two products added, we obtain 18 ft. 9' 4" as the required product.

|       | 2.      |
|-------|---------|
| Mult. | ft. in. |
| 8     | 6'      |
| by 4  | 3'      |
| <hr/> |         |
| 34    | 0'      |
| 2     | 1' 6"   |
| <hr/> |         |

Prod. 36 1' 6"

|       | 3.      |
|-------|---------|
| Mult. | ft. in. |
| 9     | 10'     |
| 7     | 6'      |
| <hr/> |         |
| 68    | 10'     |
| 4     | 11' 0"  |
| <hr/> |         |

Prod. 73 9' 0"

|       | 4.      |
|-------|---------|
| Mult. | ft. in. |
| 3     | 8'      |
| 7     | 6'      |
| <hr/> |         |
| 27    | 6'      |

|       | 5.      |
|-------|---------|
| Mult. | ft. in. |
| 7     | 3'      |
| 4     | 7'      |
| <hr/> |         |

33 2' 9"

|       | 6.      |
|-------|---------|
| Mult. | ft. in. |
| 3     | 11'     |
| 9     | 5'      |
| <hr/> |         |

36 10' 7"

|       | 7.      |
|-------|---------|
| Mult. | ft. in. |
| 4     | 6'      |
| 5     | 8'      |
| <hr/> |         |

25 6'

|       | 8.      |
|-------|---------|
| Mult. | ft. in. |
| 9     | 7'      |
| 3     | 6'      |
| <hr/> |         |

33 6' 6"

| 9.    |     |     |       | 10.   |     |      |      |      |
|-------|-----|-----|-------|-------|-----|------|------|------|
| ft.   | in. |     |       | ft.   | in. |      |      |      |
| 6     | 4'  | 3'' |       | 3     | 9'  | 11'' |      |      |
| 4     | 6'  | 4'' |       | 4     | 2'  | 3''  |      |      |
| <hr/> |     |     |       | <hr/> |     |      |      |      |
| 28    | 9'  | 2'' | 11''' | 16    | 0'  | 3''  | 3''' | 9''' |

11. How many square feet does a board 28 ft. 10' 6" long, and 3 ft. 2' 4" wide contain? *Ans.* 92 ft. 2' 10" 6'''.

12. In a board 16 ft. 9' long, and 2 feet 3 broad, how many square feet? *Ans.* 37 ft. 8' 3''.

13. There is a wall 82 ft. 6 in. high, and 13 ft. 3 in. wide. How many square feet does it contain? *Ans.* 1093 ft. 1' 6''.

14. There is a room 20 feet square and 7 ft. 6 in. high, to be plastered at 10 d. New York currency, per square yard. How many dollars will it cost? *Ans.* \$6.94. +

15. There is a yard 58 ft. 6 in. in length, and 54 ft. 9 in. in breadth. How many dollars will it cost to pave it, at 5 d. New York currency, per square yard? *Ans.* \$18.53.

16. If a floor be 59 feet 9 inches long, and 24 feet 6 inches broad, how many square yards does it contain? *Ans.* 162 yards, 5 ft. 10' 6''.

*Note 2d.*—If three dimensions, viz. length, breadth, and depth be given, the solid content is found by multiplying them successively into each other.

17. There is a pile of wood 12 feet 6 inches long, 4 feet high, and 8 feet 6 inches wide. How many cords does it contain? *Ans.* 3 cords, 41 feet.

To reduce solid feet to cords, divide by 128, that being the number of solid feet in one cord. The required dimensions of the cord, are 8 feet long, 4 feet wide, and 4 feet high; since  $8 \times 4 \times 4 = 128$ .

18. How many solid feet are there in a block 6 feet 8 inches in length, 4 feet 6 inches in height, and 3 feet 4 inches in width? *Ans.* 100 feet.

19. There is a certain pile of wood measuring 24 feet in length, 16 feet 9 inches in depth, and 12 feet 6 inches in width. How many cords are there; and how many solid feet may be daily consumed to have it last one year? *Ans.* 39 cords, 33 feet; daily allowance,  $13\frac{3}{4}$  feet, nearly.



20. How many square feet are there in a board, which measures 16 feet, 9 inches in length, and 2 feet, 3 inches in breadth?  
*Ans.* 37 feet, 8' 3".

**QUESTIONS.**—What are Duodecimals? How is the foot divided? What is each part called? How is the inch divided? What is each part called, &c.? When is the foot divided duodecimally? Repeat the table of denominations. How may duodecimals be added? What difficulty will be encountered in Multiplication of Duodecimals? Give an illustration of the difficulty. Of what denomination will the product of any two denominations be? What is the rule? What is Note 1st? What is Note 2d?

## INVOLUTION.

A number is involved by being *multiplied into itself*.

The number thus multiplied by itself is called the *root*.

A *power* of any number is the product obtained by multiplying that number into itself. The particular power produced depends on *the number of successive multiplications*; the given number always being the first power, and also the root of the succeeding higher powers. The first multiplication then produces the second power; the second multiplication, the third power; the third multiplication, the fourth power, &c., *the power obtained being always one in advance of the number of multiplications*.

**Illustration:** 2 = the first power, and is also the root of the succeeding higher powers.

$2 \times 2 = 4$ , the 2d power, or square of 2.

$2 \times 2 \times 2 = 8$ , the 3d power, or cube of 2.

$2 \times 2 \times 2 \times 2 = 16$ , the 4th power, or biquadrate of 2.

$2 \times 2 \times 2 \times 2 \times 2 = 32$ , the 5th power of 2, &c.

The power to which a number is to be raised, is frequently expressed by a small figure, called *the index of the required power*, placed on the right of that number, thus:  $4^2$  denotes the second power of 4 = 16; and  $4^3$  denotes the third power of 4 = 64; and  $9^5$  denotes the fifth power of 9 = 59049, &c.

A fraction is involved by multiplying the numerator and the denominator each into itself, the required number of times; thus the square of  $\frac{2}{3}$  is  $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$ . The square of  $\frac{5}{7}$  is  $\frac{25}{49}$ ; and the cube of the same is  $\frac{125}{343}$ , &c.

If the given quantity be a mixed number, it should first be reduced to an improper fraction, before being involved; thus the second power of  $2\frac{1}{2}$  is  $2\frac{1}{2} = \frac{5}{2}$ , and  $\frac{5}{2} \times \frac{5}{2} = \frac{25}{4} = 6\frac{1}{4}$ . Or, proper and improper fractions may both be reduced to decimals, and then involved.

If any number be raised to two different powers, the power which is obtained by multiplying these two powers together, is expressed by adding their indices, thus:  $2^2 \times 2^3 = 2^5 = 32$ ; for  $2^2 = 4$ , and  $2^3 = 8$ , and  $8 \times 4 = 32$ ; and  $2 \times 2 \times 2 \times 2 \times 2 = 32$ . Or,  $3^2 \times 3^5 = 3^7 = 6561$ , for  $3^2 = 9$ , and  $3^5 = 243$ ; and  $243 \times 27 = 6561$ .

Or, again, any power of a given number is divided by another power of the same number by subtracting the index of the divisor from the index of the dividend, thus:  $2^5 \div 2^3 = 2^2$ , for  $2^5 = 32$ , and  $2^3 = 8$ , and  $32 \div 8 = 4$ , the second power of 2. Or  $3^4 \div 3^2 = 3^2$ , for  $3^4 = 81$ , and  $3^2 = 9$ , and  $81 \div 9 = 9$ , the second power of 3.

Ex. 1. What are the square, cube, and biquadrate of 3? *Ans.*  $3 \times 3 = 9$ , the square;  $3 \times 3 \times 3 = 27$ , the cube; and  $3 \times 3 \times 3 \times 3 = 81$ , the biquadrate.

2. What are the square, cube, and biquadrate of 5? *Ans.* 25, 125, and 625.

3. What are the cube and biquadrate of 12? *Ans.* 1728 and 20736.

4. Multiply the second and third powers of 4 together. What is the product, and what power of 4 is it? *Ans.* The product, 1024, or fifth power.

5. What power of 3 is obtained by multiplying its third power and its fourth power together; and what is the number? *Ans.* 7th power, or 2187.

*Note.*—When the number to be raised to some given power consists of whole numbers and decimals, the number of decimals to be cut off in the required power is ascertained by multiplying the number of decimals in the given number by the index of the required power.

6. What is the square of 26.13?  $26.13 \times 26.13 = 682.7769$ . Now to determine how many decimals are to be cut off, we

first notice that the number of decimals in the given number is two, and also, that the index of the required power is 2, therefore,  $2 \times 2 = 4$ , the number of decimals to be cut off. Therefore, 682.7769 is the required power.

7. What is the cube of 25.4? *Ans.* 16387.064.

8. Divide  $2^6$  by  $2^3$ , and what power of 2 will be obtained? *Ans.* 8, the cube of 2.

## EVOLUTION.

Evolution is the reverse of Involution. In Involution we have the *root* given to find some required power; but in Evolution a *power* is given, and a *root* required.

The relation between roots and powers requires to be clearly understood.

A *root* of a number is obtained whenever that number is resolved into several equal factors; and a *power* of a number is obtained whenever that number taken as a root, is multiplied into itself once or more. Thus, 2 is the cube root of 8, because it may be resolved into three 2's; or because 2 raised to its third power equals 8, for  $2 \times 2 \times 2 = 8$ . Also, 8 is the square root of 64, because the second power of 8 is 64, or  $8 \times 8 = 64$ . Again, 9 is the square of 3, and 3 is the square root of 9; 27 is the cube of 3, and 3 is the cube root of 27. Roots and powers are therefore correlative terms.

The exact root of some numbers cannot be obtained. Such numbers are called *irrational powers*, and their roots are called *surd*s. Thus, no root can be obtained, which, when multiplied into itself, will produce 2; 2 is therefore an irrational power, and its root is a surd. But a number whose root can be exactly extracted is a *perfect* or *complete power*, and its root is called a *rational number*. Thus, 16 is a complete power, for 4 is its exact root; 4 therefore is a rational number.

There are two methods of expressing roots. The first and more common method is, by using the character called the *radical sign*; written thus,  $\sqrt{\quad}$ . This sign, without any accom-

panying index, always indicates the square root. If other roots are required, the same radical sign is used, with an index of the required root. Thus,  $\sqrt{9}$ , is an expression for the square root;  $\sqrt[3]{9}$ , for the cube root;  $\sqrt[4]{9}$ , for the fourth root of 9, &c.  $\sqrt{64}$ , equals 8, because  $8 \times 8 = 64$ ; and  $\sqrt[3]{64} = 4$ , because  $4 \times 4 \times 4 = 64$ , or  $\sqrt[4]{64} = 2$ , for  $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$ , &c. Hence the root is to be taken as a factor in producing its corresponding power, as many times as there are units in the index of the required root. The other mode of expressing roots is by means of fractional exponents. Thus,  $6^{\frac{1}{2}}$  expresses the square root;  $6^{\frac{1}{3}}$ , the cube root, and  $6^{\frac{1}{4}}$ , the biquadrate or fourth root of 6. The chief advantage of this mode arises from the fact, that not only *roots* of numbers may be expressed by it, but also *any required power of a given root*. The denominator of a fractional index always denotes a *root* of the quantity to which it is applied, while the numerator expresses *some power of that root*; thus,  $9^{\frac{3}{4}}$  implies that the fourth root of 9 is to be extracted, and that root raised to its third power. Again,  $64^{\frac{5}{6}}$  implies that the sixth root of 64 is to be extracted, and the root then raised to its fifth power. But the sixth root of 64 is 2, and the fifth power of 2 is 32; therefore,  $64^{\frac{5}{6}} = 32$ . Or a power higher than the given root may be expressed; thus,  $16^{\frac{3}{2}}$ , implies the third power of the square root of 16; but the square root of 16 is 4, and the third power of 4 is 64; therefore,  $16^{\frac{3}{2}} = 64$ .

When several numbers are to be added and the root of the sum obtained, it may be expressed thus:  $\sqrt{65+16}$ ; which implies that the root of the sum of 65 and 16 is to be obtained. If the vinculum over the two numbers be rejected, the expression would imply, that 16 is to be added to the square root of 65. As the expression now stands, its value is  $65+16=81$ , and  $\sqrt{81}=9$ . Or the root of the difference of two quantities may be expressed in like manner, by placing the minus sign between them; thus,  $\sqrt{90-26}$ , the value of which is  $90-26=64$ , and  $\sqrt{64}=8$ . Without the vinculum over the two quantities, the expression would imply that 26 is to be taken from the square root of 90.

*The root of the product of several numbers is equal to the*

*product of their roots.* As an illustration, take 9 and 16. Their product is 144, of which the square root is 12; that is, the root of their product is 12. The product of their roots is the same, for the square of 3 is 9, and of 4 is 16; and  $4 \times 3$  is 12. The same is true of the cube roots, or of any roots whatever.

Take the numbers 8 and 27.  $\sqrt[3]{8}=2$ ,  $\sqrt[3]{27}=3$ , and  $3 \times 2=6$ , the product of their roots. Again,  $27 \times 8=216$ , and  $\sqrt[3]{216}=6$ , the root of their products.

## EXTRACTION OF THE SQUARE ROOT.

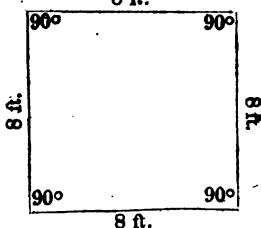
The Square Root of any number, is that number which being multiplied into itself once, will produce the number given.

The following table exhibits the square of all numbers from 1 to 12:

| Roots,   | 1 | 2 | 3 | 4  | 5  | 6  | 7  | 8  | 9  | 10  | 11  | 12  |
|----------|---|---|---|----|----|----|----|----|----|-----|-----|-----|
| Squares, | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | 100 | 121 | 144 |

A square is a figure bounded by four equal sides, and has all its angles right angles, or angles of 90 degrees. This may be seen in figure 1st. Now the area of a square, that is, the number of square feet, or rods, &c., it contains, is found by multiplying the length of any two sides together; or since the sides are all equal, by multiplying the length of any one side into itself; or by squaring it. As each of the sides of the annexed figure is 8 feet in length, it is obvious they may each be divided into 8 equal parts, each of which will be one foot in length. Let each side be thus divided and the points of division united, as

FIG. 1st.  
8 ft.



seen in figure 2d. Now since each side is 8 feet long, the divisions *aa*, *bb*, *cc*, *dd*, &c. must be just one foot each; the divisions made by the lines running at right angles to these, are also one foot each. The whole figure is, therefore, divided into 64 equal parts, each of which is just one foot square. But  $8 \times 8 = 64$ . Therefore, the area of a square is obtained by multiplying the length of the side by itself; or, in other words, by squaring it.

Fig. 2d.

|   |    |    |    |    |    |    |    |    |   |
|---|----|----|----|----|----|----|----|----|---|
| a | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | a |
| b | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | b |
| c | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | c |
| d | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | d |
| e | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | e |
| f | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | f |
| g | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | g |
| h | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | h |

Now to apply the above remarks to Evolution, suppose we have an area equal to what is given above, but placed in a different form. Suppose it to consist of a board one foot wide and 64 feet long; and that it is required to determine how large a square floor it will exactly cover.  $\sqrt{64} = 8$  feet, is the length of the side of the required floor. Hence, the area of a square is found by squaring one of its sides; and the length of its sides is found by extracting the square root of the given area.

The following is the rule for extracting the square root:

**RULE 1st.**—Separate the given number into periods of two figures each, by placing a point or dot over the place of units, another over the place of hundreds, and another over the place of tens of thousands, &c.

2d. Find by trial the greatest square root of the left hand period, and place it for the first figure of the root, after the manner of the quotient in division.

3d. Subtract the square of this root from the left hand period, and to the remainder bring down the next period of two figures for a dividend.

4th. Double the root figure already obtained, for a divisor; then, omitting the right hand figure of the dividend, divide as in Simple Division, and place the figure obtained, as the second figure of the root or quotient; and also on the right hand of the divisor.

5th. Multiply the divisor thus increased, by the figure last placed in the root, and place the product under the dividend, as in Division; then subtract, and to the remainder bring down the next period of two figures.

6th. *Double the root already found for a new divisor, with which divide as before. Continue the operation till the periods are all brought down: the number obtained will be the root required.*

*Note 1st.*—If the number, the root of which is required, consist in part of a decimal, place the first point over the unit figure, as already directed, and point off both ways from that figure, allowing two figures to each point. If the decimal consist of an odd number of figures, annex one cypher to complete the last period.

2d. If after all the periods have been brought down, there be a remainder, periods of two cyphers each may be annexed and the operation continued. The root figures obtained by thus annexing cyphers will be decimals; and must be so marked.

3d. The number of dots employed in pointing off the given number, always determines the number of figures in the required root.

Ex. 1. What is the square root of 1296?

1296 pointed according to the rule is 12<sup>96</sup>. Hence we know that the root will consist of two figures. The next step is to determine the root of 12, the left hand period. This is done by trial. If 4 be taken, it will be found too large, since  $4 \times 4 = 16$ . We will, therefore, take 3.  $3 \times 3 = 9$ , and since 9 is less than 12, it is not too large, and yet it is the greatest integral quantity, whose square is less than 12, and is therefore the root we want.

OPERATION CONTINUED.

$$\begin{array}{r} 1296(3 \\ \text{Root squared} = 9 \end{array}$$

396 remainder increased by the next period,

Now since we are to have another figure in the root, the 3 already obtained is 3 tens or 30, the square of which is  $30 \times 30 = 900$ ; the 12 is also 1200. After subtracting the 9, therefore, there remains 3, or 300, and the next period being brought down, we obtain the number 396.

The next step is to find a divisor, which by the rule is  $3 + 3 = 6$ ; therefore,

$$\begin{array}{r}
 1296(36 \\
 \underline{9} \\
 6)3
 \end{array}$$

In dividing, the right hand figure, viz. 6, is omitted.

$$\begin{array}{r}
 \text{Again, } 1296(36 \\
 \underline{9} \\
 66)396 \\
 \underline{396} \\
 000
 \end{array}$$

The last root figure is placed on the right of the divisor, making it 66, and the whole is multiplied by the root figure, that is,  $66 \times 6 = 396$ . I then subtract and nothing remains. Therefore, 36 is the root required.

The operation is proved by squaring the root; thus,  $36 \times 36 = 1296$ .

If the given number had consisted of three or four periods instead of two, the operation would have been continued by bringing down another period, and then doubling the root 36 for a new divisor.

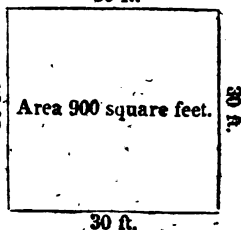
*Explanation.*—We will suppose the 1296 in the preceding operation to be so many feet of boards one foot in breadth; and that it is required to know how large a floor, exactly square, they will cover.

As has already been said, the 12 in the number 1296 is 1200, and the root 3 is so many tens, or 30. This, therefore, is the length in feet of one side of a square floor, which 1200 feet of the boards will cover, and leave a remainder of 3 or 300.

Now to find the area of a square, (see fig. 3d,) we multiply the length of any two sides together, or square the length of one side. Therefore,  $30 \times 30 = 900$ . We have then disposed of 900 of 1296 feet given, and there remains 396 feet to be so added to this figure as to preserve its square form. This is done by making equal additions upon any two adjacent sides; and hence we see the obvious reason for doubling the root, (which is the length of one side of

FIG. 3d.

30 ft.





the square,) for a divisor. But the root, figure 3d, being-doubled, gives 6 for a divisor, and this is contained in the remaining figures, 396, 6 times, (the right hand figure, viz. 6, being omitted, agreeably to the rule.) Now this figure, 6, expresses the breadth of the addition which the remaining feet of boards are sufficient to make to the original square, as seen at fig. 3d. This addition is seen at fig. 4th.

This diagram is not a perfect square; a corner remains to be filled up. We will, however, before completing it, ascertain how many of 396 feet, that remained after the first square of 900 feet was completed, are here disposed of. The length of each addition being 30 feet, and the breadth 6 feet, the area is  $30 \times 6 = 180$  sq. feet; and the area of both, consequently, is  $180 + 180 = 360$ .

But  $396 - 360 = 36$ . Therefore, 36 feet still remain to be added. The scholar, by reference to the preceding diagram, will perceive that the corner, which yet remains to be filled, to complete the square, is just 6 feet from corner to corner; therefore,  $6 \times 6 = 36$ . This addition just disposes of the remaining feet of boards, and completes the square, as may be seen at fig. 5th. The area of the original square, as seen at fig.

3d, and also of the several additions made at figures 4th and 5th, disposes of the whole of the given quantity of board; for  $900 + 180 + 180 + 36 = 1296$ . But why, in dividing, is the right hand figure of the dividend omitted?

The scholar will remember that the points placed over any number, determine the number of figures in its root. (See Note 3d.) Before performing the operation, we therefore know that the required root of the preceding number will consist of two figures. The 3, or left hand figure of

FIG. 4th.

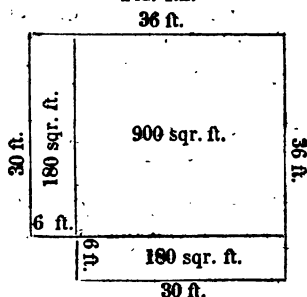
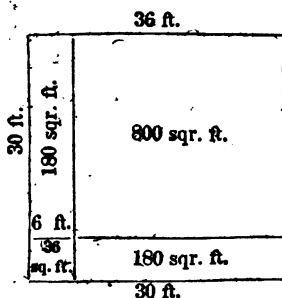


FIG. 5th.



that root, is therefore 3 tens, or 30. This number, viz. 30, is consequently the true value of the root already found, which, if doubled, will give 60 as the divisor, and not 6, as in the sum. The true value of the divisor is therefore ten times as great as represented in the operation, and hence the dividend is divided by 10; that is, its right hand figure is omitted, to make its value correspond with the apparent value of the divisor.

Another peculiar feature of the operation consists in placing the quotient, or root figure, on the right of the divisor, thereby multiplying it into itself. For this, there must also be a reason. If the scholar will examine figure 4th, he will notice a vacant corner, which, as the addition made to each of the two adjacent sides, is 6 feet, must be just 6 feet square. By placing the root figure on the right of the divisor, and multiplying it into itself, this corner is filled up and the square completed.

2. What is the square root of 9801?

$$\begin{array}{r}
 \text{OPERATION.} \\
 9801 \text{ (99 root.} \\
 81 \\
 \hline
 189 \text{ ) } 1701 \\
 \underline{1701}
 \end{array}$$

The 9 in the divisor is the last root figure, placed there by the rule.

3. What is the square root of 30138.696025?

$$\begin{array}{r}
 \text{OPERATION.} \\
 30138.696025 \text{ (173.605} \\
 1 \\
 \hline
 27 \text{ ) } 201 \\
 \underline{189} \\
 343 \text{ ) } 1238 \\
 \underline{1029} \\
 3466 \text{ ) } 20969 \\
 \underline{20796} \\
 347205 \text{ ) } 1736025 \\
 \underline{1736025}
 \end{array}$$

It will be observed from this example, that when a period is brought down, and the number obtained is not sufficient to contain the divisor, a cypher is to be placed in the root, and also on the right hand of the divisor, and the next period is then to be brought down. For arranging the points in this sum, see Note 1st.

4. What is the square root of 5499025? *Ans.* 2345.
5. What is the square root of 1522756? *Ans.* 1234.
6. What is the square root of 207936? *Ans.* 456.
7. What is the square root of 5700.25? *Ans.* 75.5.
8. What is the square root of 74770609? *Ans.* 8647.
9. What is the square root of 419112517321? *Ans.* 647389.
10. What is the square root of 10? *Ans.* 3.16227, +
11. What must be the length of each side of a square field, in rods, in order that the field may contain 40 acres? *Ans.* 80 rods.
12. A certain regiment consists of 6561 men. How many must be placed in rank and file, to form them into a square? *Ans.* 81 men.
13. A company of men spent 6 £. 13 s. 4 d., which was just as many pence for each man as there were men in the company. How many men were there, and how many pence did each man spend? *Ans.* There were 40 men, and each man spent 40 d.
14. If a man were to plant 3969 hills of corn in a square, how many rows must he have, and how many hills in a row? *Ans.* 63 of each.

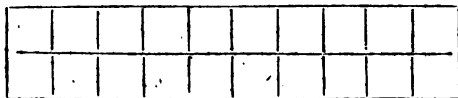
*Note 4th.*—To extract the square root of a vulgar fraction, first reduce the fraction to its lowest term, and then extract the root both of the numerator and denominator.

15. What is the square root of  $\frac{72}{128}$ ?  
 $\frac{72}{128} = \frac{9}{16}$  and the square root of 9 is 3  
 $\frac{3}{4}$  and the square root of 16 is 4, *Ans.*  $\frac{3}{4}$ .
16. What is the square root of  $\frac{98}{112}$ ? *Ans.*  $\frac{7}{4}$ .
17. What is the square root of  $\frac{112}{98}$ ? *Ans.*  $\frac{4}{7}$ .
18. There is an army containing 9216 men. How many men must be placed rank and file, to form a square which shall contain every man? *Ans.* 96.
19. What is the square root of  $\frac{7056}{2116}$ ? *Ans.*  $\frac{7}{2}$ .

## APPLICATION OF SQUARE ROOT TO PARALLELOGRAMS AND TRIANGLES.

A parallelogram is an oblong figure having its opposite sides equal and parallel. The adjoining figure represents the paral-

Fig. 6th.



lelogram. If we suppose this figure to be 10 feet long, and 2 feet broad, the area of the whole figure is evidently  $10 \times 2 = 20$  square feet. *The area of a parallelogram is therefore found by multiplying its length into its breadth.* Had the length of the above figure been 18 feet, and its breadth as given above, the area would have been  $18 \times 2 = 36$  square feet; and this equals a square figure, the sides of which are 6 feet long, for  $\sqrt{36} = 6$ . *A square, equal in area to a given parallelogram, is found by extracting the square root of the area of that parallelogram.*

20. What is the length of the sides of a square, whose area shall be equal to the area of a parallelogram 32 feet in length and 2 in breadth?

Operation:  $32 \times 2 = 64$ , and  $\sqrt{64} = 8$  feet, the side of the required square.

21. There is a parallelogram 27 feet in length and 3 in breadth. Required the sides of a square of equal dimensions. *Ans.* 9 feet.

22. Required the dimensions of a square, the area of which shall be equal to the area of a parallelogram 18 feet in length and 8 in breadth. *Ans.* 12 feet square.

*Note 5th.*—When the area of a parallelogram is given, and also the ratio of its length and breadth, the sides may be found by dividing the area by that ratio, and extracting the square root of the quotient. The root obtained will be the required breadth, by which divide the area, and the quotient will be the length.

23. If the area of a parallelogram be 288 rods, and its length be twice as much as its breadth, what is its length, and what is its breadth?

Operation:  $288 \div 2 = 144$  and,  $\sqrt{144} = 12$ , the required breadth; and  $288 \div 12 = 24$ , the required length. The reason of the above operation is obvious. The length being twice its breadth, if the parallelogram be divided in the middle, the two parts will be equal and square, and the square root of one of these parts will evidently be the breadth of the parallelogram. The same reasoning may be applied to parallelograms of any dimensions.

24. There is a field containing 30 acres, lying in the form of a parallelogram, of which the length is three times the width. What is its length, and what is its breadth? *Ans.* The length is 120 rods, and the breadth 40 rods.

*Note 6th.*—The side of a square of equal area with a geometrical figure of any form, may be found by extracting the square root of the area of that figure.

25. I have an irregular piece of land containing  $50\frac{1}{2}$  acres, which I am desirous to exchange for an equal number of acres lying in a square form. What must be the length of the sides of that square? *Ans.* 90 rods.

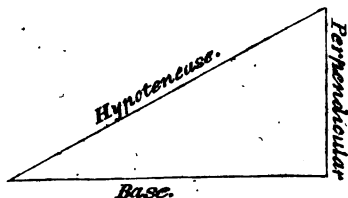
26. A certain triangular field contains 10 acres of land. What is the length of the sides of a square containing the same number of acres? *Ans.* 40 rods.

The principle of the square root may be applied to find the length of the sides of a right angled triangle, the measure of either two being given.

The square of the hypotenuse or side opposite the right angle, is always equal to the sum of the squares of the base and perpendicular. Therefore,

If the base and perpendicular be given, and the hypotenuse required, *square the given sides, and extract the square root of their sum.*

If the hypotenuse, and one of the legs of the triangle be given, to find the other leg, *square the hypotenuse, and from its square subtract the square of the given side. The square root of the remainder will be the length of the required side.*



27. There is a wall 15 feet high, and in front of it is a pavement 24 feet wide. How long a ladder is required to reach from the outside of the pavement to the top of the wall? The hypoteneuse is required, therefore,  $15 \times 15 = 225$ ; and  $24 \times 24 = 576$ ; then,  $225 + 576 = 801$ ; and  $\sqrt{801} = 28.3$ , + *Ans.*

28. A certain tree is broken off 8 feet from the ground, and, resting on the stump, touches the ground at the distance of 12 feet. What is the length of the part broken off? *Ans.* 14.42 + feet.

29. There is a fort standing by the side of a river, 24 yards high, and a line 36 yards long will just reach from the top of the fort to the opposite side of the river. What is the width of the river? *Ans.* 26.832 yards.

30. Two ships sail from the same port, one due east, and the other due north. What is the distance between them, when one has sailed 100 miles, and the other 168 miles? *Ans.* 195.5 + miles.

31. A man shot a bird sitting on the top of a steeple 80 feet high, while standing at the distance of 60 feet from its base. How far did he shoot? *Ans.* 100 feet.

32. A rope 100 feet long attached to the top of a steeple, touches the ground when drawn perfectly straight, 20 feet from its base. How high is the steeple? *Ans.* 98 feet, nearly.

33. Two boys were playing with a kite, the line of which was 520 feet in length. When the string was all out, one of them standing directly under the kite, and the other holding the string, the distance between them was 312 feet. What was the perpendicular height of the kite? *Ans.* 416 feet.

QUESTIONS.—When is a number involved? What is a root? What is a power of any number? On what does the particular power produced depend? How does the power obtained compare with the number of multiplications in producing it? How is a required power expressed? What is the figure denoting the power called? How is a fraction involved? If the given quantity be a mixed number, what must be done? If a number be raised to two different powers, how is the power obtained by multiplying these two powers together, expressed? How is any power of a given number divided by another power of the same number? When the number to be raised to a power is in part a decimal, how is the number of decimals to be cut off from the required power ascertained? What is evolution? What is a root of a number? What is a power of any number? What are irrational powers? What are surds? How many methods are there of expressing roots? What is the first method? What root is expressed by the radical sign without any index or exponent? If

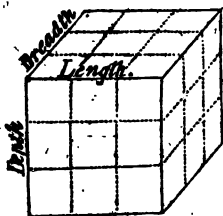
any other root is to be expressed, how is it done? How many times is the root to be taken as a factor in producing its corresponding power? What is the other mode of expressing roots? What is an advantage of this mode? When a fractional index is used, what does the denominator denote? What does the numerator? When several numbers are to be added, and the root of the sum extracted, how is the operation expressed? What does the root of the product of several numbers equal? Give an illustration. What is the square root of any number? What is a square? How is the area of a square found? Give the illustration. How is the length of the sides of a square found? What is the rule for extracting the square root? What is Note 1st? Note 2d? Note 3d? Why do we take twice the root for a divisor? Why in dividing do we omit the right hand figure of the dividend? Why do we place the quotient figure on the right of the divisor? How is the square root of a vulgar fraction extracted? The vulgar fraction may first be reduced to a decimal and the root of the decimal extracted, if preferred. What is a parallelogram? How is its area found? How may a square equal in area to a given parallelogram be found? What is Note 5th? Note 6th? To what is the square of the hypotenuse of a right angled triangle equal? If the base and perpendicular be given, how may the hypotenuse be found? If the hypotenuse and one of the legs of a triangle be given, how may the other leg be found? The base and perpendicular are called the *legs* of a triangle. How are the operations in Square Root proved? Ans. By multiplying the root into itself.

## EXTRACTION OF THE CUBE ROOT.

A CUBE is bounded by six equal plane surfaces, each of which is a square; that is, the length, breadth, and depth of a cube are equal. (See fig. 1st.) The area of each of the six equal surfaces is found by squaring the measure of its side, as has already been explained in Square Root.

If the adjoining figure represent a cubic block measuring three feet in length, breadth, and thickness, the superficial area of each face is  $3 \times 3 = 9$  square feet. Now if the divisions marked by the dotted lines on each face of the block, were extended through it, in either direction, the whole would be divided into 9 parts, each one foot square and 3 feet long, and susceptible of being divided each into three, and consequently the whole, into 27 blocks, each one

FIG. 1.



cubic foot. We have then the following general principle. *The content of a solid or cubic body is found by multiplying its length, breadth, and thickness into each other; or, what is the same in effect, by cubing one of these dimensions.*

Extracting the cube root is a process, the reverse of the preceding, that is, it is *finding the length of one of the sides of a cubic body, the solid content of that body being given; or it is finding from a given number, another number, whose cube or third power shall equal that number.*

**RULE 1st.**—*Separate the given number into periods of three figures each, by placing a point first over the unit figure, and advancing toward the left when the number consists of integers only; but to the right and left both, when it consists of integers and decimals, and make the last period of the decimal complete, by annexing cyphers, whenever necessary.*

2d. *Find by trial the greatest cube root of the left hand period, and place it as in square root; then subtract its cube from the same period, and bring down the next period of three figures to the remainder for a dividend.*

3d. *Square the root figure and multiply its square by 3 for a divisor, and see how many times it is contained in the dividend, omitting the first two right hand figures, and place the result as the second figure in the root.*

4th. *Multiply the divisor by the last quotient figure, and, placing two cyphers on the right of the product, place the result under the dividend. Also multiply the square of this same last quotient figure by the former figure or figures of the root, and also by 3; and, placing one cypher on the right of the product, write it under the preceding product. Lastly, under this, write the cube of the last quotient or root figure, and make the sum of these three numbers a subtrahend.*

5th. *Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend.*

6th. *To obtain a new divisor proceed as before, and thus continue the operation, till all the periods of the given number have been brought down.*

**Note 1st.**—*In obtaining each divisor, square the whole root obtained, and multiply that square by 3.*

**Ex. 1.** What is the cube root of 10648?



## OPERATION.

$$\begin{array}{r} 1\ 0\ 6\ 4\ 8\ (2\ 2 \\ 2^3=8 \end{array}$$

Art. 3d, Rule,  $2^3 \times 3 = 12$  (Div.)  $1\ 2\ )\ 2\ 6\ 4\ 8$  (See Art. 2d.)

$$\text{Rule, Art. 4th, } \left\{ \begin{array}{l} 12 \times 2 + 00 = 2\ 4\ 0\ 0 \\ 2^3 \times 2 \times 3 + 0 = 2\ 4\ 0 \\ 2^3 = 8 \end{array} \right.$$

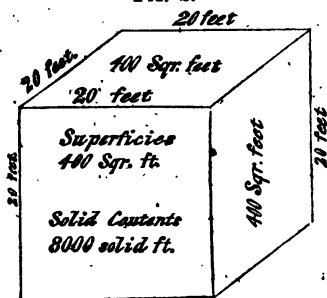
$$\begin{array}{r} \text{Subtrahend, } 2\ 6\ 4\ 8 \\ \hline 0\ 0\ 0\ 0 \end{array}$$

In dividing, the two right hand figures of the dividend are omitted. (See rule, Art. 3.) Proof,  $22^3 = 10648$ .

*Explanation.*—We will suppose the number 10648, in the preceding sum, to be so many feet of timber, one foot square, and that it is required to find how large a cubic pile they will form; that is, what will be the length, breadth, and depth of a cubic pile containing that number of solid feet. To make each step as clear as possible, we will repeat the preceding operation.

The number given when pointed, (see rule,) is divided into two periods, viz. 10 and 648. We therefore know that the required root will consist of two figures, and by trial we find the root of the first or left hand period to be 2, that is, 2 tens, or 20. Hence, 10648(20. The same as is seen in the above operation, excepting that a cypher is placed on the right of the root figure 2, to give it its true value. This gives the linear measure of a cubic block which the 10 (10,000) of the given number will make. Now, since 20 is the linear measure of the cube, (that is, the direct and not diagonal measure from corner to corner,) it is also the linear measure of each of the six equal square faces of that cube. Therefore,  $20 \times 20 = 400$ , the area of each face; and  $400 \times 20 = 8000$ , the number of cubic feet required to make a cubic body, whose linear measure is 20 ft.

FIG. 2.



(See fig 2.) By this step we have then disposed of 8000 of the 10648 solid feet. Hence,

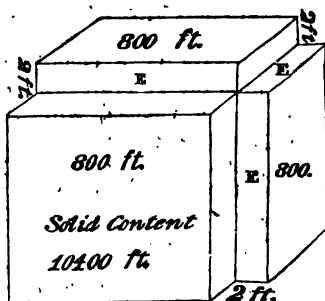
$$\begin{array}{r} 1\ 0\ 6\ 4\ 8\ (2\ 0 \\ 8\ 0\ 0\ 0 \\ \hline 2\ 6\ 4\ 8 \end{array}$$

(This same effect is obviously produced in the first solution of this sum, by cubing the 2, and subtracting it from the left hand period, 10, and then bringing down the next period, 648, to the remainder.) There now remains 2648 feet to be so added to the block already formed, that the whole shall be a perfect cube. This is done by making equal additions on any three of the equal faces *which lie contiguous to each other*. The reason of this is obvious. A solid body has length, breadth, and thickness; and in a cubic body, these dimensions are all equal, and by making the additions as here directed, *they are equally increased*.

The scholar will now understand why three times the square of the root obtained is taken as a divisor. The *square of the root* is the superficial area of one of the sides or faces of the cubic block, and this multiplied by 3 gives the area of three faces or sides, which is the number of sides to which equal additions are to be made. Hence, dividing the quantity to be added to the cube now obtained by this area, determines the thickness of the addition. This, in the sum now under consideration, is 2 feet, as seen at fig. 3. But these additions are evidently limited in size to the original block; consequently, the corners E, E, E, (fig. 3,) remain to be filled before a perfect cube is produced.

Now, to determine the quantity here added. Each face of the original cube contains 400 square feet, (see fig. 2,) and this multiplied by 2, the depth of the addition, gives 800 solid feet as the content of the addition made to each face; the whole addition therefore is  $800 \times 3 = 2400$  solid feet, and  $2648 - 2400 = 248$  solid feet, the quantity yet remaining to be disposed of.

FIG. 3.



## OPERATION CONTINUED.

$$\begin{array}{r}
 10648 \overline{)22} \\
 \underline{8} \\
 2^3 \times 3 = 12 \text{ div.} \quad - - - 12 \overline{)2648} \\
 \underline{\phantom{0000}}
 \end{array}$$

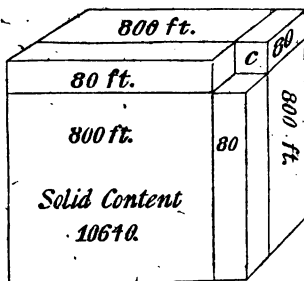
Solid con. of 3 additions,  $12 \times 2 + 00$ , (Rule,) 2400

The reason for placing two cyphers on the right of the divisor multiplied by the root figure, is obvious. By referring to a previous statement of this sum, it will be seen that the root figure 2, which, when squared and multiplied by 3, forms the divisor, is 2 tens, or 20. Hence,  $20 \times 20 = 400$ , and  $400 \times 3 = 1200$ , which would be the divisor, were the full value of the root figure expressed. This deficiency in the divisor is made up by omitting the two right hand figures of the dividend in dividing, and by placing two cyphers on the right of the product of the divisor multiplied by the root figure.

Our next step will be to fill the vacant corners as seen at E, E, E, (fig. 3.)

The rule, after directing the preceding step, says, "*Also multiply the square of this same last quotient figure by the former quotient figure or figures, and also by 3, and placing one cypher on the right of the product, write it under the preceding product.*" This operation

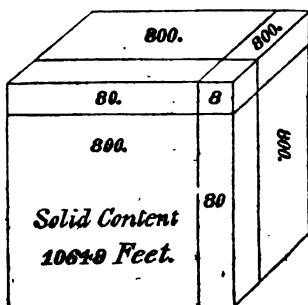
FIG. 4.



fills the corners here alluded to. For since the last quotient figure 2, is the thickness of the addition made, its square, viz. 4, is the measure of the corner to be filled, and the preceding quotient figure is the length of each corner; hence,  $4 \times 2 + 0 = 80$ , the solid content of one corner, (see fig. 4.) The cypher here is added because 2, the preceding quotient figure, is 2 tens, or 20. The three corners therefore require  $80 \times 3 = 240$  feet to fill them. The corner, C, still remains to be filled, and 8 feet of timber also remain, for  $10648 - 10640 = 8$ . This vacant corner, C, measures exactly 2 feet

in each of its three dimensions ; hence,  $2^3=8$ , is the solidity of that corner ; and this exactly disposes of the remaining timber. The cube completed is seen at fig. 5. The contents of the different parts of the completed block are, fig. 2,  $8000+$  ; fig. 3,  $2400+$  ; fig. 4,  $240+$  ; fig 5,  $8=10648$  feet.

FIG. 5.



*Note 1st.*—In Square Root we were directed to point off the given number into periods of two figures each ; and in Cube Root, the direction is, to allow three figures to each period. The following is the reason :—the square of any number always consists of *twice as many figures as the number itself, or one less than twice as many.* The cube of any number always consists of *three times as many figures as the number itself, or one or two less than three times as many.* That is, in Square Root, the left hand period may consist of *one or two figures* ; and in Cube Root, the same period may consist of *one, two, or three figures.*

*Illustration.*— $13^2=169$ , one less than twice the number of figures squared, and to extract its root it would be thus pointed, 169.  $46^2=2116$ , twice the number of figures squared.  $13^3=2197$ , two less than three times the figures cubed, and the left hand period consists of one figure only.  $25^3=15625$ , one less than three times the figures cubed, and the left hand period consists of two figures.  $99^3=970299$ , three times the number of figures cubed, and the left hand period consists of three figures.

In the following solution, the scholar will carefully compare each step of the operation with the rule. The first thing to be done, is to form the periods, (Art. 1st, Rule.) The first figure of the root is then to be determined, its cube subtracted, and to the remainder, the next period of three figures to be brought down. (Art. 2d, Rule.) He must then proceed to obtain a divisor, as directed by Art. 3d, and, lastly, to determine the solidity of the several additions made, and to make the result a subtrahend. (Art. 4th.)

Ex. 2. What is the cube root of 12812904 ?

| OPERATION.                              |                             |
|-----------------------------------------|-----------------------------|
|                                         | 12812904(234                |
| $2^3$ - - -                             | 8                           |
| $2^3 \times 3 = 12$ (Divisor) - - -     | 12)4812                     |
| $12 \times 3 + 00$ - - -                | 3600                        |
| $3^3 \times 2 \times 3 + 0 = 540$ - - - | 540                         |
| $3^3$ - - -                             | 27                          |
|                                         | 4167 = Subtrahend.          |
| $23^3 \times 3 = 1587$ (Divisor) -      | 1587)645904 = New dividend. |
| $1587 \times 4 + 00$ - - -              | 634800                      |
| $4^3 \times 23 \times 3 + 0$ - - -      | 11040                       |
| $4^3$ - - -                             | 64                          |
|                                         | 645904 = Subtrahend.        |
|                                         | 000000                      |

Proof,  $234^3 = 12812904$ .

It is obvious from the first division by 12, that the divisor is not contained in the dividend, in all instances, as many times as it would be in simple division.

In dividing it must be remembered to omit the first two figures on the left hand of the dividend.

3. What is the cube root of 250047? *Ans.* 63. Proof,  $63^3 = 250047$ .

4. What is the cube root of 970299? *Ans.* 99. Proof as before.

5. What is the cube root of 1.953125? *Ans.* 1.25.

6. What is the cube root of 22069810125? *Ans.* 2805.

7. What is the cube root of 183250432? *Ans.* 568.

8. What is the cube root of 84.027672? *Ans.* 4.38.

9. What is the cube root of 6859? *Ans.* 19.

10. What is the cube root of 205379? *Ans.* 59.

11. What is the cube root of 432081216? *Ans.* 756.

11. There is a cubic rock containing 8000 solid feet. What is the distance from corner to corner? *Ans.* 20.

13. What is the difference between half of a solid foot, and a solid half foot? *Ans.* 3 solid half feet.

14. What is the superficial area of one of the faces of a cubic block containing 4096 solid feet? *Ans.* 256 square feet.

15. What is the side of a cubical mound, equal to one, 144 feet long, 108 feet broad, and 24 feet deep? *Ans.* 72 feet. Multiply together the several dimensions of the given mound, and extract the cube root of their product.

*Note 2d.*—All solid bodies are to each as the cubes of their similar sides or diameters.

16. If a ball weighing 8 lb. be 6 inches in diameter, what will be the diameter of another ball of the same material, weighing 64 lb.?

$$8 : 64 :: 6^3 : 1728 \quad \sqrt[3]{1728} = \text{Ans. } 12 \text{ inches.}$$

17. If a ball 6 inches in diameter, weigh 8 lb. what is the weight of another ball of the same kind, measuring 12 inches in diameter? *Ans.* 64 lb.  $6^3 : 12^3 :: 8 : 64$ .

18. What would be the value of a globe of silver, one foot in diameter, if a globe of the same, one inch in diameter, be worth \$6? *Ans.* \$10368.

19. If a globe of silver, one inch in diameter, be worth \$6, what is the diameter of another globe of the same metal, worth \$10368? *Ans.* 12 inches.

20. How many globes one foot in diameter, would be required to make one globe 27 feet in diameter? *Ans.* 19683.

21. Suppose the diameter of the sun to be 110 times as large as that of the earth; how many bodies like the earth would be required to make one as large as the sun? *Ans.* 1331000.

22. If a man dig a square cellar, that will measure 5 feet each way, in one day, how long would it take him to dig one measuring 15 feet each way? *Ans.* 27 days.

**QUESTIONS.**—What is a cube? How is the area of each face of a cubic body found? How is the content of a cubic body found? What is the extraction of the cube root? How is the number whose root is to be extracted, to be pointed? Of which period is the root first found? What is then done with this root? How many figures are to be brought down to what remains? How is a divisor found? By what do you multiply the divisor? How many cyphers do you place on the right of the product? Where do you place the product? What farther is done with the quotient or root figure? What does the sum of all these products form? What is the fifth step of the rule? How is a second divisor obtained? What is Note 1st? Can we know of how many figures the root will consist? Of what is the root figure the linear measure? How much of the whole timber is disposed of

by subtracting the cube of the quotient figure from the left hand period, in the operation taken for explanation? How many feet remain to be added? To how many sides of a cube must equal additions be made to preserve its cubic form? Why is three times the square of the root taken for a divisor? What is determined by dividing? How many solid feet are disposed of by the first addition made to the three faces of the cube? Explain how the solid content of the addition to each face is obtained. Why, in multiplying the divisor by the root figure, are two cyphers placed on the right of the product? How is the deficiency of the divisor made up in dividing? What operation as directed by the rule fills up the corners left vacant? Why is the square of the quotient figure multiplied by the previous root figures, and a cypher placed on the right hand of the product? How much of the remaining timber is required to fill the three vacant corners? What is the measure of the corner left vacant, after the preceding additions were made? How is it filled? Why in Square Root do we divide the given number into periods of two figures each? Why is the given number divided into periods of three figures each, in Cube Root? Of how many figures does the square of any number consist? Of how many does the cube? How are the roots proved? Ans. By raising the root to a power of the same name as the root itself.

## ARITHMETICAL PROGRESSION.

Any series of numbers more than two, increasing or decreasing by a constant and uniform difference, is called *Arithmetical Progression*, or *Arithmetical Series*.

The series formed by a continual addition of any number, (called the common difference,) is called the *ascending series*. Thus, 3, 5, 7, 9, 11, 13, &c. is an ascending series, of which the common difference is 2. The reverse of this forms the *descending series*; that is, a series decreasing by a continual subtraction of the common difference. Thus, 13, 11, 9, 7, 5, 3, &c. is a descending series.

The numbers constituting the series are called *terms*. The *first* and *last* term of the series are called *extremes*; all the intervening terms are called the *means*, and the number constantly added or subtracted, is called the *common difference*. When the first term and common difference are given, the series may be indefinitely extended.

In every arithmetical progression, *five particulars are to be noticed*; of which, if any *three* be given, the remaining two may be found. The five particulars are, *the first term, the last term, the common difference, the number of terms, and the sum of all the terms.*

CASE 1st.—THE FIRST TERM, THE NUMBER OF TERMS, AND THE LAST TERM GIVEN, TO FIND THE COMMON DIFFERENCE.

Ex. 1. The first term of an arithmetical series is 2; the number of terms, 13; and the last term, 38. What is the common difference?

The series commences with 2, as the first term; therefore,  $38 - 2 = 36$ , is the amount of all the additions made to this number. But the whole number of terms being 13, and one of these being given, 12 terms must be formed by adding the common difference. Therefore,  $36 \div 12 = 3$ , the common difference required. The whole series, therefore, is, 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38; thirteen in number.

We have then the following rule for solving sums like the preceding:

RULE.—*Divide the difference of the extremes by the number of terms, less one. The quotient will be the common difference.*

2. A man in feeble health, commenced a journey and traveled 9 days. On the first day he traveled only 3 miles, but afterwards continued to gain each day an equal number of miles on the journey of the preceding day, till the last day, on which he traveled 43 miles. The daily increase is required? *Ans.* 5 miles per day; that is,  $43 - 3 \div 8 = 5$ .

3. I owe a debt which by agreement I am to pay at 17 different periods. The first payment is to be \$20, and the last, \$100. Required the common difference of the several payments. *Ans.* \$5.

4. A man had 10 sons whose ages differed alike; the youngest of whom was 2 years old, and the oldest 29. What was the difference of their ages? *Ans.* 3 years.

CASE 2d.—THE FIRST TERM, THE COMMON DIFFERENCE, AND THE LAST TERM GIVEN, TO FIND THE NUMBER OF TERMS.

Ex. 1. If the extremes be 3 and 51, and the common difference 6, what is the number of terms?



$51-3=48$ , the amount of all the additions made to the first term, 3; and since each addition is 6,  $48 \div 6 = 8$ , the number of additions; that is, the number of terms formed by adding the common difference to the first term; therefore,  $8+1=9$ , the whole number of terms, or answer required.

From the above we derive the following rule:

**RULE.**—*Divide the difference of the extremes by the common difference, and add one to the quotient.*

2. A man commenced a journey, and traveled the first day only 4 miles; after which he gained each day 6 miles on the journey of the preceding day, and on the last day he traveled 94 miles. How many days did he travel? *Ans.* 16.

Operation:  $94-4=90$ , and  $90 \div 6+1=16$ .

3. A man commenced a journey in great haste, and traveled 63 miles the first day; but being unable to continue at the same rate, the second day he traveled only 59 miles; and thence continued to lose 4 miles per day, till the last day of his journey, on which he traveled 11 miles. How many days did he travel? *Ans.* 14 days.

4. A man set out on a journey for the improvement of his health; the first day he traveled 10 miles, and the last, 65 miles, making each day an advance of 5 miles on the journey of the preceding day. How many days did he travel? *Ans.* 12 days.

**CASE 3d.**—THE FIRST TERM, THE LAST TERM, AND THE NUMBER OF TERMS GIVEN, TO FIND THE SUM OF ALL THE TERMS.

Ex. 1. Bought 30 yards of cloth, paying 20 cents for the first yard and 50 cents for the last. What was the whole cost, allowing each succeeding yard to increase in price by a constant access?

The price of each succeeding yard increases by a constant access; therefore, the price of the last is as much above the average price as the price of the first is below it; hence, one half of the price of the first and last yard is the average price. Therefore,  $20+50=70$  and  $70 \div 2=35$  cents, average price; hence,  $30 \times 35 = \$10.50$ , whole cost.

We have the following rule:

**RULE.**—Multiply half the sum of the extremes by the number of terms; the product will be the answer.

2. Paid 4 cents for the first, and \$1.21 for the last yard of a piece of cloth containing 86 yards. What was the whole cost? *Ans.* \$53.75.

3. How many strokes does a regular clock strike in 24 hours? *Ans.* 156.

4. If a person walk 3 miles the first, and 91 miles the last day of his journey, how far will he have walked, allowing him to have been on his journey 24 days? *Ans.* 1128 miles.

**QUESTIONS.**—What is arithmetical progression? What is the ascending series? What is the descending series? What is denoted by the word, *terms*? What terms are the extremes? What are the means? What is the number constantly added or subtracted called? How many particulars require to be noticed? What are they? What is Case 1st? What is the rule? What is Case 2d? What is the rule? What is Case 3d? What is the rule?

## GEOMETRICAL PROGRESSION.

Any series of numbers, increasing by a common multiplier, or decreasing by a common divisor, is called *Geometrical Progression*, or *Geometrical Series*. The common multiplier of the *ascending series*, and the common divisor of the *descending series*, is called the *ratio* of the series, or the *common ratio*. Thus, if we take 3 as a first term, and multiply it continually by 2, as a common ratio, we obtain the series, 3, 6, 12, 24, 48, 96, 192, 384, 768, &c., in which series each term is obtained by multiplying the preceding term by 2. We have here an ascending series. Or the series may be 768, 384, 192, 96, 48, 24, 12, 6, 3, &c., in which, each term is obtained by dividing the preceding term by 2. This forms a descending series. The several numbers thus produced constitute the *terms* of the series; the *first* and *last* of which are called the *extremes*; and the intervening ones are called the *means*.

In geometrical, as in arithmetical progression, there are *five things* to be considered ; of which, if any three be given, the other two may be found. The five things are, *the first term, the last term, the number of terms, the sum of all the terms, and the ratio.*

**CASE 1st.**—THE FIRST TERM, THE RATIO, AND THE NUMBER OF TERMS GIVEN, TO FIND THE LAST TERM.

**RULE.**—*Raise the ratio to a power whose index is one less than the number of terms, and multiply this power by the first term. The product will be the number required.*

**Ex. 1.** The first term of a geometrical series is 2, and the ratio 3. What is the 12th term ?

Since the given term 2, is the first of the required series of 12 terms, and since each succeeding term is found by multiplying the preceding one by 3, 3 is evidently to be taken as multiplier, or factor, eleven times ; that is, any term of a series is equal to the first term multiplied by the ratio raised to a power *one less than the number of terms*. Thus,  $2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3 \times 2 = 354294$ , *Ans.* or twelfth term.

2. A person purchased a house having 8 doors, and agreed to pay for the whole, whatever value might be attached to the eighth door, by allowing \$4 for the first, \$16 for the second, and \$64 for the third door, &c. What did his house cost him ? *Ans.*  $4^7 \times 4 = \$65536$ .

3. A boy purchased 12 oranges, and agreed to pay 1 cent for the first, 4 cents for the second, 16 cents for the third, &c. What was the value of the twelfth orange ? *Ans.* \$41943.04.

4. A man wishing to get his horse shod, agreed to allow 3 cents for the first nail, 9 cents for the second, and 27 cents for the third, &c. ; and to pay for the whole the value of the last nail, of which there were 32. What was the cost of shoeing his horse ? *Ans.* \$18530201888518.41.

**Note 1st.**—It is obvious from what was said of Involution, that, if the ratio be raised to two or three different powers, whose indices, when added together, equal the index of the required power, the product of these several powers will be the power, or number required. Thus, in the second example, the

answer is obtained thus :  $4^3 \times 4^4 \times 4 = \$65536$ , the answer as before.

5. A person sold 20 yards of cloth as follows : for the first yard he received 3 d., for the second, 9 d., for the third, 27 d., &c. What was the cost of the twentieth yard? *Ans.* 14528268 £. 6 s. 9 d.

6. A man purchased 12 horses ; for the first horse he gave only 4 cents, for the second, he gave 16 cents, for the third, 64 cents, &c., and thus in a quadruple ratio to the last. What did the twelfth horse cost him? *Ans.* \$167772.16.

**CASE 2d.—THE EXTREMES AND RATIO BEING GIVEN, TO FIND THE SUM OF ALL THE TERMS.**

*RULE.—Divide the difference of the extremes by the ratio less 1, and the quotient increased by the greater extremes, will be the sum of the series.*

**Ex. 1.** The extremes of a geometrical series are 2 and 1458. What is the sum of all the terms, the ratio being 3?

1458—2=1456, the difference of the extremes; and 3—1=2, the ratio less 1. Therefore,  $1456 \div 2 = 728$ ; and  $728 + 1458 = 2186$ , the sum of all the terms, *Ans.*

2. A farmer has sheep in 6 different pastures. In the first pasture there are 3, and in the sixth, 729. If the ratio of increase be 3, how many sheep has he in all his pastures? *Ans.* 1092.

3. There is a cherry tree with 10 branches; on the first branch there are only 2 cherries; on the tenth branch there are 524288. Now the ratio of increase from one branch to another being 4, how many cherries are there on the tree? *Ans.* 699050.

**CASE 3d.—THE FIRST TERM, THE RATIO, AND THE NUMBER OF TERMS GIVEN, TO FIND THE SUM OF THE SERIES.**

*RULE.—Find the last term by Case 1st, and the sum of the series, or of all the terms, by Case 2d.*

**Ex. 1.** A gentleman sold 20 yards of cloth, receiving for

the first yard, 3 d. ; for the second yard, 9 d. ; and for the third yard, 27 d. How much did he receive for the whole, at that rate ?

Case 1st,  $3^9 \times 3 = 3486784401$ , the last term. Case 2d,  $\frac{3486784401 - 3}{2} = 1743392199. + 3486784401 = 5230176600$  the sum of all the terms in pence = 21792402 £. 10 s.

2. What would 12 horses cost, if 4 cents were allowed for the first, 16 cents for the second, and 64 cents for the third horse, &c. the value thus increasing in a quadruple ratio to the last or twelfth horse ? *Ans.* \$223696.20.

3. A gentleman gave his daughter, on the day of her marriage, one dollar, promising to triple it on the first day of each month in the year. What was the amount of her portion ? *Ans.* \$265720.

CASE 4th.—THE EXTREMES AND NUMBER OF TERMS GIVEN, TO FIND THE RATIO.

*RULE.*—Divide the greater extreme by the less, and the quotient will be that power of the ratio, which is equal to the number of terms less 1. The corresponding root will, therefore, be the ratio.

Ex. 1. The first term of a geometrical series is 2, and the last term 354294, and the number of terms 12. What is the ratio ?

$354294 \div 2 = 177147$ , and the eleventh root of this number is the ratio required ; therefore,  $\sqrt[11]{177147} = 3$ , the ratio.

2. The first term of a certain series is 4, and the last 65536, and the number of terms 8. What was the ratio ? *Ans.* 4.

*QUESTIONS.*—What is Geometrical Progression ? What is the ratio of the series ? What are the terms of the series ? What terms of a series are called extremes ? And what are called means ? In geometrical progression, how many things are to be considered ? How many of these must be given to find the others ? What are the five things given ? What is Case 1st ? What is the rule for Case 1st ? What is Note 1st ? What is Case 2d ? What is the rule ? What is Case 3d ? What is the rule ? What is Case 4th ? What is the rule ?

## ALLIGATION.

*Alligation is the method of mixing several simples of different qualities, so as to obtain a compound of a mean or middle quality.*

**CASE 1st.**—WHEN THE QUANTITIES AND PRICES OF SEVERAL SIMPLS ARE GIVEN, TO FIND THE MEAN PRICE OF THE MIXTURE.

**RULE.**—*Find the total value of the several kinds to be mixed, and divide the amount of this value by the whole number of articles.*

**Ex. 1:** A farmer mixed together 8 bushels of rye worth \$0.50 per bushel; 12 bushels of corn worth \$0.65 per bushel; and 6 bushels of oats worth \$0.30. What was the value of one bushel of the mixture?

8 bushels of rye at 50 ct. = \$4.00; 12 bushels of corn at 65 ct. = \$7.80; and 6 bushels of oats at 30 ct. = \$1.80. And  $8 + 12 + 6 = 26$  bushels, and  $\$4.00 + \$7.80 + \$1.80 = \$13.60$ , and  $\$13.60 \div 26 \text{ bushels} = \$0.523$ , + price of one bushel of the mixture.

2. A grocer mixed 6 lb. of tea at \$1.20 per lb.; 12 lb. at \$1.60; and 8 lb. at \$1.80. What was the value of one lb. of the mixture? *Ans.* \$1.569.

3. If 15 bushels of wheat worth \$1.40 per bushel, be mixed with 12 bushels of rye at \$0.60 per bushel, and ten bushels of oats at \$0.35, what is the value of one bushel of the mixture? *Ans.* \$0.856. +

4. If 6 lb. of gold, 20 carats fine, be mixed with 12 lb. at 18 carats fine, what is the fineness of the mixture? *Ans.*  $18\frac{1}{2}$  carats fine.

5. If 6 gallons of wine at \$0.67 per gallon, 7 gallons at \$0.80 per gallon, and 5 gallons at \$1.20 per gallon, be mixed together, what will be the value of one gallon of the mixture? *Ans.* \$0.867. +

**CASE 2d.**—THE PRICES OF SEVERAL COMMODITIES BEING GIVEN, TO DETERMINE HOW MUCH OF EACH COMMODITY MUST BE TAKEN, TO FORM A COMPOUND OF A CERTAIN PROPOSED MEDIUM VALUE.

**RULE.**—Write down the prices of the several simples under each other, placing that price which is least in value uppermost, and the remaining prices in the order of their values.

Connect, by a line, any price less than the given mean price, with one that is greater, and continue thus to do till they are all connected; then place the mean price on the left, and separate it from the other numbers by a perpendicular line. Write the difference between the proposed price of the mixture, and the price of each simple, opposite the number or numbers with which that simple is connected; and, finally,

Notice whether more than one difference stands opposite any one price; if so, their sum will express the quantity of that price to be taken; but if only one difference stands there, that will be the quantity required.

**Note.**—One difference at least must stand against each price.

**Ex. 1.** How much corn at 48 cents, barley at 36 cents, and oats at 24 cents per bushel, must be taken to make a compound worth 30 cents per bushel?

|                |  |    |   |                              |
|----------------|--|----|---|------------------------------|
| Mean price, 30 |  | 24 | } | 6+18=24 bushels at 24 cents. |
|                |  | 36 |   | 6 . . . . . 36 cents.        |
|                |  | 48 |   | 6 . . . . . 48 cents.        |

The difference between 30, the mean, and 24, is placed opposite of both 36 and 48, as it is connected with them both; and the difference between 30, the mean, and 36, and also between 30 and 48, are both placed opposite 24, because these numbers are both linked with 24, and the sum of their differences determines the number of bushels required of that price. Of the oats, therefore, 24 bushels are required, and of the corn and barley, only 6 bushels of each.

**2.** I have four kinds of sugar valued at 8, 12, 15, and 18 cents per pound. How much of each kind must be taken to make a mixture worth 14 cents per pound?

|    |  |    |   |                                |
|----|--|----|---|--------------------------------|
| 14 |  | 8  | } | 4 number of pounds at 8 cents. |
|    |  | 12 |   | 1 . . . . . 12 cents.          |
|    |  | 15 |   | 2 . . . . . 15 cents.          |
|    |  | 18 |   | 6 . . . . . 18 cents.          |

3. A grocer mixed together three kinds of tea, valued at 6, 9, and 10 shillings per pound, so that the compound was worth 8 shillings per pound. How much of each sort did he take? *Ans.* 3 lb. at 6 s., 2 lb. at 9 s., and 2 lb. at 10 s.

4. A merchant has three kinds of wine. For the one kind he charges 3 s. 4 d., for the second 5 s., and for the third 7 s. per gallon. How much of each is required to form a mixture worth 6 s. per gallon? *Ans.* 12 gal. at 3 s. 6 d., 12 gal. at 5 s., and 44 gal. at 7 s.

5. How much gold at 16, 19, 21, and 24 carats fine, will be required to form a compound of 20 carats fine? *Ans.* 4 parts of 16, 1 of 19, 1 of 21, and 4 of 24 carats fine.

**CASE 3d.**—THE PRICE OF EACH OF SEVERAL SIMPLES, THE QUANTITY OF ONE AND THE PRICE OF THE COMPOUND BEING GIVEN, TO FIND HOW MUCH OF EACH OF THE OTHER SIMPLES IS REQUIRED.

**RULE.**—*Link the several prices together, as in the last case, and find their differences; then multiply the given quantity by the differences standing severally against the other quantities, and divide the product by the difference standing against itself. Or say, as the difference opposite the given quantity is to the given quantity, so are the other differences severally to their required quantities.*

**Ex. 1.** How much barley at 30 cents, rye at 36 cents, and corn at 48 cents per bushel, must be mixed with 12 bushels of oats at 18 cents per bushel, so that the compound may be worth 22 cents per bushel?

$$\begin{array}{r|l} \text{Mean price, 22.} & \left. \begin{array}{l} 30 \\ 36 \\ 48 \\ 18 \end{array} \right\} \end{array} \left. \begin{array}{l} \dots\dots 4 = 4. \\ \dots\dots 4 = 4. \\ \dots\dots 4 = 4. \\ 8 + 14 + 26 = 48. \end{array} \right\}$$

The price of the given quantity is 48; therefore,  $48 : 12 :: 4 : 1$ ; the quantity required at 30 cents per bushel. The remaining statements and answers are the same, since the differences are all the same. Therefore, one bushel at 30 cents, one at 36 cents, and one at 48 cents, would be required to be mixed with 12 bushels at 18 cents, to form a mixture worth 22 cents.

2. A grocer has three kinds of beer for sale, valued at 7 s.,



5 s., and 3 s. per gallon, which he proposes to mix with 20 gallons of a superior quality, worth 6 s. per gallon, so that the mixture may be sold at 4 s. per gallon. How much of the first three kinds must he take? *Ans.* 120 gal. at 3 s., and 20 gal. at 5 s. and 7 s.

3. How much tea at 80, 60, and 40 cents per lb. must be mixed with 30 lb. at \$1.00 per lb. so that the mixture may be sold at 70 cents per lb.? *Ans.* 10 lb. at 80 cents and 60 cents, and 30 lb. at 40 cents.

4. How much water of no value must be mixed with 100 gal. of wine at 7 s. 6 d. per gal., to reduce the price to 6 s. 3 d. per gallon? *Ans.* 20 gal.

CASE 4th.—THE PRICE OF THE SIMPLES BEING GIVEN, AND ALSO THE COMPOUND TO BE FORMED, TO FIND HOW MUCH OF EACH SIMPLE MUST BE TAKEN.

RULE.—Connect the prices of the simples as in the preceding cases, and find the amount of the differences; then say, as the amount of the differences is to each of the differences taken separately, so is the whole compound to the part required.

Ex. 1. A compound of 15 gallons, which shall be worth 8 shillings per gallon, is to be made of three sorts of wine, valued at 5, 7, and 12 shillings per gallon. How much of each kind will be required?

$$\begin{array}{r|l}
 8 & \left. \begin{array}{l} 5 \\ 7 \\ 12 \end{array} \right\} \left\{ \begin{array}{l} - - - - 4 - - - - 4 \\ - - - - 4 - - - - 4 \\ - - - - 3+1 - - - - 4 \end{array} \right. \\
 & \hline
 & 12
 \end{array}$$

Then,  $12 : 4 :: 15 : 5$ , *Ans.* 5 lb. of each kind are required.

Proof,  $5 \text{ s.} \times 5 = 25 \text{ s.}$   $7 \text{ s.} \times 5 \text{ s.} = 35 \text{ s.}$ ; and  $12 \text{ s.} \times 5 \text{ s.} = 60 \text{ s.}$ ; and  $25 + 35 + 60 = 120 \text{ s.}$ ; and  $120 \div 8 = 15 \text{ gallons.}$

2. I have four sorts of tea, of which the first kind is worth 1 s. per lb.; the second kind, 3 s.; the third, 6 s.; and the fourth, 10 s. How much of each kind will be required to make a compound of 120 lb. worth 4 s. per lb.? *Ans.* 60 lb. at 1 s.; 20 lb. at 3 s.; 10 lb. at 6 s.; and 30 lb. at 10 s.

3. How much of each of four kinds of coffee, worth 8, 12, 18, and 22 cents per lb. will be required to make a compound

of 120 lb. worth 16 cents per lb. ? *Ans.* 36 lb. at 8 cents ; 12 lb. at 12 cents ; 24 lb. at 18 cents ; 48 lb. at 22 cents.

4. A gold beater has gold 15, 17, 18, and 22 carats fine, of which he wishes to make a compound of 40 oz. 20 carats fine. How much of each kind must he take ? *Ans.* 25 oz. 22 carats fine ; and 5 oz. of 15, 17, and 18 carats fine.

5. How much water of no value, and how much wine at 90 cents per gallon, must be taken to make 100 gallons, worth 60 cents per gallon ? *Ans.* 33 $\frac{1}{2}$  gallons of water, and 63 $\frac{1}{2}$  gallons of wine.

QUESTIONS.—What is Alligation ? What is Case 1st ? What is the rule ? What is Case 2d ? What is the rule ? What is the note ? What is Case 3d ? What is the rule ? What is Case 4th ? What is the rule ?

## POSITION.

Position is a rule by which answers are obtained to such questions as cannot be solved by the common direct rules, by assuming any convenient number or numbers, and then working according to the nature of the question.

### SINGLE POSITION.

When the question can be solved by the assumption of a *single number*, the operation is called *Single Position*.

The following sum will serve for an illustration.

Ex. 1. A teacher being asked how many scholars he had, replied, "If I had once, one half, one third, and one fourth as many more as I now have, I should have 185." How many had he ? We will suppose the number to be 24. As he first doubles his number, 24 must be doubled. To this amount, one half his original number, 12, must also be added. He then increases his number by one third of his original

number, viz. 8, and also by one fourth, viz. 6. Whole amount, 74.

Now it is evident that we have not supposed the right number, otherwise the amount would have been 185, as given in the sum. We have, however, *increased the number* we supposed, viz. 24, by the *same or similar additions*, as the *teacher did the true number of his scholars*; consequently, 74, the number we obtained, must have the same ratio to 24, the number assumed, as 185 has to the real number of scholars in the school. Therefore,  $74 : 24 :: 185 : \text{the number required}$ ; viz. 60. Proof,  $60 + 60 + 30 + 20 + 15 = 185$ .

We have then the following rule :

**RULE.**—*Take any convenient number and proceed with it according to the conditions of the question, and observe the result ; then say, as the number thus obtained is to the given number, so is the assumed number to the true one.* Or the numbers may be canceled by *arranging the terms as directed in Simple Proportion*.

**Ex. 2.** A man being asked how much money he had, replied, that  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{6}$  of his money added, made \$57. How much money had he ? *Ans.* \$60.

3. What number is that, which, being multiplied by 9 and divided by 4, the quotient will be 27 ? *Ans.* 12.

4. A man borrowed a sum of money on interest, which in 10 years amounted to \$1800 at 6 per cent.; what was the sum ? *Ans.* \$1125.

5. Two boys were playing at marbles. Says one to the other,  $\frac{1}{2}$ ,  $\frac{1}{4}$ , and  $\frac{1}{6}$  of my marbles added together make 45, and if you can now tell how many I have, you may have them. How many had he ? *Ans.* 60.

6. A boy wishing to try the skill of his companions in figures, said he had a pile of apples, of which, if he gave  $\frac{1}{2}$  to A.,  $\frac{1}{4}$  to B., and  $\frac{1}{6}$  to C., there would remain 28 for D.; and requested them to tell him how many there were in all. What was the number ? *Ans.* 112.

7. A person being asked his age, said that if  $\frac{2}{3}$  of the years he had lived were multiplied by 7, and  $\frac{2}{3}$  of them added to the product, the sum would be 292. How old was he ? *Ans.* 60 years.

8. A. saves  $\frac{1}{3}$  of his income, but B. who has the same income, spends twice as fast as A., and thereby contracts a debt of \$120 annually. What is their income? *Ans.* \$360.

9. The sum of A., B., and C's ages, is 132 years. B's age is  $1\frac{1}{2}$  the age of A; and C's age is twice as great as B's. What are their respective ages? *Ans.* A's age is 24; B's, 36; and C's, 72 years.

QUESTIONS.—What is Position? What is Single Position? What is the rule? How may the operations be canceled?

## DOUBLE POSITION.

By Double Position, we solve such sums as require two suppositions.

In this rule, the numbers supposed to be the true ones bear no certain or definite proportion to the required answers.

*RULE.*—Assume any two convenient numbers and proceed with each according to the conditions of the question, and compare the result of each with the sum or result given in the question, and find their differences. Call each difference an error.

Multiply the first assumed number by the last error; and the last assumed number by the first error.

If both errors are too great or too small, divide the difference of these products by the difference of the errors, and the quotient will be the number sought. But if one of the errors be too large, and the other too small, divide the sum of the products by the sum of the errors.

*Note.*—The errors are said to be too large or too small, when by operating on each supposed number according to the nature of the question, the number obtained is greater or less than the corresponding number in the sum.

Ex. 1. Three men found a purse of money containing \$80,

which they agree to divide in such a manner, that A. shall have \$5 more than B, and that B. should have \$10 more than C. What was each man's share of the money?

Suppose, 1st, that C. had - \$15  
 then B. had by the conditions, 25  
 and A., - - - - - 30

\$70, a sum of money less than  
 that found; therefore,  $\$80 - \$70 = \$10$ , 1st error.

Again, suppose that C. had - \$20  
 B. of course must have had - 30  
 and A., - - - - - 35

\$85, a sum of money greater  
 than that found; therefore,  $\$85 - \$80 = \$5$ , 2d error.

If now the above operations be compared with the rule and the note following, it will be seen that the first error is too small, and the last one too large; therefore, 15, the number first supposed  $\times 5$ , the last error  $= 75$ ; and 20, the number last supposed  $\times 10$ , the first error  $= 200$ ; and  $200 + 75 = 275$ , the sum of the products; and  $10 + 5 = 15$ , the sum of errors. Therefore,  $275 \div 15 = \$18.333+$ , C's share; and  $\$18.333 + \$10 = \$28.333+$ , B's share; and  $\$28.333 + \$5 = \$33.333$ , A's share.

2. Four individuals having \$100 to divide among themselves, agree that B. should have \$4 more than A.; C., \$8 more than B.; and D. twice as much as C. What was each man's share?

1st, suppose A. had \$6  
 then B. had - - - 10  
 C., - - - - - 18  
 and D., - - - - 36

2d, suppose A. had \$8  
 then B. had - - 12  
 C., - - - - - 20  
 and D., - - - - 40

\$70

\$80

and  $100 - 70 = 30$ , 1st error. Hence,  $100 - 80 = 20$ , 2d error.

Here both errors are too small, therefore,  $6 \times 20 = 120$ ; and  $8 \times 30 = 240$ ; then,  $240 - 120 = 120$ , the difference of the products; and  $30 - 20 = 10$ , the difference of errors. Therefore,  $120 \div 10 = 12$ , A's share;  $12 + 4 = 16$ , B's share; and  $16 + 8 = 24$ , C's share; and  $24 + 24 = 48$ , D's share. Proof,  $12 + 16 + 24 + 48 = 100$ .

3. Three men hired a piece of wall built, for which they paid \$500. Of this, A. paid a certain part; B. paid \$10 more than A., and C. paid as much as A. and B. both. What did each man pay? *Ans.* A. paid \$120; B. \$130; and C. \$250.

Sums like the preceding are solved with ease by analysis. Since we have the sum they all paid, we know that C. paid \$250, because he has paid as much as the other two, that is, one half of the whole. Therefore, A. and B. together paid \$250. But B. paid \$10 more than A., hence,  $250 - 10 = 240$ , twice the number of dollars A. paid, and  $240 \div 2 = 120$ , A's share; then,  $120 + 10 = 130$ , B's share; and  $120 + 130 = 250$ , C's share.

4. Two persons lay out equal sums of money in trade. A. gains 120 £., and B. loses 80 £. A's money was then treble B's. With what sum did they commence? *Ans.* 180 £.

5. A farmer hired a laborer 40 days, on condition that he should receive 20 cents for every day he wrought, and forfeit 10 cents every day he was idle. At the expiration of the 40 days he received \$5. How many days did he work, and how many was he idle? *Ans.* He wrought 30 days, and was idle 10 days.

6. What is the length of a fish whose head is 10 inches long, his tail as long as his head and half the length of his body, and his body as long as his head and tail both? *Ans.* 80 inches.

7. Two persons, A. and B., have the same income. A. saves  $\frac{1}{4}$  of his, but B., by spending \$150 per annum more than A., at the end of 8 years finds himself \$400 in debt. What was their income, and how much did each spend annually? *Ans.* Income, \$400. A. spends \$300, and B. \$450.

8. A man bequeathed his property to his three sons, on the following conditions; viz. to A., one half, wanting \$50; to B., one third; and to C., the remainder, which was \$10 less than B's share. How much did each son receive, and what was the whole estate? *Ans.* A. received \$130; B. \$120; and C. \$110. The whole estate was \$360.

9. A farmer bought a certain number of oxen, cows, and calves; for which he paid 130 £. For every ox he paid 7 £.; for every cow, 5 £.; and for every calf, 1 £. 10s. There were two cows for every ox, and three calves for every cow. How many were there of each kind? *Ans.* 5 oxen, 10 cows, and 30 calves.

10. A person after spending \$10 more than  $\frac{1}{2}$  of his annual

income, had \$35 more than  $\frac{1}{2}$  of it remaining. What was his income? *Ans.* \$150.

11. A person has two horses; he also has a saddle worth 10 £. If the saddle be placed on the first horse, the horse and saddle are worth twice as much as the second horse; but the value of the second horse with the saddle is 13 £. less than the value of the first horse. How much is each horse worth? *Ans.* The first is worth 56 £., and the second, 33 £.

QUESTIONS.—What is Double Position? What relation do the supposed numbers bear to the true ones? What is the rule? When are the errors said to be too large or too small?

## PROMISCUOUS EXAMPLES.

Ex. 1. If 460 be multiplied by 36, and the product divided by 9, what will the quotient be? *Ans.* 1840.

2. What number is that which, when increased by  $\frac{1}{2}$  of itself, will be 126? *Ans.* 72.

3. What number multiplied by  $\frac{3}{4}$  will produce 16? *Ans.* 21 $\frac{1}{3}$ .

4. What fraction multiplied by 15 will produce  $\frac{1}{2}$ ? *Ans.*  $\frac{1}{30}$ .

5. What number multiplied by 32 will produce 2912? *Ans.* 91.

6. What number divided by 21 will give 65 as a quotient? *Ans.* 1365.

7. How many nails are required to shoe 27 horses, each shoe requiring 8 nails? *Ans.* 864.

8. In the counter of a merchant there are four drawers, in each drawer, 4 divisions, and in each division, \$23.75. How many dollars do the four drawers contain? *Ans.* \$380.00.

9. Two men depart from the same place and travel the same way; one travels 36 miles per day, and the other 42. What will be the distance between them at the end of the 8th day, and how far will each have traveled? *Ans.* 48 miles apart, the one having traveled 288, and the other 336 miles.

10. A person owning  $\frac{3}{4}$  of a ship, sold  $\frac{1}{4}$  of his share for \$474. What was the value of the whole ship, at the same rate? *Ans.* \$1264.

11. How many men must be employed to finish a piece of work in 15 days, which would require 5 men 24 days? *Ans.* 8 men.

12. A person being asked the time of day, answered, "The time past noon is equal to  $\frac{1}{4}$  the time till midnight." What was the time? *Ans.* 36 minutes past 5.

13. In a certain school,  $\frac{1}{2}$  the scholars learn to read and write;  $\frac{1}{4}$  learn geography;  $\frac{1}{8}$  learn grammar; and 16 study astronomy. What was the number in the school? *Ans.* 128.

14. What is the whole length of a pole,  $\frac{1}{3}$  of which stands in the ground, 16 feet in the water, and  $\frac{1}{4}$  in the air? *Ans.* 213 feet, 4 inches.

15. There is a room 12 feet long, 8 feet wide, and 7 feet high. How much paper, 2 feet wide, will be required to paper the same? *Ans.* 46 yards, 2 feet.

16. My horse and saddle are both worth 36 £. 12 s., and my horse is worth 7 times as much as my saddle. What is the value of each? *Ans.* My horse is worth 32 £. 0 s. 6 d., and my saddle, 4 £. 11 s. 6 d.

17. There is a cistern having 3 faucets, the largest of which will empty it in one hour, the second in 2 hours, and the third in 3 hours. In what time will they all empty it, if opened at the same time? *Ans.*  $32\frac{2}{11}$  minutes.

18. Divide 1500 acres of land between A., B., and C., so that A. shall have 150 acres more than B., and B. 100 acres more than C. *Ans.* A. has 633 $\frac{1}{3}$ , and B. 483 $\frac{1}{3}$ , and C. 383 $\frac{1}{3}$ .

19. A certain pasture will feed 324 sheep 7 weeks. How many must be turned away, in order that it may be sufficient for the remainder 9 weeks? *Ans.* 72.

20. A merchant bought 120 gallons of molasses for \$45. How must he sell the same per gallon, to gain 15 per cent.? *Ans.* 0.43.+

21. If a family of 8 persons consume \$200 worth of provision in 9 months, how much will 18 persons consume in a year? *Ans.* \$600.

22. A man left his son a fortune,  $\frac{1}{3}$  of which he spent in 3 months; and in 6 months more he spent  $\frac{1}{4}$  of the remainder, when he had only \$1500 remaining. What was his fortune? *Ans.* \$13500

23. A young man received 350 £. as his share of his fa-



ther's estate, which was  $\frac{3}{4}$  of his elder brother's portion; and the elder brother's portion was  $\frac{1}{2}$  of the whole estate. What was the whole estate? *Ans.* 2333 £. 6s. 8d.

24. If 356 persons consume 75 barrels of provision in 9 months, how many barrels will 500 men consume in the same time? *Ans.*  $102\frac{4}{3}$  barrels.

25. A. can mow an acre of grass in  $5\frac{1}{2}$  hours; B. can mow 2 acres in 9 hours. In what time will they both mow  $12\frac{1}{2}$  acres? *Ans.*  $30\frac{1}{6}$  hours.

26. If 6 men build a wall 20 feet long, 6 feet high, and 4 feet thick, in 16 days, in what time will 24 men build one 200 feet long, 8 feet high, and 6 feet thick? *Ans.* 80 days.

27. A farmer being asked how many sheep he had, replied, that in one pasture he had  $\frac{1}{4}$  of his whole flock, in another  $\frac{1}{6}$ , in another  $\frac{1}{8}$ , and  $\frac{1}{12}$  in another, and that a fifth pasture contained 450 sheep. How many did the five pastures contain? *Ans.* 1200.

28. If  $\frac{5}{8}$  of a gallon of wine cost  $\frac{5}{8}$  of a pound, New York currency, what will  $\frac{5}{8}$  of a tun cost in dollars and cents? *Ans.* \$262.50.

29. If  $\frac{3}{4}$  of an ounce cost  $\frac{7}{8}$  of a shilling, how many dollars, each 8s. will a pound cost? *Ans.* \$2.62 $\frac{1}{2}$ .

30. Bought 36 bags of rice, each weighing 84 lb., tret 4 lb. per 104. What will the whole neat weight amount to in federal money, at 8d. New York currency, per pound? *Ans.* \$242.307. +

31. In a certain orchard,  $\frac{1}{4}$  of the trees bear apples,  $\frac{1}{4}$  pears,  $\frac{1}{4}$  plums, and 60 bear peaches, and 40 cherries. How many trees are there? *Ans.* 1200.

32. Sold goods to the amount of \$560, by which I lost 18 per cent., whereas I ought to have gained 12 per cent. What was my real loss? *Ans.* 204.877. +

33. What is that number of beggars to whom if I give 3 pence apiece I shall want 8 pence more than I now have, but if I give them 2 pence apiece, I shall have 3 pence left. *Ans.* 11.

34. How much sugar at 9d. per lb. must be given in exchange for 492 lb. of rice at 3d. per lb.? *Ans.* 164 lb.

35. Reduce  $\frac{1}{2}$  of  $\frac{3}{4}$  of  $\frac{5}{8}$  of  $\frac{7}{12}$  of  $\frac{3}{14}$  of  $\frac{21}{4}$ , to a simple fraction. *Ans.*  $\frac{1}{16}$ .

36. What is the premium on \$1800, at 15 per cent.? *Ans.* \$270.

37. A father of 12 children said he was 24 years old when his oldest child was born, and that just a year and a half inter-

vened between the birth of each two of his children. What was the age of the father, at the birth of his youngest child? *Ans.* 40 $\frac{1}{2}$ .

38. Bought 16 bales of goods in London for 96 £.; paid 4 £. for shipment to New York. How ought I to sell the same in federal money to gain 20 per cent.? *Ans.* \$533.333.+

39. Bought a quantity of Irish linen in Dublin for 50 £., and paid 10 £. for the shipment of the same. How must I sell the whole in federal money to gain 30 per cent.? *Ans.* \$319.80.

40. What is the interest on \$462.50 for 3 years, 6 months, and 12 days? *Ans.* \$98.05.

41. Suppose there to be two silver cups, having one cover, which weighs 5 oz.; and suppose them to be such, that if the cover be placed on the smaller cup, the whole weighs twice as much as the greater cup; but if it be placed on the greater cup, the whole weighs three times as much as the smaller cup. What is the weight of each cup? *Ans.* The smaller cup weighs 3 oz., and the larger, 4 oz.

42. A. can do a piece of work in 6 days; B. can do the same in 11 days. In what time will they both accomplish the work, if they labor together? *Ans.* 3 $\frac{5}{11}$  days.

43. A. can do a piece of work in 7 days; B. in 12 days; C. in 6 days; and D. in 4 days. In what time will they accomplish the same work, if they all labor upon it at the same time? *Ans.* 1 $\frac{5}{12}$  day.

44. A person being asked the time of day, replied that it was between 5 and 6 o'clock, and that the hour and minute hand were precisely together. What was the time? *Ans.* 27 $\frac{3}{11}$  minutes past 5 o'clock.

45. How many square feet are there in a board 16 feet 8 inches long, and 9 inches broad? *Ans.* 12 $\frac{1}{2}$  square feet.

46. The linear measure of a cubic block being 8 inches, how many cubic blocks, each one solid inch, does it contain? *Ans.* 512.

47. If a person own  $\frac{2}{3}$  of a ship, and sell  $\frac{1}{3}$  of his share for \$1500, what is the value of the whole ship, at the same rate? *Ans.* \$6000.

48. Divide  $\frac{3}{4}$  of  $\frac{8}{9}$  of  $\frac{7}{12}$  of  $\frac{15}{16}$  by  $\frac{4}{5}$  of  $\frac{6}{7}$  of  $\frac{1}{2}$ . *Ans.* 1 $\frac{5}{16}$ .

49. In a thunder storm, I observed the time between the flash of the lightning and the report to be one minute and a half. How far distant was the lightning, allowing sound to travel 1142 feet in a second? *Ans.* 19 $\frac{1}{4}$  miles.

50. Bought 84 apples at the rate of 2 for a penny, and 114

at the rate of 3 for a penny; the whole of which I afterwards sold at the rate of 9 for 4 pence. Did I gain or lose, and how much? *Ans.* I gained 8 d.

51. A line 44 yards in length will just reach from the top of a steeple to the opposite side of the street, which is 24 yards wide. How high is the steeple? *Ans.* 36.87+ yards.

52. A tree 36 feet high stands by the side of a stream 27 feet wide. How many feet from the top of the tree to the opposite side of the stream? *Ans.* 45.

53. There is a cistern having two pipes leading into it, one of which will fill it in 30 minutes, and the other in 45 minutes. In what time will both, running together, fill the same? *Ans.* 18 minutes.

54. How many bricks 9 inches long and 4 inches broad will pave a yard 40 feet square? *Ans.* 6400.

55. A certain cistern has two pipes leading into it, and one leading out of it. Of the two leading into it, one will fill it in 50 minutes, and the other in 75 minutes; while the one leading from it, will empty it in 60 minutes. Now if all three be opened at the same time, in what time will the cistern be filled? *Ans.* 1 hour.

56. What is the difference between 6 dozen dozen, and half a dozen dozen? *Ans.* 792.

57. If  $\frac{1}{2}$  of  $\frac{1}{4}$  of  $\frac{1}{2}$  of a ship be worth  $\frac{2}{3}$  of  $\frac{1}{5}$  of  $\frac{1}{4}$  of her cargo, valued at \$2400, what is the value of both ship and cargo? *Ans.* \$5415.384.+

58. If 11 men can build a house in 5 months by working 12 hours per day, in what time will they complete it if they work only 8 hours per day? *Ans.* 7 $\frac{1}{2}$  months.

59. Bought a pipe of wine for \$84, from which 12 gallons leaked out. Now what shall I gain, if I sell the remainder at 12 $\frac{1}{2}$  cents a pint? *Ans.* \$30.

60. A. alone can do a piece of work in 12 days, and in connection with B., in 8 days. In what time can B. perform the same alone? *Ans.* 24 days.

61. If A. can do a piece of work in 12 days, and B. in 24 days, in what time will they both do it? *Ans.* 8 days.

62. Three men, A., B., and C., can do a piece of work in 18 days. A. alone can do it in 36 days; B. in 54 days. In what time can C. do it? *Ans.* 108 days.

63. Four men can do a piece of work in 15 days. A. alone can do it in 40 days, B. in 60 days, and C. in 80 days. In what time will D. do the work alone? *Ans.* 80 days.

64. A gentleman left his son a fortune,  $\frac{1}{3}$  of which he spent in 3 months;  $\frac{2}{3}$  of  $\frac{1}{3}$  of the remainder lasted him 6 months longer, when he had only \$1200 left. What was his whole fortune? *Ans.* \$6400.

65. A farmer bought a yoke of oxen, a horse and a cow, for \$250. For the oxen he paid twice as much as for the horse, and for the horse three times as much as for the cow. What did he pay for each? *Ans.* For the oxen, \$150; for the horse, \$75; and \$25 for the cow.

66. Three men start at the same time to travel round an Island 80 miles in circumference, and agree that each shall continue to travel at the same rate till they all come together again, and that the first shall travel 5 miles, the second, 6 miles, and the third, 7 miles per day. In what time will the three come together, and how far will each have traveled? *Ans.* 80 days. The first will have traveled 400 miles; the second, 480 miles; and the third, 560 miles.

67. How will it affect the distance traveled by each, provided they travel 5, 7, and 9 miles respectively? *Ans.* The distance of the first will be 400 miles; of the second, 560; and of the third, 720.

68. Divide 1200 acres of land between A., B., and C., so that B. may have 75 acres more than A., and C. 95 acres more than B. What will be the share of each one? *Ans.* A's share will be 318 $\frac{1}{3}$ ; B's, 393 $\frac{1}{3}$ ; and C's, 488 $\frac{1}{3}$  acres.

69. Three men met at an Inn, two of whom brought provision with them. The third not having brought any, proposed that they should eat together, and then he would pay his proportion. The proposal being accepted, A. produced 5 loaves, and B. 4 loaves, all of which they ate up, and C. as his share, paid 9 pieces of money. With this, A. and B. were satisfied, but could not agree as to the division. What should each have received? *Ans.* A. 6, and B. 3 pieces of the money.

70. If A. can reap a field of grain in 12 days, and B. in 16 days, in what time will both do it, working together? *Ans.* 6 $\frac{2}{3}$  days.

71. What number must be added to  $\frac{1}{3}$  of 4560, to make the same 500? *Ans.* 369 $\frac{2}{3}$ .

72. If the head of a fish be 9 inches long, its tail as long as its head and half the length of its body, and its body as long as its head and tail both, how long is the fish? *Ans.* 72 inches.

73. If 6 pairs of hose are equal in value to 4 pieces of Hol-

land, and 6 pieces of Holland to 14 yards of satin, and 12 yards of satin to 8 pieces of lace, and 9 pieces of lace to 8 £. 2 s., how many pairs of hose may be bought for 2 £. 16 s. ? *Ans.* 24 pairs.

74. A man was hired 50 days, on condition that for every day he worked, he should receive 6 s., and for every day he was idle, he should forfeit 2 s. At the expiration of the time, he received \$27.50. How many days did he work, and how many was he idle? *Ans.* He labored 40 days, and was idle 10 days.

*Note.*—The currency of the preceding sum is that of New York.

75. A hare starts 40 yards in advance of a greyhound, and is not perceived by him till she has been up 40 seconds. She scuds away at the rate of 10 miles per hour, and the hound pursues after her at the rate of 18 miles per hour. In what time will the hound overtake the hare, and how far will he have run? *Ans.* The required time is  $60\frac{5}{12}$  sec.; the distance run, 530 yards.

76. If 8 men can build a wall 15 rods long in 10 days, how many men will be required to build 45 rods of wall in 5 days? *Ans.* 48 men.

77. A man when he married was three times as old as his wife; after they had been married 15 years, his age was only double that of his wife's. How old were they when they married? *Ans.* The man was 45 years, his wife 15 years old.

78. There is in a pasture a certain number of sheep, cows, and oxen; there are twice as many sheep as cows, and three times as many cows as oxen, and the whole number is 80. How many are there of each kind? *Ans.* 8 oxen, 24 cows, and 48 sheep.

79. Sold coffee at 15 cents per pound, and thereby lost 10 per cent. on the first cost. Afterwards sold a quantity of the same for \$525, by which I gained 40 per cent. What was the quantity sold, and what the price per pound? *Ans.* Quantity, 20 cwt. 10 lb.; price,  $23\frac{1}{2}$  cents.

80. Two sons, one 11 and the other 16 years of age, received a bequest of \$10000, to be so divided between them that the shares being put on interest at 5 per cent. should amount to equal sums, when they became respectively 21 years of age. What were the shares of each? *Ans.* The elder received \$5454 $\frac{6}{11}$ , and the younger, \$4545 $\frac{5}{11}$ .

## APPENDIX.

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**Prob. 1.** To find the greatest common measure of two or more numbers.

*Note 1st.*—The greatest common measure of two or more numbers, is the *greatest number that will divide them separately without remainders.*

**RULE.**—*If two numbers only are given, divide the greater of them by the less, and if nothing remain, that divisor is the common measure; but if there be a remainder, divide the preceding divisor by it; and so continue to divide each preceding divisor by the last remainder, till the division is effected without remainder; the last divisor will be the common measure required. When more than two numbers are given, find the common measure of any two of them first, and then of that common measure, and either of the remaining numbers. This process carried through all the numbers will give their greatest common measure.*

**Ex. 1.** What is the greatest common measure of 72 and 108?

OPERATION.

$$\begin{array}{r}
 72 \overline{) 108} (1 \\
 \underline{72} \\
 36 \overline{) 72} (2 \\
 \underline{72} \\
 00
 \end{array}$$

36 is, therefore, the common measure required. Proof,  $108 \div 36 = 3$ , and no remainder.  $72 \div 36 = 2$ , and no remainder.

**2.** What is the greatest common measure of 27 and 99?  
*Ans.* 9.

3. What is the greatest common measure of 25, 45, and 90 ?  
*Ans.* 5.

4. What is the greatest common measure of 16, 32, 48, and 96 ? *Ans.* 16.

Prob. 2. To determine how many different positions any given number of objects may assume with regard to each other.

*RULE.*—*Represent the number of objects by the figures 1, 2, 3, 4, 5, &c. making the numbers of figures equal to the number of objects. The product of these figures will determine the number of changes.*

Ex. 1. How many changes may be made by the first three letters of the alphabet ?

Operation :  $1 \times 2 \times 3 = 6$ , *Ans.* These changes are as follows : 1st, a, b, c ; 2d, a, c, b ; 3d, b, a, c ; 4th, b, c, a ; 5th, c, b, a ; 6th, c, a, b.

2. How many changes may be made by the first 6 letters of the alphabet ? *Ans.* 720.

3. At a certain boarding house there are 12 boarders. How many different positions may they occupy at the table ? *Ans.* 479001600.

4. Five men engaged board at a tavern for as many days as the landlord might be able to seat them in different positions. How long did they remain ? *Ans.* 120 days.

Prob. 3. To find the area of a square.

*RULE.*—*Multiply the length of one of the sides by itself ; or, square its linear measure. (See fig. 2d, Square Root.)*

Ex. 1. There is a room just 8 feet square, what is the area ?

Operation,  $8 \times 8 = 64$  square feet, *Ans.*

2. What is the area of a floor 19 feet square ? *Ans.* 361 square feet.

3. What is the area in square inches of a board 3 feet square ? *Ans.* 1296 square inches.

Prob. 4. To find the area of a parallelogram.

*RULE.*—*Multiply the length of the parallelogram by its breadth. (See fig. 6th, Square Root.)*

**Ex. 4.** How many square feet are there in a floor 16 feet long and 12 broad?  $16 \times 12 = 192$  square feet, *Ans.*

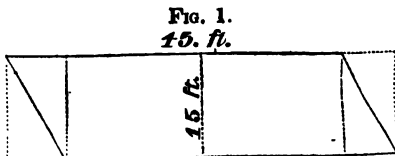
*Note 1st.*—If the dimensions of a field in the form of a square or parallelogram are given in rods, the area is reduced to acres by dividing the square rods by 160.

2. How many acres are there in a piece of land 80 rods in length, and 40 in breadth?  $80 \times 40 = 3200$ , and  $3200 \div 160 = 20$  acres, *Ans.*

3. What is the number of acres in a piece of land 63 rods long and 49 rods wide? *Ans.*  $19\frac{47}{160}$ .

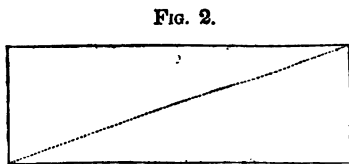
*Note 2d.*—If the parallelogram be not right-angled, the length must be multiplied into the perpendicular distance between the sides. (See fig. 1st.)

The dotted line in the center is the perpendicular distance required; therefore,  $45 \times 15 = 675$ , the area of the figure.



**Prob. 5.** To find the area of triangles.

The area of any triangle is just equal to one half the area of a square or parallelogram of the same height. This is obvious with regard to right-angled triangles, and true of all others. The diagonal of a square or parallelogram divides it into two equal right-angled triangles, and since the area of the whole figure is found by multiplying the length into the breadth, the area of its half, that is, of either of the triangles into which it is divided by the diagonal, is found by multiplying the same length or base into one half the breadth.



We have then the following rule :

**RULE.**—If the triangle be right-angled, multiply its base into half its perpendicular height. But if it be not right-angled, drop a perpendicular from one of the angles to the opposite side



or base; then multiply the base or side upon which the perpendicular falls, by one half the perpendicular.

Ex. 1. What is the area of a triangular piece of land whose base is 40 rods, and whose perpendicular is 30 rods?  $30 \div 2 = 15$ , and  $40 \times 15 = 600$  square rods, *Ans.*

*Note.*—The same result is obtained by multiplying the perpendicular by one half the base. Thus,  $40 \div 2 = 20$ , and  $30 \times 20 = 600$ .

2. There is a room 26 feet long and 18 feet high. If a line be drawn from one of the upper corners to its opposite lower corner, the side of the room will be divided into two equal triangles. What is the area of each triangle, and also of the side of the room? *Ans.* Each triangle contains 234 square feet, and the side, 468.

3. How many acres are there in a triangular piece of land, whose base is 42 rods and whose perpendicular height is 36 rods? *Ans.*  $4\frac{1}{2}$  acres.

Prob. 6. Given the diameter of a circle, to find its circumference.

The diameter of a circle is to its circumference, as 7 to 22; or, as 1 to 3.141592. Therefore,

**RULE.**—Multiply the diameter of a circle by 3.141592, and the product will be the circumference; or, multiply the diameter by 22, and divide the product by 7.

Ex. 1. What is the circumference of a wheel whose diameter is 8 feet.  $8 \times 22 \div 7 = 25.14 +$  feet, *Ans.* Or,  $8 \times 3.141592 = 25.132736$ , nearly the same as before.

2. What is the circumference of a circle whose diameter is 6 feet? *Ans.* 18.85 feet.

3. What is the circumference of a circle whose diameter is 42 feet? *Ans.* 132 feet.

Prob. 7. Given the circumference of a circle, to find the diameter.

**RULE.**—Multiply the circumference by 7 and divide the product by 22; or, (what will produce nearly the same result,) divide the circumference by 3.141592.

**Ex. 1.** If the circumference of a wheel be 26 feet, what is the diameter?  $26 \times 7 \div 22 = 8.27+$  feet, *Ans.*

2. What is the diameter of a wheel whose circumference is 50 feet? *Ans.* 15.9+ feet.

**Prob. 8.** To find the area of a circle.

**RULE.**—*Multiply half the diameter by half the circumference.*

**Ex. 1.** If the diameter of a circle be 6 feet, what is its area?  $6 \times 22 \div 7 = 18.857$  feet, the circumference. Therefore,  $18.857 \div 2 = 9.4285$ , one half the circumference, and  $6 \div 2 = 3$ , one half the diameter; then,  $9.4285 \times 3 = 28.2855$ ,+ the area in feet and decimals.

2. If the diameter be 20 feet, what is the area of the circle? *Ans.* 314.285 feet?

3. What is the area of a circle 66 feet in circumference? *Ans.* 346.5

**Prob. 9.** To find the superficial area of a globe.

The superficial area of a globe is four times that of a circle of the same diameter. Therefore,

**RULE.**—*Find the area of a circle of the same diameter as the given globe, and multiply that area by 4.*

**Ex. 1.** What is the superficial area of a globe 16 inches in diameter?  $16 \times 22 \div 7 = 50.2757$ .  $50.2757 \div 2 = 25.13785$ , half the circumference,  $16 \div 2 = 8$ , half the diameter; therefore,  $25.13785 \times 8 \times 4 = 804.4+$  square inches, the required area.

2. If the earth be 8000 miles in diameter, and 25000 in circumference, how many square miles on its whole surface. *Ans.* 200000000.

**Prob. 10.** To find the solid content of a globe.

**RULE.**—*Find the superficial area by the preceding problem, and multiply that area by  $\frac{1}{6}$  of the diameter of the globe.*

**Ex. 1.** If the diameter of the earth be 8000 miles, and the circumference 25000 miles, how many cubic miles does it contain.

The superficial area was found by the preceding problem to be 200000000; and  $\frac{1}{2}$  the earth's diameter is  $8000 \div 6 = 1333\frac{1}{3}$ ; therefore,  $200000000 \times 1333\frac{1}{3} = 266666666666\frac{2}{3}$  cubic miles, *Ans.*

Prob. 11. To find the solid content of a cylinder.

A cylinder is a round body of uniform diameter, and of any assignable length. Each end of a cylinder is therefore a circle.

**RULE.**—Find the area of one end of the cylinder, and multiply that area by its length.

Ex. 1. What is the solid content of a cylinder 6 feet in length and 3 feet in diameter? The area of the end, as found by the preceding rule, is 7.06; and  $7.06 \times 6 = 42.36$ .

2. What is the solid content of a log, the uniform diameter of which is 2 feet, and the length 30 feet? *Ans.* 94.247 solid feet.

3. What fraction of a cord is there in a log 10 feet long, and 4 feet in diameter? *Ans.*  $\frac{127}{180}$ .

Prob. 12. In a stick of timber of uniform thickness, to find what length will make a solid foot.

**RULE.**—Find the area of one end in inches, and divide 1728 by it; the quotient will be the required length in inches.

Ex. 1. If a stick of timber be 16 inches square, what length will be required to make a solid foot?  $16 \times 16 = 256$ , the area of one end, and  $1728 \div 256 = 6\frac{3}{4}$  inches, *Ans.*

2. How much length of plank 3 inches thick and 24 inches wide, will be required to make a solid foot? *Ans.* 2 feet.

Prob. 13. To find the solid content of cones and pyramids.

A pyramid or cone is a solid whose base is a plain surface, and which gradually and uniformly tapers till it comes to a point. It may be either round, square, or triangular.

The solid content of such figures is  $\frac{1}{3}$  as much as the content of a cylinder of the same length; therefore,

**RULE.**—Multiply the area of the base by one third of the perpendicular height.

Ex. 1. What is the solid content of a cone 60 feet high,

the base of which is 6 feet in diameter?  $8 \times 22 \div 7 = 25.14$  circumference, and  $25.14 \div 2 \times 4 = 50.28$ , area of the base; and  $50.28 \times 20$ , (one third the perpendicular height)  $= 100.56$ , the solid content.

2. What is the solid content of a pyramid, whose base is 4 feet square, and whose height is 15 feet? *Ans.* 80 solid feet.

3. How many cubic inches in a cone 18 inches high, and 12 inches in diameter at the base? *Ans.* 678.6.

4. There is a pyramid whose base is 30 feet in diameter, and the height of it is 90 feet. What is its solid content? *Ans.* 21205.73+ solid feet.

Prob. 14. To find the solidity of the frustum of a cone or pyramid.

A *frustum* is that part of a cone or pyramid that remains after a part has been cut off from the apex or top.

**RULE.**—Find the diameter of each end of the cone or pyramid, and multiply them together; then to the product add one third of the square of the difference of the diameters, and multiply the sum by the decimal .785398; the product will be the mean area between the two bases; therefore, multiply this mean area by the length of the frustum.

Ex. 1. What is the solid content of a frustum of a cone 30 feet in length, whose base measures 4 feet in diameter, and whose top measures 2 feet in diameter,  $4 \times 2 = 8$ . The difference of diameters is  $4 - 2 = 2$ , and  $2^2 = 4$ , and  $4 \div 3 = 1\frac{1}{3}$ . Therefore,  $8 + 1\frac{1}{3} \times .785398 = 7.33$ , nearly, the mean diameter; and  $7.33 \times 30 = 219.9$ , the required content.

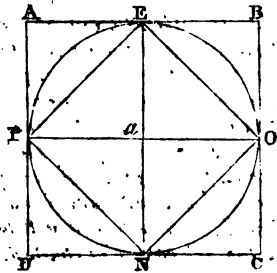
2. What is the content of a stick of timber, of which the larger end measures 2 feet in diameter, and the smaller end, 1 foot, the length being 40 feet? *Ans.* 73.3 feet, nearly.

Prob. 15. Given the length and diameter of a log or stick of round timber, to determine the solid content of the greatest square timber that can be obtained from it.

The area of the greatest square that can be described upon the end of the given log must first be determined; that is, the area of a square inscribed in a circle, the diameter of which equals the diameter of the log. How then is the area of an inscribed square, that is, of a square the corners of which all

terminate in the circumference of a circle, to be found, the diameter of that circle being given?

If the scholar will examine the adjoining figure, he will see that the square  $ABCD$ , which is circumscribed about the circle, is equal to the square of the diameter of the circle, since the diameter  $EN$  equals the side of  $AD$ ; and  $AD$  squared gives the area of the square  $ABCD$ ; also, that the inscribed square  $PEON$ , is just one half of the circumscribed square, since each of the triangles



$EaO$ ,  $OaN$ ,  $NaP$ , and  $PaE$ , into which the inscribed square is divided, is precisely one half of each of the four squares,  $APaE$ ,  $EaOB$ ,  $OaNE$ , and  $NaPD$ , into which the circumscribed square,  $ABCD$ , is divided. If therefore a fourth part of the inscribed square equals one half of a fourth part of the circumscribed square, the whole inscribed square equals one half of the circumscribed square. Hence,

**RULE.**—Square the diameter of the small end of the log and multiply one half of this square by the length of the log; the product will be the content in solid feet, if the length and diameter are both given in feet; but if one of them be expressed in inches, divide this product by 144; and if both, by 1728, and the quotient will express the solid content in feet.

**Ex. 1.** How many solid feet of square timber may be sawn from a log 2 feet in diameter, and 20 feet in length?  $2^2=4$ , and  $4 \div 2=2$ ; then,  $2 \times 20=40$  solid feet, *Ans.*

2. How many solid feet of square timber may be obtained from a log 10 inches in diameter, and 15 feet long? *Ans.*  $5\frac{5}{4}$  solid feet.

3. How many square feet of timber are there in a log 43 feet long, allowing the diameter to be 1 foot 6 inches at the small end; and how many solid feet of slab would be taken off in squaring it, allowing the diameter at the large end to be 2 feet? *Ans.* 48.37 + solid feet of timber, and 55.76 solid feet of slab.

Prob. 16. To find the capacity of casks in gallons, &c.

**RULE.**—Take the length of the cask between the heads, and also its interior bung and head diameter, in inches. Multiply the difference between the head and bung diameter by .62, when the staves are of ordinary curvature; but by .86, if the curvature be more than ordinary, and by .58, if they be curved less than usual, and add the product to the head diameter; the sum will be the mean diameter. Multiply the square of the mean diameter by the length, and this product by 84 for wine, and by 28 for beer measure, and from the product cut off four decimals. The number obtained will be the gallons, and the decimal of a gallon, the cask will contain.

Ex. 1. How many gallons of wine are there in a cask whose bung diameter is 34 inches, head diameter 30 inches, and whose length is 60 inches?

Operation:  $34 - 30 = 4$ , the difference between the head and bung diameter; then,  $4 \times .62 = 2.48$ , and  $2.48 + 30 = 32.48$ , mean diameter, and  $32.48 \times 32.48 = 1054.9504$ , square of mean diameter.  $1054.9504 \times 60 = 63297.0240 \times 84 = 215.20988160$ , or nearly 215.2 gallons, Ans.

2. Suppose the bung diameter of a cask of wine to be 24 inches, and the head diameter to be 20 inches, and the length 40 inches; how many gallons will it contain? Ans. 68½ gallons, nearly.

Prob. 17. To find the tonnage of double and single decked vessels.

The following is the rule established by Congress:

**RULE.**—“If the vessel be double-decked, take the length thereof from the fore part of the main stern to the after part of the stern-post, above the upper deck; the breadth thereof at the broadest part above the main wales, half of which breadth shall be accounted the depth of such vessel, and then deduct from the length, three fifths of the breadth. Multiply the remainder by the breadth, and the product by the depth, and divide this last product by 95; the quotient whereof shall be deemed the true contents or tonnage of such ship or vessel. But if such ship

*or vessel be single-decked, take the length and breadth, as above directed, deduct from said length three fifths of the breadth, and take the depth from the under side of the deck plank to the ceiling in the hold; then multiply and divide as aforesaid, and the quotient shall be deemed the tonnage."*

*Note.*—Carpenters multiply together the length of the keel, the breadth at the main-beam, and the depth of the hold, each dimension being taken in feet; and divide the product by 95, for the tonnage of single-decked vessels. But for double-decked vessels, they take half the breadth at the beam for the depth of the hold, and then work as before.

Ex. 1. Suppose the length of a double-decked vessel to be 80 feet, and its breadth to be 30 feet. What is its tonnage?

Operation:  $30 \div 2 = 15$ , the depth of the vessel;  $\frac{2}{3}$  of  $30 = 18$ , and  $80 - 18 = 62$ ; and  $62 \times 30 = 1860$ ; then,  $1860 \times 15 = 27900$ ; and, lastly,  $27900 \div 95 = 293\frac{1}{2}$  tons, *Ans.*

2. Required the tonnage of a double-decked vessel, whose length is 95 feet, and whose breadth is 35 feet? *Ans.* 477 $\frac{1}{2}$ .

3. What is the tonnage of a single-decked vessel, which is 75 feet in length, 30 feet in breadth, and 10 feet in depth? *Ans.* 180 tons.

*Note.*—The tonnage of a ship of war is found by dividing the continued product of the length, breadth, and depth by 100.

4. What is the tonnage of a sloop of war, whose length is 96 feet, breadth 32 feet, and depth 15 feet? *Ans.* 460.8 tons.

Prob. 18. The difference of longitude between two places being given, to find the difference in time.

As the circumference of the earth is divided into 360 degrees, and as the sun appears to pass round the earth in 24 hours, it is evident that  $360 \div 24 = 15$  degrees of the earth's circumference must pass under the sun every hour, and consequently 1 degree every 4 minutes, and 1' every 4 seconds. We have then the following general rule:

**RULE.**—If the given degrees be more than 15, divide them by 15; the quotient will be the hours. But if the degrees be less than 15, multiply them by 4, and the product will be minutes.

*Multiply also the given minutes of motion by 4, and the product will be seconds of time.*

Ex. 1. Warsaw is situated about  $17^{\circ}$  east longitude from Brussels. What is the difference in time?  $17 \div 15 = 1\frac{1}{3}$  hours, or 1 hour, 8 minutes.

It is obvious that the sun will rise first at the most eastern of any two places. Hence, when the time of one place is given, and it is required to find the time of some other place, *the difference of time must be added to the given time, when that place is situated to the east, and subtracted when it is situated to the west of the place, at which the time is given.*

2. Turin is situated  $8^{\circ}$  east from London, and Moscow  $38^{\circ}$  east from the same place. What is the hour at Moscow, when it is 9 o'clock at Turin? *Ans.* 11 o'clock.

3. When it is 9 o'clock at Philadelphia, what time is it at Council Bluff,  $21^{\circ}$  west of Philadelphia? *Ans.* 36 minutes past 7 o'clock.

4. What is the difference of time between Washington and Harmony, a missionary station west of Missouri, the difference of longitude being  $18^{\circ}$ ? *Ans.* 1 hour, 12 minutes.

Prob. 19. The difference in time between any two places given, to find the difference of longitude.

This is the reverse of the preceding; therefore,

**RULE.**—*Multiply the hours by 15 and the products will be degrees. If minutes and seconds of time be given, divide them by 4; the quotient of the minutes will be degrees, and that of the seconds, will be minutes of motion.*

Ex. 1. If the difference of time between two places be 3 hours and 32 minutes, what is the difference of longitude?  $3 \times 15 = 45^{\circ}$ , and  $32 \div 4 = 8^{\circ}$ ; and  $45^{\circ} + 8^{\circ} = 53^{\circ}$ , *Ans.*

2. What is the difference of longitude between two places, if the time of one place be 5 hours, 15 minutes in advance of the time of the other place? *Ans.*  $78^{\circ} 45'$ .

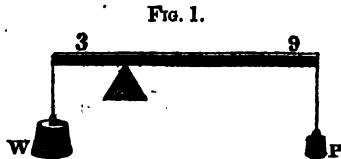


## MECHANICAL POWERS.

When one body is made to communicate motion to another, the body communicating the motion is called *a power*, and the one to which the motion is communicated, is called *a weight*. The various instruments employed to increase the effect of a given power, are called *Mechanical Powers*.

The mechanical powers are six in number, viz. the *Lever*, the *Pulley*, the *Wheel and Axle*, the *Inclined Plane*, the *Screw*, and the *Wedge*.

The lever is a bar, movable about a fixed point. It is represented at Fig. 1. P represents the power, and W the weight, and 3 and 9 the comparative length of the arms of the lever.



When the power and weight are such that the products obtained by multiplying each into the length of the arm to which it is attached, are equal, they will remain in equilibrium, that is, they will balance each other.

In fig. 1, the arm to which the power is applied is three times as long as that to which the weight is applied. Consequently, the power is required to be only one third as heavy as the weight, in order that they reciprocally balance each other. We hence perceive that the power is to the weight, as the arm to which the weight is applied, is to that to which the power is applied.

Prob. 20. Given the two arms of a lever, and the power, to find what weight that power will balance.

**RULE.**—Multiply the length of the arm to which the power is applied, by the power, and divide the product by the other arm.

Ex. 1. There is a lever 16 feet in length divided by the fulcrum or point of support into two arms, one of which is 12

feet, and the other 4 feet in length. What weight on the short arm will balance 20 lb. suspended at the end of the long arm?  $20 \times 12 = 240$ , and  $240 \div 4 = 60$  lb. *Ans.*

2. The arms of a lever are, the one 30 feet, and the other 3 feet in length. What weight will a power of 160 lb. at the extremity of the long arm, balance at the extremity of the short arm? *Ans.* 1600 lb.

3. How many lb. will a power of 9 lb. placed 15 feet from the fulcrum of a lever, support at the extremity of the other arm, 2 feet in length? *Ans.*  $67\frac{1}{2}$  lb.

Prob. 21. Given the arms of the lever and the weight, to find the power.

*RULE.*—Multiply the weight by the length of the arm to which it is suspended, and divide the product by the other arm. The quotient will be the power required.

Ex. 1. A weight of 80 lb. is attached to an arm of a lever 3 feet in length; what power must be applied to the other arm, 27 feet in length, to balance it?  $80 \times 3 = 240$ , and  $240 \div 27 = 8\frac{8}{9}$  lb. *Ans.*

2. A weight of 20 tons is suspended to an arm of a lever 6 inches in length. What weight at the extremity of the other arm, 40 feet in length, will balance the same? *Ans.* 5 cwt.

Prob. 22. Given the power, the weight, and the length of the lever, to find the fulcrum.

*RULE.*—Add together the numbers representing the power and weight, and say, as their sum is to the whole lever, so is the number representing the power to the length of the arm to which the weight is to be applied; or, so is that representing the weight to the length of the arm to which the power is to be applied.

Ex. 1. Where must be the fulcrum in a lever 16 feet in length, so that a power of 20 lb. shall balance a weight of 60 lb?  $20 + 60 = 80$ . Therefore,  $80 : 60 :: 16 : 12$ , *Ans.* or length of the arm to which the power is to be applied, and  $16 - 12 = 4$ , length of the other arm.

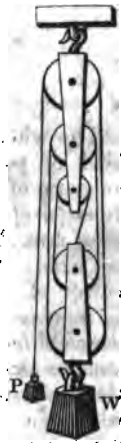
2. If the power be 72 lb., the weight 1728 lb., and the lever 36 feet, where must be the fulcrum, in order that they shall balance each other? *Ans.*  $1\frac{3}{8}$  feet from the weight.

9. If the length of a lever be 10 feet, the power 170 lb., and the weight to be balanced 1530 lb., where must be the fulcrum?  
*Ans.* 9 feet from the power.

## PULLEY.

A pulley consists of a wheel movable about its axis, and so arranged as to be put in motion by means of a cord passing over it. The movable pulley is the only one by which a gain in power is effected. A double movable pulley is represented at fig. 2. By such a pulley an equilibrium is produced, when the power is to the weight, as one to the number of ropes sustaining the weight. If a single movable pulley be employed, the weight is sustained by two ropes; and the power will be to the weight, as 1 to 2. If two movable pulleys be employed, the weight is sustained by four ropes; and the power will be to the weight, as 1 to 4, &c.; that is, each movable pulley is sustained by two ropes, and consequently doubles the effect of the power employed.

FIG. 2.



Prob. 23. Given the number of movable pulleys and the power, to find the *weight*.

**RULE.**—Multiply the power applied by twice the number of movable pulleys.

Ex. 1. If in a system of pulleys there be three movable pulleys, what weight will a power of 72 lb. balance?  $72 \times 6 = 432$  lb. *Ans.*

2. What weight will a power of 15 lb. balance by a system of 6 movable pulleys? *Ans.* 180 lb.

3. By the aid of a system of 12 movable pulleys, how many pounds will a man sustain who is capable of applying a power of 150 lb.? *Ans.* 3600.

Prob. 24. Given the number of movable pulleys, and the weight to be balanced, to find the required *power*.

**RULE.**—*Divide the weight by twice the number of movable pulleys.*

**Ex. 1.** How many pounds would be required by the aid of two movable pulleys, to sustain 800 lb. ? *Ans.* 200 lb.

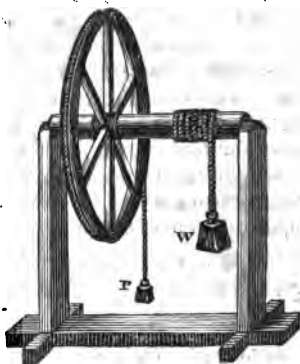
**2.** By the aid of 10 movable pulleys, how many lb. would be required to balance 2000 lb. ? *Ans.* 100 lb.

**3.** What weight would balance 10 pounds of silver by the aid of a system of 100 pulleys ? *Ans.* 12 pwt.

## WHEEL AND AXLE.

The wheel and axle form a kind of perpetual lever, the long arm of which is the *semi-diameter of the wheel*, and the short arm, the *semi-diameter of the axle*. Consequently, the power of the wheel and axle is increased either *by making the wheel larger*, while the axle remains unaltered, or *by making the axle smaller*, while the wheel remains the same. The power must always be applied to the wheel, and the weight to the axle, when an increase of action is to be gained. In the use of the wheel and axle, an equilibrium is produced, when the power is to the weight as the *semi-diameter of the axle* is to the *semi-diameter of the wheel*,

FIG. 3.



**Prob. 25.** Given the diameter of the wheel, the diameter of the axle, and the power, to find the weight.

**RULE.**—*Multiply the diameter of the wheel by the power applied, and divide the product by the diameter of the axle.*

**Ex. 1.** If the diameter of the axle be six inches, and that of the wheel 6 feet, what weight attached to the axle, will 16 lb. attached to the wheel, balance ? *Ans.* 192 lb.

2. If the diameter of a wheel be 20 feet, and that of the axle 2 feet, and a power of 400 lb. be applied to the wheel, what weight will be balanced at the axle? *Ans.* 4000 lb.

Prob. 26. Given the diameter of the wheel, the diameter of the axle, and the weight to be balanced, to find the required power.

**RULE.**—*Multiply the diameter of the axle by the weight, and divide the product by the diameter of the wheel.*

Ex. 1. If the diameter of the axle be 6 inches, and the diameter of the wheel 12 feet, what power will balance a weight of 360 lb? *Ans.* 15 lb.

2. If the diameter of the axle be 2 feet, and the diameter of the wheel 20 feet, what power will balance a weight of 4000 lb.? *Ans.* 400 lb.

## THE INCLINED PLANE.

An inclined plane is a plane that makes with the ground or floor some *certain angle less than a right angle*. In the application of this instrument, the ratio of the power and weight is always the same as that of the *height and length of the plane*.

Prob. 27. Given the length and height of the plane, and also the power, to determine the weight.

**RULE.**—*Multiply the power by the length of the plane, and divide the product by its perpendicular height.*

Ex. 1. If the length of a plane be 16 feet, and its height 4 feet, what weight will a power of 32 lb. sustain? *Ans.* 128 lb.

2. What weight will 164 lb. sustain on a plane 112 feet in length and 3 feet in height? *Ans.* 6122 $\frac{2}{3}$  lb.

Prob. 28. Given the length and height of the plane, and also the weight, to find what power will sustain it:

**RULE.**—*Multiply the weight by the height of the plane, and divide the product by the length.*

Ex. 1. What power will balance 128 lb. on an inclined plane, the length of which is 16 feet, and the height 4 feet? *Ans.* 32 lb.

2. Suppose on a railroad there is an inclined plane 150 rods in length, and rising to the perpendicular height of 50 feet; what power will be required to sustain a weight of 84000 lb. *Ans.* 28000 lb.

## THE WEDGE.

The wedge is composed of two inclined planes, whose bases unite and form the base of the wedge. The power applied to the wedge is to the effect produced at the side of the wedge, as the thickness of the head to the length of the wedge, no allowance being made for friction.

Prob. 29. Given the thickness of the head, the length of the side, and the power acting upon the head of a wedge, to determine the force produced on the side.

*RULE.*—Multiply the length of the wedge by the power applied, and divide the product by the thickness of the head.

Ex. 1. If the length of a wedge be 12 inches, the thickness of the head 3 inches, and the force applied, 64 lb., what will be the resistance at the side? *Ans.* 256 lb.

2. If the length of a wedge be 32 inches, the thickness of the head 2 inches, and the force applied 1600 lb., what will be the resistance at the side? *Ans.* 25600 lb.

Prob. 30. Given the length of the side, the thickness of the head, and the resistance upon the side of a wedge, to find the force acting upon the head.

*RULE.*—Multiply the resistance at the side by the thickness of the head, and divide the product by the length of the side of the wedge.

Ex. 1. If the length of the wedge be 32 inches, the thickness of the head 2 inches, and the resistance at the side be 25600 lb. what must be the force upon the head, no allowance being made for friction? *Ans.* 1600 lb.

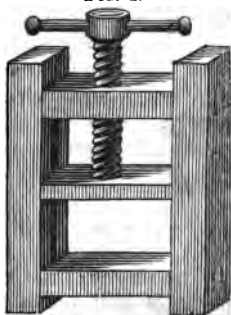
2. If the resistance at the side of a wedge be 20000 lb., the length of the wedge 20 inches, and the thickness of the head 3 inches, what force is required to be applied to counteract the resistance at the sides? *Ans.* 3000 lb.

## THE SCREW.

The screw is a cylindrical piece of wood, or metal, with a spiral thread running round it with a gradual and uniform inclination. The thread, therefore, forms an *inclined plane* of the same length as the thread itself, and whose height equals the length of the screw.

It is then obvious that the power of the screw depends in part upon the fineness of the threads, since the finer they are, the greater will be the length of the plane, while its height remains the same. The power of the screw also depends in part on the *length of the lever* employed to move it. It is, therefore, a *compound power*, involving in its action both the principle of the lever and the inclined plane, and is consequently more efficient than either of the other mechanical powers. In the action of this instrument, an equilibrium between the power and weight is produced, when the *power is to the weight as the distance between two contiguous threads of the screw is to the circumference of the circle described by the power in one revolution.*

FIG. 4.



Prob. 31. Given the distance between the threads of the screw, the length of the lever, and the power applied to find the weight.

**RULE.**—*Multiply the circumference of the circle described by one revolution of the lever by the power applied, and divide the product by the distance between the threads of the screw.*

**Note.**—The lever is the semi-diameter of the circle it describes; the circumference of the circle may therefore be found by Prob. 6th.

Ex. 1. If the threads of a screw be 2 inches apart, the lever 40 inches in length, and a power of 60 lb. be applied to the end of the lever, what weight will be required to produce an equilibrium? *Ans.* 7542 lb. 13 oz. 11 $\frac{1}{2}$  drams.

2. If the distance between the threads of a screw be 1 inch, the length of the lever 16 inches, and the power applied to the end of the lever be 40 lb., what weight will produce an equilibrium? *Ans.* 4022.8 + lb.

Prob. 32. Given the weight, the length of the lever, and the distance between the threads of the screw, to find the power required to produce an equilibrium.

*RULE.*—Multiply the given weight by the distance between the threads of the screw, and divide the product by the circumference of the circle described by one revolution of the lever.

Ex. 1. How many pounds applied to the end of a lever 16 inches long, will balance 4022.8 lb. upon a screw, whose threads are one inch apart? *Ans.* 40 lb.

2. How many pounds applied to the end of a lever 30 inches long, will produce an equilibrium with 4000 lb. attached to a screw whose threads are 2 inches apart? *Ans.* 42 $\frac{1}{2}$  lb. nearly.

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## STEAM POWER.

The power of the steam engine is usually estimated by comparing it with that of the horse. There are, however, various circumstances, which tend to render the power of the horse an indefinite standard; such as the natural strength of the animal,—the training it may have received to enable it to exert its strength to the best advantage,—its position while exerting its strength,—the length of time the exertion is to be continued, and the velocity of motion required. Before the term, “horse power,” could be employed to communicate any uniform idea of force exerted, it was therefore necessary to determine by experiment, what was the medium power of the horse, making due allowance for varying circumstances.

Since, however, the power required to move any given body in a horizontal direction is dependent on various circumstances besides its actual weight, and as a diminished force only is re-



quired to continue a body in motion, when motion in that direction is once communicated, a medium power could be obtained only by repeated experiments on the power of the horse in raising weights in a perpendicular direction, like raising a stone from a well, while the animal was made to act in a horizontal direction through the agency of a single pulley. Experiments were accordingly instituted, the result of which has been to establish the fact, that a one horse power is a power competent to raise 2250 lb. one mile in a perpendicular direction in 6 hours of time. This fact being thus established, the term, "horse power," has acquired a distinctness of meaning, which renders it a convenient standard by which to estimate the power of machinery, especially when propelled by steam.

The weight given, viz. 2250 lb., is a little more than a ton; an amount entirely beyond the power of any horse to move as above stated. The velocity of one mile in 6 hours of time, is, on the other hand, far less than the natural velocity of the horse. How are we then to understand the estimation of horse power here presented? It will be remembered that in all machinery by which a *gain of power* is effected, the power applied to put the machinery in motion, always moves through a greater space than the body moved by the same machinery; or, in other words, "what is gained in power is lost in time." This fact presents us with a full explanation of the matter. A horse with a moderate burden will easily move 3 miles per hour, and in 6 hours,  $6 \times 3 = 18$  miles. Now, suppose the horse to act through machinery so constructed, that while he moves at the rate of 18 miles in 6 hours, the weight attached to and acted upon by the same machinery, moves only one mile; how great a power would the horse be required to exert to raise 2250 lb. that distance? Evidently, a power equal to  $\frac{1}{18}$  of 2250 lb. or 125 lb., a weight entirely within the power of a horse of ordinary strength to raise at the velocity proposed. This reduces the statement of a single horse power to a form easy to be comprehended; viz. it is a power competent to raise 125 lb. 18 miles in 6 hours, or 3 miles per hour.

Prob. 33. To determine the horse power required to raise a given weight any specified distance per hour.

*RULE.*—Multiply the weight reduced to pounds by the distance through which it is to be moved, and make the product a dividend. Then, having reduced 3 miles to the same denomina-

tion with the given distance, multiply it by 125, and make the product a divisor. Divide, and the quotient will express the required horse power.

Ex. 1. What amount of horse power will be required to raise 14 T. 14 cwt. 2 qr. 16 lb., 60 feet per hour?

Operation: 14 T. 14 cwt. 2 qr. 16 lb. = 33000 lb.; and  $33000 \times 60 = 1980000$ , the dividend. 3 miles = 15840 feet; and  $15840 \times 125 = 1980000$ , the divisor; then,  $1980000 \div 1980000 = 1$ , Ans.

Note.—If the time given be less than one hour, the required power must be *proportionally greater*. Had the time in the preceding sum been 20 minutes, instead of one hour, that is,  $\frac{1}{3}$  of the time given, *three times as much power would have been required*. Hence, the power required is found by multiplying 1 by the direct ratio of 125 and the given weight, and by the inverse ratio of 1 hour and the given time.

2. What amount of horse power is necessary to raise 60 tons 3 feet per minute? Ans. 12.2+ horse power.

Prob. 34. Given the horse power, the distance, and the time, to determine the weight that may be raised.

RULE.—Multiply 125 by the given horse power; the product will be the weight that power will raise 3 miles per hour. Multiply this by 3 miles reduced to the same denomination as the given distance, and divide the product by that distance, and the number obtained will determine the pounds that may be raised the given distance in one hour; then, lastly, multiply by the given time in minutes, and divide the product by 60; the quotient will be the weight required.

Ex. 1. What weight will a 5 horse power raise 120 feet in 20 minutes of time?

Solution:  $125 \times 5 = 625$ ; then, since 3 miles equals 15840 feet,  $625 \times 15840 = 9900000$ ; and  $9900000 \div 120 = 82500$ ; and  $82500 \times 20 \div 60 = 27500$  lb., Ans.

2. What weight will an 8 horse power raise 360 feet in 90 minutes of time? Ans. 29 T. 9 cwt. 1 qr. 4 lb.

## PROBLEMS IN INTEREST.

Prob. 35. Time, rate per cent., and amount given, to find the principal.

RULE.—*Divide the given amount by the amount of \$1 or 1 £. for the given rate and time.*

Ex. 1. What sum of money will amount to \$360, at 6 per cent., in 3 years and 4 months?

The amount of \$1 for 3 years and 4 months, is \$1.20 ; and  $\$360 \div \$1.20 = \$300$ , *Ans.*

2. What sum of money will, in 4 years, 6 months, at 6 per cent. per annum, amount to \$1016 ? *Ans.* \$800.

Prob. 36. Time, rate per cent., and interest given, to find the principal.

RULE.—*Divide the given interest by the interest of one dollar for the given time and rate per cent.*

Ex. 1. If a man's annual interest be \$1200, what is his capital, the rate per cent. being 6 ? *Ans.* \$20000.

2. How much money on interest, at 6 per cent., will gain \$36 in 1 year and 8 months ? *Ans.* \$360.

Prob. 37. Given the principal, interest, and time, to find the rate per cent.

RULE.—*Divide the given interest by the interest of the given principal, at one per cent. for the given time.*

Ex. 1. A man having \$4000 on interest, at the expiration of one year received \$240 interest on the same. What rate per cent. did he receive ? *Ans.* 6 per cent.

2. If \$200 be paid as interest on \$2000 for 2 years and 6 months, what is the rate per cent. ? *Ans.* 4 per cent.

Prob. 37. Given the principal, rate per cent., and interest, to find the time.

RULE.—*Divide the given interest by the interest of the principal for one year ; the quotient will be the time.*

Ex. 1. Paid \$108 interest on a note of \$600. How long had the note been on interest, the rate per cent. being 6? *Ans.* 3 years.

2. Paid \$400 interest on a note of \$800, the rate per cent. being 4. How long had the note been on interest? *Ans.* 12 years and 6 months.

## ANNUITIES.

An *annuity* is a sum of money payable annually for any given number of years, or forever.

When any sum of money, payable annually, has remained unpaid for any number of years, the amount due *is the sum of all the annuities which are in arrears, together with the interest on each annuity for the time it has remained unpaid.*

Prob. 38. Given the annuity, the rate per cent., and the time of arrears, to find the amount due.

*RULE.*—Find the amount of each annuity for the time it has remained unpaid, at the given rate per cent.; the sum of the amounts thus obtained will be the sum required.

Ex. 1. A man, who was in the receipt of an annual pension of \$200, was required to forego the payment four years in succession. What was due at the expiration of that time, the rate per cent. being 6?

The annuity of the fourth year would receive no interest, as it was not due till the year expired, and consequently amounted to only \$200. The annuity of the third year was entitled to one year's interest, and therefore amounted to \$212. The annuity of the second year was entitled to two years interest, and amounted to \$224. The annuity of the first year was entitled to three years interest, and amounted to \$236. Therefore,  $200 + 212 + 224 + 236 = \$872$ , *Ans.*

2. What is the value of an annuity of \$600, which has remained unpaid for 8 years, interest at 6 per cent. being allowed? *Ans.* \$5808.

Prob. 39. Given the annuity and rate per cent., to find its present worth for any number of years.

**RULE.**—*Divide each annuity by the amount of \$1 or 1 £. for the time before it becomes due; the quotient will be the present worth of the several annuities, and their sum will be the amount required.*

**Ex. 1.** What is the present worth of an annuity of \$600, for three years to come, 6 per cent. being allowed for present payment?

The present worth of the first year is  $\$600 \div \$1.06 = 566.037$ ; of the second year,  $\$600 \div \$1.12 = \$535.714$ ; and of the third year,  $\$600 \div \$1.18 = \$508.474$ . Then,  $\$566.037 + \$535.714 + \$508.474 = \$1610.225$ , *Ans.*

**2.** What is the present worth of an annuity of 30 £. for 5 years to come, at 4 per cent.? *Ans.* 134 £. 5 s. 5 d. +

## ANNUITIES AT COMPOUND INTEREST.

The amount of an annuity at compound interest, is obtained by *computing the compound interest of the several payments, and then finding their sum.* Operations of this nature may obviously be solved by geometrical progression, by making \$1 the first term of a geometrical series, and the amount of \$1 for one year at the given rate per cent., the ratio. The number of terms will always be the same as the number of years. (See Geometrical Progression.)

**Ex. 1.** What will an annuity of \$60 per annum amount to in 4 years, at 6 per cent.? \$1 is the first term, \$1.06 the ratio. Therefore,

$$\frac{1.06^4 - 1}{.06} \times 60 = \$262.476, + \text{Ans.}$$

We have then the following general principle. *Raise the ratio (which is always found by adding the per cent. to \$1) to a power equal to the number of years; from this, subtract 1, then divide the remainder by the ratio less 1, (that is, by the decimal part of the ratio only,) and multiply the quotient by the annuity.*

2. What is the amount of an annuity of \$400, which has remained unpaid for 20 years, at 6 per cent. compound interest? *Ans.* 14714.23.+

Or, the answers to such sums may be found by *multiplying the amount of one dollar for the given number of years and rate per cent. as found in the following table, by the annuity :*

TABLE

SHOWING THE AMOUNT OF \$1 AT 5 OR 6 PER CENT. COMPOUND INTEREST, FOR ANY NUMBER OF YEARS BETWEEN ONE AND FORTY.

| yr. | 5 per cent. | yr. | 6 per cent. | yr. | 5 per cent. | yr. | 6 per cent. |
|-----|-------------|-----|-------------|-----|-------------|-----|-------------|
| 1   | 1.000000    | 1   | 1.000000    | 21  | 35.719252   | 21  | 39.992727   |
| 2   | 2.050000    | 2   | 2.060000    | 22  | 38.505214   | 22  | 48.392290   |
| 3   | 3.152500    | 3   | 3.183600    | 23  | 41.430475   | 23  | 46.995828   |
| 4   | 4.310125    | 4   | 4.374616    | 24  | 44.501999   | 24  | 50.815577   |
| 5   | 5.525631    | 5   | 5.637093    | 25  | 47.727099   | 25  | 54.864512   |
| 6   | 6.801913    | 6   | 6.975319    | 26  | 51.113454   | 26  | 59.156383   |
| 7   | 8.142008    | 7   | 8.393838    | 27  | 54.669126   | 27  | 63.705766   |
| 8   | 9.549109    | 8   | 9.897468    | 28  | 58.402583   | 28  | 68.528112   |
| 9   | 11.026564   | 9   | 11.491316   | 29  | 62.322712   | 29  | 73.639798   |
| 10  | 12.577893   | 10  | 13.180795   | 30  | 66.438847   | 30  | 79.058186   |
| 11  | 14.206787   | 11  | 14.971643   | 31  | 70.760790   | 31  | 84.801677   |
| 12  | 15.917127   | 12  | 16.869941   | 32  | 75.298829   | 32  | 90.889778   |
| 13  | 17.712983   | 13  | 18.882138   | 33  | 80.063771   | 33  | 97.343165   |
| 14  | 19.598632   | 14  | 21.015066   | 34  | 85.066959   | 34  | 104.183755  |
| 15  | 21.578564   | 15  | 23.275970   | 35  | 90.220307   | 35  | 111.434780  |
| 16  | 23.657492   | 16  | 25.672528   | 36  | 95.836323   | 36  | 119.120867  |
| 17  | 25.840366   | 17  | 28.212880   | 37  | 101.628139  | 37  | 127.268119  |
| 18  | 28.132385   | 18  | 30.905653   | 38  | 107.709546  | 38  | 135.904206  |
| 19  | 30.539004   | 19  | 33.759992   | 39  | 114.095023  | 39  | 145.058458  |
| 20  | 33.065954   | 20  | 36.785591   | 40  | 120.799774  | 40  | 154.761966  |

3. What is the amount of an annuity of \$150, which has remained unpaid for 12 years, compound interest 6 per cent.? *Ans.* \$2530.49.+

The amount of \$1 for 12 years in the above table, is \$16.869941, and  $\$16.869941 \times 150 = \$2530.49, +$  *Ans.*

4. What is the worth of an annual salary of \$1000, which

has remained unpaid for 15 years, compound interest at 6 per cent. ? *Ans.* \$23275.97.

5. What is the value of an annuity of \$150, to continue 30 years, at 5 per cent. compound interest ? *Ans.* \$9965.827.+

## TABLE

SHOWING THE PRESENT WORTH OF AN ANNUITY OF \$1 AT 5 OR 6 PER CENT. FOR ANY NUMBER OF YEARS BETWEEN ONE AND FORTY.

| yr. | 5 per cent. | yr. | 6 per cent. | yr. | 5 per cent. | yr. | 6 per cent. |
|-----|-------------|-----|-------------|-----|-------------|-----|-------------|
| 1   | 0.952381    | 1   | 0.943396    | 21  | 12.821153   | 21  | 11.764077   |
| 2   | 1.859410    | 2   | 1.833393    | 22  | 13.163003   | 22  | 12.041582   |
| 3   | 2.723248    | 3   | 2.673012    | 23  | 13.488574   | 23  | 12.303379   |
| 4   | 3.545950    | 4   | 3.465106    | 24  | 13.798642   | 24  | 12.550358   |
| 5   | 4.329477    | 5   | 4.212364    | 25  | 14.093945   | 25  | 12.783356   |
| 6   | 5.075692    | 6   | 4.917324    | 26  | 14.375185   | 26  | 13.003166   |
| 7   | 5.786373    | 7   | 5.582381    | 27  | 14.643034   | 27  | 13.210534   |
| 8   | 6.463213    | 8   | 6.209794    | 28  | 14.898127   | 28  | 13.406164   |
| 9   | 7.107822    | 9   | 6.801692    | 29  | 15.141074   | 29  | 13.590721   |
| 10  | 7.721735    | 10  | 7.360087    | 30  | 15.372451   | 30  | 13.764831   |
| 11  | 8.306414    | 11  | 7.886875    | 31  | 15.592810   | 31  | 13.929086   |
| 12  | 8.863252    | 12  | 8.388844    | 32  | 15.802677   | 32  | 14.084043   |
| 13  | 9.393573    | 13  | 8.852683    | 33  | 16.002549   | 33  | 14.230230   |
| 14  | 9.898641    | 14  | 9.294984    | 34  | 16.192904   | 34  | 14.368141   |
| 15  | 10.379658   | 15  | 9.712249    | 35  | 16.374194   | 35  | 14.498246   |
| 16  | 10.837770   | 16  | 10.105895   | 36  | 16.546852   | 36  | 14.620987   |
| 17  | 11.274066   | 17  | 10.477260   | 37  | 16.711287   | 37  | 14.736780   |
| 18  | 11.689587   | 18  | 10.827603   | 38  | 16.867893   | 38  | 14.846019   |
| 19  | 12.085321   | 19  | 11.158116   | 39  | 17.017041   | 39  | 14.949075   |
| 20  | 12.462216   | 20  | 11.469921   | 40  | 17.159086   | 40  | 15.046297   |

To find the value of an annuity by the preceding table, multiply the value of one dollar as given in the preceding table for the given number of years and rate per cent. by the given number of dollars.

Ex. 1. What sum of money will purchase an annuity of \$400, to continue 12 years, at 6 per cent. discount ?

The value of \$1 for 12 years is \$8.388844, and  $8.388844 \times 400 = \$3355.537$ , + *Ans.*

2. What is the present value of an annuity of \$1200, to continue 16 years, at 6 per cent discount? *Ans.* 12127.074. +

3. How much must be paid for an annuity of \$75, to continue 30 years, at 6 per cent. discount? *Ans.* \$1032.352. +

## ASSESSMENT OF TAXES.

A tax is a sum of money collected from the citizens of a State, county, or town, for defraying the expenses necessarily incurred in the administration of justice, and in works of common utility.

The sum each man is required to pay, depends, for the most part, on the amount of his property. To this the poll tax forms the only exception; which is a tax required of every male inhabitant of a State, who has attained the age of twenty-one years, *independently of the property he may possess.*

In levying a tax upon any community, first take a complete list of all the property of the town on which the tax is to be laid, and also of the number of polls to be taxed. Next, determine the amount of the poll taxes by multiplying the number of polls by the tax on each. This being determined, subtract it from the whole sum to be raised, and the remainder will be the sum to be raised on the property of the town, both real and personal. Then to ascertain the percentage, divide the tax to be raised by the whole amount of taxable property, and the quotient will be the tax on one dollar. To find what tax any individual pays, multiply his inventory by this quotient, or tax per dollar.

Ex. 1. What will be A's tax, if he own real estate to the amount of \$1340, and personal property to the amount of \$874, if the town in which he lives, the inventory of which is \$32265, raise a tax of \$1129.95, there being 270 polls which are taxed 60 cents each, and for two of which he pays?

270 polls  $\times$  60 = \$162.00, amount of the poll taxes. Therefore, \$1129.95 — \$162.00 = \$967.95, the sum to be levied on the property. Hence,  $\$967.95 \div \$32265 = 3$  cents, the tax on one dollar. Then,  $\$1340 \times .03 = \$40.20$ , and  $\$874 \times .03 = \$26.22$ , and two polls at 60 cents each = \$1.20. Therefore,  $\$40.20 + \$26.22 + \$1.20 = 67.62$ , *Ans.*



After finding what the tax is per dollar, a table may be formed like the following, which will expedite the operations :

TABLE.

|                   |                    |                      |
|-------------------|--------------------|----------------------|
| Tax on \$1 is .03 | Tax on \$10 is .30 | Tax on \$100 is 3.00 |
| " " 2 " .06       | " " 20 " .60       | " " 200 " 6.00       |
| " " 3 " .09       | " " 30 " .90       | " " 300 " 9.00       |
| " " 4 " .12       | " " 40 " 1.20      | " " 400 " 12.00      |
| " " 5 " .15       | " " 50 " 1.50      | " " 500 " 15.00      |
| " " 6 " .18       | " " 60 " 1.80      | " " 600 " 18.00      |
| " " 7 " .21       | " " 70 " 2.10      | " " 700 " 21.00      |
| " " 8 " .24       | " " 80 " 2.40      | " " 800 " 24.00      |
| " " 9 " .27       | " " 90 " 2.70      | " " 900 " 27.00      |
|                   |                    | " " 1000 " 30.00     |

Now, to determine A's tax from this table,  $\$1340 + \$874 = \$2214$ .

The tax on \$2000 = \$60.00

" " 200 = 6.00

" " 10 = .30

" " 4 = .12

Two polls at 60 ct. = 1.20

\$67.62 Ans. as before.

2. From the preceding table, calculate B's tax, whose real estate is valued at \$5620, and his personal property at \$1162, and who pays 3 poll taxes, each 75 cents. Ans. \$205.71.

3. Form a table and calculate the tax of A., whose property is valued at \$1500; of B., whose property is \$2000; of C., whose property is \$1200; of D., whose property is \$3000, and of E., whose property is estimated at \$3500; supposing them to live in a town whose inventory is \$1000000, and on which a tax of \$4280 is to be levied, the number of polls being 400, and paying each 70 cents. Ans. A's tax will be \$6; B's, \$8; C's, \$4.80; D's, \$12; and E's, \$14.

4. A tax of \$3000 is to be raised on a certain town, the inventory of which is \$60000; the number of polls 250, taxed each 75 cents. What is the amount of A's tax, whose property is valued at \$2500, and who pays for two polls? Ans. \$118.68 $\frac{1}{2}$ .

